Essays on Financial Stability

Inaugural-Dissertation
zur Erlangung des Grades eines Doktors
der Wirtschafts- und Gesellschaftswissenschaften
durch die
Rechts- und Staatswissenschaftliche Fakultät
der Rheinischen Friedrich-Wilhelms-Universität Bonn

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Bonn 2015
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Tag der mündlichen Prüfung: 27. Februar 2015

Diese Dissertation ist elektronisch publiziert auf dem Hochschulschriftenserver der ULB Bonn: http://hss.ulb.uni-bonn.de/diss_online
Acknowledgments

Writing this thesis was a great challenge for me as well as a great opportunity. Many individuals and institutions have enabled and supported the process of this work, and I would like to thank them at this point.

I am particularly grateful to my supervisor, Martin Hellwig, for his invaluable advice and support. While giving me freedom to pursue my own research agenda, he always pointed out interesting aspects and research topics, and he generously shared his impressive knowledge of facts and his economic intuition. I have been inspired by his scientific approach to both highly theoretical topics as well as applied matters which are of high practical relevance.

I am also very grateful to Hendrik Hakenes for several inspiring discussions about this work, which were especially helpful for improving its formal rigidity. Moreover, I would like to thank him for acting as the second assessor of my thesis.

My dear friend and coauthor Stephan Luck deserves my deepest gratitude for our collaboration. Stephan, it was a great pleasure to do research with you over the last three years! I am very thankful for your persistence, for your enthusiasm and your eye for economic and political relevance, for the countless inspiring discussions and long hours of tedious technical work, for your indulgence and relentless encouragement. I am looking back on a productive and pleasant collaboration that I am looking forward to continue!

All chapters of this thesis have benefited from various discussions and comments. I would like to thank Tobias Adrian, Patrick Bolton, Florian Buck, Jean-Edouard Colliard, Christoph Engel, Falko Fecht, Christian Hellwig, Konstantin Milbradt, Sebastian Pfeil, Jean-Charles Rochet, Eva Schliephake, Isabel Schnabel, Julian Schumacher, and Ansgar Walther. Furthermore, I would like to thank my fellow graduate students as well as my colleagues at the MPI, particularly Dominik Grafenhofer, Wolfgang Kuhle, and Alexander Morell.

I would also like to thank Brian Cooper for thorough proof-reading and
stylistic advice, and I would like to thank Rafael Aigner for generously providing me with the \LaTeX{} environment for this thesis.

I conducted the research for my thesis at the Bonn Graduate School of Economics, with funding from Deutsche Forschungsgemeinschaft, and at the Max Planck Institute for Research on Collective Goods, with funding through the Max Planck Research Award of the Alexander von Humboldt Foundation and the Max Planck Society.

I would like to thank the BGSE and the MPI for offering such a great learning and research environment, and I would like to thank my fellow PhD students as well as my colleagues for the pleasant time! I am also thankful for the support by the administration of the BGSE and the MPI. Monika Stimpson deserves special thanks for helping me with various technical and non-technical matters!
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<tr>
<td>ABCP</td>
<td>Asset-Backed Commercial Papers</td>
</tr>
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<td>ABS</td>
<td>Asset-Backed Securities</td>
</tr>
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<td>ASF</td>
<td>Available Stable Funding</td>
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<td>D&amp;D</td>
<td>Diamond and Dybvig (1983)</td>
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<td>DGS</td>
<td>Deposit Guarantee Scheme</td>
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<td>DI</td>
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<td>FCIC</td>
<td>Financial Crisis Inquiry Commission</td>
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<td>FSB</td>
<td>Financial Stability Board</td>
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<td>HQLA</td>
<td>High Quality Liquid Assets</td>
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<td>LCR</td>
<td>Liquidity Coverage Ratio</td>
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<td>MMF</td>
<td>Money Market (Mutual) Fund</td>
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<td>NAV</td>
<td>Net Asset Value</td>
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<tr>
<td>NSFR</td>
<td>Net Stable Funding Ratio</td>
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<td>RMBS</td>
<td>Residential Mortgage Backed Securities</td>
</tr>
<tr>
<td>RSF</td>
<td>Required Stable Funding</td>
</tr>
<tr>
<td>SIV</td>
<td>Structured Investment Vehicle</td>
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<td>SoC</td>
<td>Suspension of Convertibility</td>
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<td>SPV</td>
<td>Special Purpose Vehicle</td>
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<td>SRM</td>
<td>Single Resolution Mechanism</td>
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<td>SSM</td>
<td>Single Supervisory Mechanism</td>
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Introduction

