

# Contents

|  |           |
|--|-----------|
| <b>Introduction</b>  | <b>1</b>  |
| From geometry to arithmetic . . . . .                      | 1         |
| Arakelov geometry . . . . .                                | 2         |
| The delta invariant . . . . .                              | 4         |
| Statement of results . . . . .                             | 7         |
| Main ideas of the proof . . . . .                          | 12        |
| Overview . . . . .   | 14        |
| Acknowledgement . . . . .                                  | 15        |
| <b>1 Invariants</b>  | <b>16</b> |
| 1.1 Invariants of abelian varieties . . . . .              | 16        |
| 1.2 Invariants of Riemann surfaces . . . . .               | 18        |
| 1.3 Invariants of hyperelliptic Riemann surfaces . . . . . | 21        |
| <b>2 Integrals</b>   | <b>24</b> |
| 2.1 Integrals of theta functions . . . . .                 | 24        |
| 2.2 Integrals of the Arakelov–Green function . . . . .     | 25        |
| <b>3 The hyperelliptic case</b>                            | <b>29</b> |
| 3.1 Decomposition of theta functions . . . . .             | 29        |
| 3.2 Comparison of integrals . . . . .                      | 30        |
| 3.3 Explicit formulas for the delta invariant . . . . .    | 32        |
| 3.4 A generalized Rosenhain formula . . . . .              | 34        |
| <b>4 The general case</b>                                  | <b>37</b> |
| 4.1 Forms on universal families . . . . .                  | 37        |
| 4.2 Deligne pairings . . . . .                             | 40        |
| 4.3 Graphs and Terms . . . . .                             | 46        |
| 4.4 Main result . . . . .                                  | 56        |
| 4.5 Bounds for theta functions . . . . .                   | 57        |
| 4.6 The Arakelov–Green function . . . . .                  | 60        |

|          |   |           |
|----------|---|-----------|
| <b>5</b> | <b>The case of abelian varieties</b>                        | <b>63</b> |
| 5.1      | The delta invariant of abelian varieties . . . . .          | 63        |
| 5.2      | Asymptotics . . . . .                                       | 64        |
| <b>6</b> | <b>Applications</b>   | <b>68</b> |
| 6.1      | Bounds of heights and intersection numbers . . . . .        | 68        |
| 6.2      | Explicit Arakelov theory for hyperelliptic curves . . . . . | 72        |
|          | <b>Bibliography</b>   | <b>77</b> |