This thesis consists of three chapters that analyze the stability of financial institutions, particularly that of banks. All chapters are based on joint work with my friend and colleague Stephan Luck. The focus lies on self-fulfilling liquidity crises that are associated with maturity transformation conducted by financial intermediaries such as banks. The need for such maturity transformation arises because, on the one hand, investors have a demand for liquid assets and, on the other hand, investment projects require stable long-term funding. As pointed out by Diamond and Dybvig (1983), banks are institutions that can efficiently intermediate in this environment. The maturity mismatch induced by such intermediation makes banks prone to self-fulfilling liquidity crises, i.e., bank runs or roll-over freezes. We show that the government has a distinct role in ensuring the functioning of efficient maturity transformation (Chapter 1). We also show that if a single government’s power is limited, supranational agreements can help to mitigate this limitation (Chapter 2). Finally, we address the problem of regulatory arbitrage and show that safeguarding measures become ineffective if there is an opportunity to shift intermediation into a shadow banking sector (Chapter 3).

Chapter 1 is based on Luck and Schempp (2014b) and discusses the optimal provision of liquidity. It asks whether financial intermediaries can optimally provide liquidity, or whether the government has a role in creating liquidity by supplying government securities. We discuss a model in which intermediaries optimally manage liquidity with outside rather than inside liquidity: instead of holding liquid real assets that can be used at will, banks sell claims on long-term projects to investors. While increasing efficiency, liquidity management with private outside liquidity is associated with a rollover risk. This rollover risk either keeps intermediaries from providing liquidity optimally, or it makes the economy inherently fragile. In contrast to privately produced claims, government bonds are not associated with coordination problems unless there is a prospect of the government defaulting. Therefore, efficiency and stability can be enhanced if liquidity management relies on public outside liquidity.

The main results of the first chapter are derived under the assumption
that a government cannot default because it can commit future liquidity via taxation. It is thus the only institution that can credibly promise to provide liquidity in the future. However, this ability depends on the assumption that the government has access to a sufficiently large and stable tax base. In the next chapter, we assume that the tax base is endogenous and depends on the performance of the financial sector and of the whole economy.

Chapter 2 is based on Luck and Schempp (2014c) and provides a model that unifies the notion of self-fulfilling banking crises and sovereign debt crises. In this model, a bank run can be contagious by triggering a sovereign default, and vice versa. A deposit insurance scheme can eliminate the adverse equilibrium only if the government can repay its debt and credibly insure deposits, irrespective of the performance of the financial sector. Moreover, we analyze how banking crises and sovereign defaults can be contagious across countries. We give conditions under which the implementation of a banking union, including a supranational deposit insurance, prevents crises effectively and at no cost. Finally, we discuss the current proposals for a banking union in the euro area and argue that it should be extended by a supranational Deposit Guarantee Scheme.

Chapter 3 is based on Luck and Schempp (2014a) and addresses the regulation of financial intermediation. While deposit insurance is effective at excluding panic-based runs, it may also introduce moral hazard on the part of banks and make regulation necessary. We study a banking model of maturity transformation in which regulatory arbitrage induces the coexistence of regulated commercial banks and unregulated shadow banks. We derive three main results: First, the relative size of the shadow banking sector determines the stability of the financial system. If the shadow banking sector is small relative to the capacity of secondary markets for shadow banks’ assets, shadow banking is stable. In turn, if the sector grows too large, it becomes fragile: an additional equilibrium emerges that is characterized by a panic-based run in the shadow banking sector. Second, if regulated commercial banks themselves operate shadow banks, the parameter space in which a run on shadow banks may occur is reduced. However, once the threat of a crisis reappears, a crisis in the shadow banking sector spreads to the commercial banking sector. Third, in the presence of regulatory arbitrage, a safety net for banks may fail to prevent a banking crisis. Moreover, the safety net may be tested and may eventually become costly for the regulator.

All three chapters are concerned with the self-fulfilling elements of financial crises. We find that the government has a distinct role in dealing with such fragilities. During the 2007-09 financial crisis, fiscal authorities and cen-
Central banks tried to stabilize the financial system by engaging in bail-outs and by providing guarantees for distressed institutions. While such ex-post measures can be useful once a financial system is in a state of crisis, this thesis contributes to the understanding of how ex-ante measures can prevent such crises in the first place. Our models show that a fiscally strong government can ensure efficient liquidity provision by issuing government bonds. If a single country is fiscally weak, we show how a banking union that includes a supranational Deposit Guarantee Scheme may be mutually beneficial for its participants. Finally, we show how regulatory arbitrage can reintroduced panic-based runs even in the presence of deposit insurance for regulated banks, and how this poses severe restrictions on the government’s ability to stabilize the financial system.
Outside Liquidity, Rollover Risk, and Government Bonds

1.1 Introduction

Empirical evidence suggests that investors value the liquidity of government bonds (see, e.g., Longstaff, 2004; Krishnamurthy and Vissing-Jorgensen, 2012). There are various explanations for why incomplete financial markets and financial frictions give rise to a demand for liquidity, and for government securities as means to provide such liquidity. Government bonds may be valuable to investors as a simple medium of transfer across time, e.g., to enhance risk-sharing (see, e.g., Gale, 1990) or to improve investment by alleviating frictions (see, e.g., Woodford, 1990; Saint-Paul, 2005). Demand for government securities may especially arise when private liquidity provision is limited, e.g., if moral hazard and commitment problems restrict the pledgeable income of private agents. Publicly issued claims may guarantee the provision of liquidity and reduce the need to set liquid real assets aside (Holmström and Tirole, 1998, 2011). Moreover, it lies in the nature of government bonds that they mitigate the adverse selection problems typically associated with liquidity provision because they are free from private information (Gorton and Pennacchi, 1990; Gorton and Ordoñez, 2013).

This chapter provides a simple but novel explanation for why government securities are especially suited to manage liquidity needs: government securi-
ties are less prone to coordination failures than privately issued claims, i.e., less exposed to rollover risk.

In the run-up to the recent financial crisis, financial intermediaries satisfied liquidity needs by transforming long-term real investments into liquid claims instead of setting liquid real assets aside. However, when the crisis unfolded as a consequence of various shocks in the housing market, privately produced assets stopped being liquid – leaving financial markets and intermediaries in turmoil. The crisis ultimately appears as an inability of the private sector to provide liquidity efficiently to the economy.

In our model, financial intermediaries\textsuperscript{2} optimally provide liquidity not through holding liquid real assets that can be used at will (inside liquidity). Instead, they optimally rely on liquidity that investors provide in exchange for claims on future returns of long-term real investments (referred to as private outside liquidity). The key friction of our model is that at the time of initial investment, it impossible to contract with the potential providers of private outside liquidity such as wholesale funding. While the reliance on outside liquidity increases profitable long-term investment, it may be also associated with a rollover risk. We argue that this rollover risk is inherent in liquidity management with privately produced claims. We show that the rollover risk may either make intermediaries refrain from providing liquidity optimally in the first place, or it may make the economy inherently fragile. In turn, under the assumption that the government never defaults, public claims are free from such risk. Satisfying liquidity needs by selling government securities in exchange for outside liquidity (referred to as public outside liquidity) may thus enhance efficiency and stability.

We derive our results from a banking model in the tradition of Diamond and Dybvig (1983, henceforth D&D). Demand for liquid assets arises from an idiosyncratic liquidity risk on the part of consumers. Financial intermediaries provide optimal risk-sharing to consumers by offering demand-deposit contracts. However, we alter the D&D setup by assuming that banks can sell claims on their future returns to investors in the interim period in exchange for outside liquidity. Banks use the proceeds to serve early withdrawing consumers. This model feature is reminiscent of Holmström and Tirole (1998, 2011) and Bolton et al. (2011).

The model’s implications are the following: First, the presence of investors

\textsuperscript{1}See, e.g., Hellwig (2008), Brunnermeier (2009), Krishnamurthy (2010), and Caballero (2010).
\textsuperscript{2}We use the terms “bank” and “financial intermediary” interchangeably throughout the chapter.
who may buy claims on future returns generally allows a reduction of the holdings of liquid real assets in order to manage liquidity. Banks can conduct more productive, but illiquid long-term investments. Second, we find that intermediaries might not be able to manage liquidity optimally with privately produced claims. Relying on outside liquidity by investors in exchange for privately produced claims exposes an intermediary to the risk of a rollover freeze. There is strategic complementarity between investors in their decisions to purchase claims on intermediaries’ future returns. If no investor purchases claims, the intermediary will be forced to conduct costly liquidation. This in turn may make it optimal to refuse a rollover. Importantly, the rollover risk – unlike the classical bank run problem – cannot be eliminated by a classic deposit insurance or by a suspension of convertibility. This caused by the friction that outside liquidity is not contractible in the initial period. The potential rollover freeze in turn may make intermediaries either reluctant to implement the first-best, or it may make the economy inherently fragile.

As a third result, we show that in the presence of potential coordination failures between investors, the existence of public claims increases welfare. These claims allow intermediaries to implement the optimal allocation without exposing the economy to the risk of a rollover freeze. The reason is simple: under the assumption that the government never defaults, government securities are never subject to a coordination problem, i.e., there is no strategic complementarity between the investors in their decisions to purchase government bonds. In contrast to privately produced assets, the value of government securities is independent of the decision of investors to purchase the security or not. By using government bonds to manage liquidity, banks can reduce inefficient reliance on inside liquidity while avoiding rollover risk. Consequently, government borrowing may have non-Ricardian effects (see, e.g., Barro, 1974).

Finally, we discuss the assumption that the government can always repay its debt. We show that once the government’s ability to repay depends on the banking sector, a run on the banking sector may be complemented by a run on government debt if there is public supply of liquidity. In this case, the positive effects of public liquidity provision may vanish. We analyze the interplay of sovereign defaults and banking crises in more depth in Chapter 2.

We use the term “outside liquidity” in the sense of Holmström and Tirole (2011), Bolton et al. (2011), and Gourinchas and Jeanne (2012). The concept of inside and outside liquidity is to some degree reminiscent to the definition of inside and outside money (see, e.g., Lagos, 2006), but there are subtle differences. Outside money is money that is not anyone’s liability, and that is thus a net asset for the private sector. In contrast, inside money is created
within the private sector, and is thus some private agent’s liability. Similarly, inside liquidity is the liquidity that is created within a specified sector, while outside liquidity is supplied by agents or institutions outside this sector. In contrast to the definition of outside money, outside liquidity is mostly defined “from the point of view of the financial sector”.\(^3\) In Bolton et al. (2011), inside liquidity denotes the intermediary’s cash reserves, whereas the intermediary can raise outside liquidity by selling assets to long-term investors (hedge funds and pension funds). Thus, outside liquidity is the label for liquidity that investors supply to banks (and thus to consumers).

This chapter is closely related to the literature on the government’s role in providing safe assets for the purpose of liquidity management. As in the seminal paper by Holmström and Tirole (1998), we allow the economy to reduce the holdings of real assets and to issue claims on future returns in order to manage liquidity needs. In contrast to Holmström and Tirole, the limitation of private liquidity supply originates not from agency problems, but from coordination problems. In terms of our results, this chapter is close to a series of recent papers (Greenwood et al., 2012; Gourinchas and Jeanne, 2012; Gorton and Ordoñez, 2013). With Gorton and Ordoñez (2013), we share the notion that government bonds are more liquid than privately produced assets and make the economy more stable. However, their reasoning is based on the information sensitivity of assets.\(^4\) They show that liquidity provision by privately produced assets may make an economy fragile, as seemingly safe assets may become illiquid when they become information-sensitive. Government bonds in turn are less information-sensitive and thus more liquid. With the paper by Greenwood et al. (2012) we have in common that the government has a comparative advantage in bearing refinancing risk relative to the private sector, and thus public provision of liquidity is welfare-enhancing. However, their focus is on the maturity of different securities. Finally, Gourinchas and Jeanne (2012) provide a macroeconomic model with inside and outside liquidity. As in our setup, a crisis occurs when private liquidity provision is insufficient and the role of public securities for financial stability is emphasized.

The results of this chapter can also be interpreted in the light of the theory of liquidity mismatch. Brunnermeier et al. (2013) argue that maturity transformation and the associated maturity mismatch are not problematic per se. Fragility arises only if maturity transformation also induces a liquidity mismatch. While financing a 20 year government bond with demand deposits

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\(^3\)Definition in Gourinchas and Jeanne (2012); other definitions are similar.

\(^4\)See Dang et al. (2013a) and Dang et al. (2013b) on information (in)sensitivity of assets and financial crises.
1.1 Introduction

is an extreme form of maturity mismatch, it does not constitute a liquidity mismatch as long as there is a liquid market for government bonds. In our model, the government bonds on the banks’ balance sheets neither change the mechanism of maturity transformation nor the liquidity mismatch, but it substantially reduces the liquidity mismatch.

We also relate our results to recent empirical findings. In our model, liquidity benefits from government bonds have real effects, consistent with the evidence that investors value these attributes (Krishnamurthy and Vissing-Jorgensen, 2012). Moreover, public provision of liquidity reduces the fragility in our setup, which is in line with the finding that financial crises are more likely when little public debt is available (Krishnamurthy and Vissing-Jorgensen, 2013) and financial crises seem to be related to excessive private debt rather than public debt (Jordà et al., 2013; Schularick, 2014).

This chapter is also very closely related to theories of banking, in which intermediaries optimally rely less on inside liquidity and more on sales of claims on long-term investments, such as the model by Bolton et al. (2011). This model is concerned with the timing of trade in the presence of uncertainty and asymmetric information, while we focus on the coordination failures that may be associated with outside liquidity.

Finally, this chapter contributes to the literature on liquidity provision by financial intermediaries. D&D have argued that financial intermediaries can provide optimal risk-sharing to consumers and allow them to benefit from profitable long-term investments by offering demand-deposit contracts. In contrast, we argue that the ability of financial intermediaries to provide liquidity is limited. We are far from being the first to address the problems of liquidity provision by intermediaries. The banking literature has already produced various arguments. It has been argued that the ability of banks to provide risk-sharing in the presence of financial markets is very limited (Jacklin, 1987; Farhi et al., 2009). Especially when consumers are able to adjust their portfolio, liquidity provision may be harmed (von Thadden, 1998). Moreover, banks may be unable to implement the first-best through demand-deposit contracts in the presence of macroeconomic interest rate risk (Hellwig, 1994). Under aggregate risk and in the presence of moral hazard, financial intermediaries may not be able to insure firms against liquidity shocks either (Holmström and Tirole, 1998). The creation of liquidity through interbank

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5 On the optimality of intermediaries of liquidity provider, see also, e.g., Gorton and Pennacchi (1990), Calomiris and Kahn (1991), Diamond and Rajan (2001), and Kashyap et al. (2002).

6 See also Diamond (1997) and Fecht (2004).

7 See also Allen and Gale (1998) on this point.
Chapter 1 Outside Liquidity, Rollover Risk, and Government Bonds

Trade may also be limited if banks are unable to diversify the liquidity risk of their consumers (Bhattacharya and Gale, 1987).

Our argument, however, is neither based on agency problems nor on aggregate uncertainty. We argue that liquidity management with privately issued claims creates a coordination problem between those investors who could provide liquidity. A memorable insight from the seminal contributions by Bryant (1980) and D&D is that liquidity provision may be associated with the existence of run equilibria and make an economy inherently fragile.\footnote{Following the seminal contributions by Bryant and Diamond and Dybvig, a vast literature on bank runs evolved. See, e.g., the literature regarding information-based runs (Jacklin and Bhattacharya, 1988), models with positive probability of bank runs (Postlewaite and Vives, 1987; Chari and Jagannathan, 1988; Allen and Gale, 1998; Rochet and Vives, 2004; Goldstein and Pauzner, 2005), models with interbank contagion (Allen and Gale, 2000; Dasgupta, 2004; Uhlig, 2010), runs in repurchase agreements (Martin et al., 2014), and dynamic runs (He and Xiong, 2012).} Importantly, the rollover problem in our setup differs from the classical bank run problem. We show that the coordination problem cannot be eliminated by a deposit insurance nor by a suspension of convertibility. Ultimately, the rollover risk associated with optimal private liquidity provision may prevent the implementation of the optimal allocation in the first place. This chapter stands in contrast to models arguing that banks are especially suited to provide liquidity because of their fragile capital structure. Amongst others, Calomiris and Kahn (1991) and Diamond and Rajan (2001, 2005) argue that the fragile nature of bank balance sheets disciplines bank managers and thus allows overcoming commitment problems associated with liquidity provision. In contrast, we argue that the potential rollover risk may cause banks to refrain from supplying liquidity in an optimal fashion in the first place.

We proceed as follows: In Section 1.2, we introduce the general setup and derive the first-best allocation and show how it can be implemented by banks. In Section 1.3, we investigate how the first-best and its implementation change if we introduce outside liquidity. Section 1.4 shows how the rollover risk associated with privately liquidity supply influences the stability and efficiency of banks. In Section 1.5, we demonstrate why the provision of public liquidity by the government is superior to the private case. Finally, we use our model to evaluate the liquidity regulation proposed in Basel III in Section 1.6.

1.2 Intermediation with Inside Liquidity

Consider an economy that goes through a sequence of three dates, $t \in \{0, 1, 2\}$. There is a single good that can be used for consumption as well as for invest-
1.2 Intermediation with Inside Liquidity

Moreover, there are two investment technologies that we refer to as assets. The economy is populated by risk-averse consumers who face an idiosyncratic liquidity risk.

Consumers
There is a continuum of \textit{ex ante} identical consumers with mass one. Each consumer is endowed with \(e_0\) units of the good in \(t = 0\). There are two types of consumers, denoted by \(\theta_i \in \{0, 1\}\). The type determines the consumer’s intertemporal preference for consumption in periods one and two. With probability \(\pi\), consumer \(i\) is an “impatient consumer” who needs to consume in \(t = 1\), denoted by \(\theta_i = 1\). With probability \((1 - \pi)\), she is a “patient consumer” who is indifferent between consumption at both dates, denoted by \(\theta_i = 0\). Initially, consumers do not know their type; their probability of being type 1 is identical and independent. In period one, each consumer privately learns his type. This private revelation can be considered as a liquidity shock.

A consumption profile \((c_1, c_2)\) gives a consumer \(i\) a utility of

\[
U(c_1, c_2, \theta_i) = \theta_i u(c_1) + (1 - \theta_i) u(c_1 + c_2),
\]

where the “baseline” utility \(u : \mathbb{R}^+ \to \mathbb{R}\) is an increasing and strictly concave function that is twice continuously differentiable and satisfies Inada conditions, \(u'(0) = +\infty\) and \(u'(+\infty) = 0\). For each consumer, the ex-ante expected utility is given by \(EU(c_1, c_2) = \pi u(c_1) + (1 - \pi) u(c_1 + c_2)\).

Notice that the attributes “patient” and “impatient” characterize the consumer’s exogenous type which determines his preference, denoted by \(\theta_i\). In contrast, the attributes “late” and “early” will characterize the timing of consumption which is endogenous: An “early consumer” consumes in \(t = 1\), while a “late consumer” consumes in \(t = 2\).

Assets
There are two different assets (investment technologies) available in \(t = 0\): a short asset (storage technology), and a long asset (production technology). The short asset transforms one unit of the good at time \(t\) into one unit of the good at \(t + 1\), effectively storing the good. The long asset promises a higher expected return in the long run. However, this asset is considered to be illiquid as it can only be liquidated with a substantial discount in \(t = 1\).

The long asset is represented by a continuum of investment projects. An investment project is a metaphor for an entrepreneur who is endowed with a production technology but has no endowment of goods for investment. Each consumer has access to exactly one project (or equivalently is matched with
Chapter 1 Outside Liquidity, Rollover Risk, and Government Bonds

exactly one entrepreneur). Each investment project yields a stochastic return of \( R_i \) units in \( t = 2 \) for each unit invested in \( t = 0 \). The return \( R_i \) is the realization of an independently and identically distributed random variable \( \bar{R}_i \), characterized by a probability distribution \( F \). \( F \) is continuous and strictly increasing on a compact interval with minimum \( R > 0 \) and maximum \( \bar{R}_i \), with \( \text{E}[R_i] = \bar{R}_i > 1 \). We assume that the realization of an investment project’s long-term return \( R_i \) is privately revealed to the project’s financier in \( t = 1 \). As we will shortly see, the idiosyncratic risk implies that financial intermediaries dominate a financial markets solution in terms of welfare. Finally, an investment project may be physically liquidated prematurely at a rate \( \ell \in (0, 1/R) \) in \( t = 1 \), yielding a liquidation return of \( \ell R_i \) units. The liquidation return of a project thus depends on the project’s stochastic long-term return. However, the ratio of liquidation return to long-term return is constant and equal to \( \ell \).

1.2.1 First-Best Allocation

The allocation of consumption across different consumer types and different periods is denoted by \( \{c_1(\theta), c_2(\theta)\}_{\theta \in \{0, 1\}} \). The unconstrained optimum results from the social planner’s first-best problem, which is given by

\[
\max_{\{c_1(\theta), c_2(\theta)\}_{\theta \in \{0, 1\}}} \pi u(c_1(1)) + (1 - \pi) u(c_1(0) + c_2(0))
\]

subject to

\[
\pi \left( c_1(1) + \frac{c_2(1)}{R} \right) + (1 - \pi) \left( c_1(0) + \frac{c_2(0)}{R} \right) \leq e_0.
\]

Equation (1.3) is the feasibility condition, resulting from the initial investment constraint in \( t = 0 \) and the two budget constraints in period one and two.

In the first-best, it holds that

\[
c_2(1) = c_1(0) = 0.
\]

The late consumption levels of patient and the early consumption level of impatient consumers are then given by the following first-order condition and budget constraint:

\[
u'(c_1(1)) = R u'(c_2(0)),
\]

\[
\pi c_1(1) + (1 - \pi) \frac{c_2(0)}{R} = e_0.
\]
The optimal allocation is thus characterized by the trade-off between insurance against liquidity risk (investment in storage) and productive investment (investment in the long assets).

1.2.2 DIAMOND & DYBVIG (1983)

The model described above resembles the essential features of the framework of the D&D model. Therefore, we briefly review the key results of the seminal D&D model. The first important result is that a competitive financial market generally fails to implement the first-best allocation. In contrast, a competitive banking sector or a representative bank can implement the first-best. It is assumed that the law of large numbers applies on the bank level. That is, there is neither uncertainty on the fraction of consumers being impatient, $\pi$, nor on the return of the portfolio of long assets, $R$.

A bank that aims at maximizing consumers’ expected utility thus needs to maximize (1.2) subject to the feasibility constraint, and because the type of consumers is private information, the constrained efficient program contains two additional restrictions. The allocation of consumption must be such that no consumer has an incentive to misreport his type in the interim period:

$$u(c_1(1)) \geq u(c_1(0)), \quad (1.7)$$
$$u(c_1(0) + c_2(0)) \geq u(c_1(1) + c_2(1)). \quad (1.8)$$

Constraint (1.7) ensures that an impatient consumer has no incentive to misreport, while (1.8) ensures that a patient consumer does not want to misreport. Adding constraints (1.7) and (1.8) to the first-best problem, however, does not change the solution because the constraints are not binding in the first-best. This implies that the first-best is in fact implementable given the friction of unobservable types. The second-best thus coincides with the first-best.

The proposed mechanism, a bank representing a contestable banking sector, proceeds as follows (see also Figure 1.1): In $t = 0$, the endowment of all consumers is collected. In exchange the bank offers a demand-deposit contract that allows a consumer to withdraw $c_1^D$ units in $t = 1$ and $c_2^D$ units in $t = 2$. The bank chooses $c_1^{DD}$ and $c_2^{DD}$ such that $u'(c_1^{DD}) = Ru'(c_2^{DD})$ which is the FOC for the first-best allocation, see Equation (1.5). $R > 1$ and concavity of $u$ imply that $c_1^{DD} \leq c_2^{DD}$, and thus Equations (1.7) and (1.8) are satisfied and it is incentive-compatible for patient consumers to withdraw only in $t = 2$. The bank invests $e_0 - I = \pi c_1^{DD}$ in the storage technology and the remaining funds $I = e_0 - \pi c_1^{DD}$ in the long asset, which implies that Equation (1.6) holds.
Chapter 1 Outside Liquidity, Rollover Risk, and Government Bonds

Figure 1.1: Diamond and Dybvig (1983). The graph illustrates investment and the flow of goods (solid arrows). Initially, consumers deposit their endowment at the bank. Banks invest the endowment in the long and in the short asset. In periods one and two, early and late consumers are served with the returns of the short and the long asset, respectively.

The representative bank is thus able to implement the first-best allocation.\(^9\)

The second important result is that there is also a second type of equilibrium in \(t = 1\). If all patient consumers desire withdrawing at once, the bank will be left with assets of \(\pi c_1^{DD} + \ell(1 - \pi c_1^{DD}) < 1\) which is typically strictly less than its total liabilities in \(t = 1\), which amount to \(c_1^{DD}\).\(^{10}\) The bank will therefore be insolvent in \(t = 1\) and no funds for patient consumers will be left over in \(t = 2\). It is thus optimal for all patient consumers to withdraw and a bank run may constitute an equilibrium in the interim period. In fact, there are two subgame-perfect Nash-Equilibria in \(t = 0\), one in which the bank is established, and a second one in which consumers refuse to deposit funds in the banks as they expect a bank run in \(t = 1\).

Finally, the third result of the D&D model is that the adverse run equilibrium can be eliminated by two policy measures: either the banks should commit to suspending convertibility after paying out an overall amount of \(\pi c_1^{DD}\), or the government should provide a deposit insurance which guarantees \(c_1^{DD}\) units for each consumer in \(t = 1\), irrespective of the banks being solvent or not. In both cases, the adverse equilibrium can be eliminated at

\(^9\)Note that the optimality of the banking solution relies on a no-trading restriction of consumers in \(t = 1\); see Jacklin (1987) and more recently Farhi et al. (2009).

\(^{10}\)The run equilibrium exists whenever \(c_1^{DD} \geq 1\). This condition is typically satisfied through the assumption that the coefficient of relative risk-aversion is larger than one, i.e., \(-cu''(c)/u'(c) > 1\) for every \(c\).
1.3 Intermediation with Outside Liquidity

We now introduce a new type of agents whom we refer to as “investors”. Investors can provide banks or consumers with liquidity in the interim period which we refer to as “outside liquidity”. In the following, we analyze how the optimal allocation is altered by allowing for interim outside liquidity, and how the new first-best allocation may be implemented by a banking sector.

Assume that there is a continuum of investors with mass \( \alpha > 1 \). Investors have no endowment initially, but with a probability \( \frac{1}{\alpha} \) an investor \( j \) has an endowment of \( e_{1,j} = e_1 \) in period one. Otherwise, her endowment is zero. Therefore, the mass of investors that has a positive endowment is equal to one, and the overall endowment of all investors is \( e_1 = \int e_{1,j} d\mu \). We assume that \( e_1 > \pi R e_0 \). As we will see, this condition assures that the supply of outside liquidity is never limited by a binding resource constraint.

The key friction of private outside liquidity is the following: We assume that it is the investor’s private information whether she has a positive endowment in period one. Thus, investors cannot write enforceable contracts in \( t = 0 \), which are contingent on whether they have a positive endowment in \( t = 1 \). Because investors cannot contract in \( t = 0 \), we will only consider their behavior form period \( t = 1 \) onwards. Furthermore, we have to consider only those investors who have a positive endowment.

We assume that investors have no market power, and that they are indifferent between consuming in periods one and two. Their utility is given by \( v(\hat{c}_1 + \hat{c}_2) \), where \( \hat{c}_t \) is her consumption in period \( t \) and \( v : \mathbb{R}^+ \rightarrow \mathbb{R} \) is a strictly increasing function. Consequently, they are willing to invest their complete endowment \( e_1 \) as long as the gross return in \( t = 2 \) is at least \( e_1 \).

1.3.1 First-best with Outside Liquidity

We now derive the new first-best allocation, given that outside liquidity is available in the interim period. The social planner’s objective function, specified by the maximization problem (1.2), remains unchanged. The objective is

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11 Observe that for suspension of convertibility to be an effective measure there must not be aggregate uncertainty about the actual fraction of consumers who withdraw early. Moreover, suspension of convertibility is also ineffective if withdrawing depositors are paid out by new depositors, see the extension of the D&D setup to an overlapping generation setting by Qi (1994).
maximizing the consumers’ welfare, whereas investors’ utility does not enter our welfare measure. However, we assume that even the social planner cannot transfer funds from investors to consumers without restrictions. Because investors’ welfare does not directly enter into the objective function, we require that investors must be willing to participate.\footnote{In our setup, a comparison of consumers’ and investors’ utility does not appear meaningful. We are neither interested in the allocation of risk, nor in redistribution of wealth between the two groups of agents. We interpret the investor’s participation constraint rather as a resource constraint that as a friction. It thus appears adequate to refer to the optimum as the “first-best”.} The aggregate transfer from investors to consumers in period one is denoted by \(d_1 \leq e_1\), and \(d_2\) denotes the reverse transfer in period two. Investors’ participation constraint is given by \(d_2 \geq d_1\). It is straightforward that this constraint will be binding in the optimum, i.e., \(d_1 = d_2\) will hold in the following.

The first-best program with outside liquidity is slightly different from the one in the previous section. We now explicitly consider the budget constraints in each period. The variable \(I \in [0, e_0]\) again denotes the investment in the long asset, and an amount \(e_0 - I\) is invested in storage. Let \(d\) denote the amount of interim liquidity (i.e., the amount of liquidity that is transferred between investors and consumers), where \(d = d_1 = d_2 \leq e_1\).

The budget constraints for the two periods are given by

\[
e_0 - I + d \geq \pi c_1(1) + (1 - \pi)c_1(0), \quad (1.9)
\]
\[
RI \geq \pi c_2(1) + (1 - \pi)c_2(0) + d. \quad (1.10)
\]

Constraint (1.9) ensures that, in \(t = 1\), the payments to consumers do not exceed the sum of inside liquidity (storage) and interim outside liquidity. Constraint (1.10) ensures that, in \(t = 2\), the sum of payments to consumers and the repayment of interim outside liquidity does not exceed the return from investment in the long asset.

**Proposition 1.1 (First-best).** In the presence of outside liquidity, the consumers’ first-best consumption allocation is given by \(c_1(1) = c_2(0) = Re_0\), and \(c_1(0) = c_2(1) = 0\). It is attained by choosing \(I = e_0\) and \(d = \pi Re_0\).

The fundamental insight of Proposition 1.1 is that the D&D allocation, in which consumption levels in both periods are strictly less than \(Re_0\), can strictly be improved upon.\footnote{This result is reminiscent of the finding by Qi (1994), who shows that storage may be redundant in a overlapping-generation version of the D&D model.} The social planner can make full use of the productive long asset because the supply of liquidity in the interim period

\[\text{Proposition 1.1 (First-best).} \quad \text{In the presence of outside liquidity, the consumers’ first-best consumption allocation is given by } c_1(1) = c_2(0) = Re_0, \text{ and } c_1(0) = c_2(1) = 0. \text{ It is attained by choosing } I = e_0 \text{ and } d = \pi Re_0. \]
1.3 Intermediation with Outside Liquidity

removes the need to invest in storage. In the model with outside liquidity, the trade-off between liquidity insurance (provided through the short-asset, i.e., inside liquidity) and return (long-asset) can be solved such that consumption is perfectly smoothed by making full use of the productive technology.

Observe that the first-best allocation is not unique if the endowment of investors strictly exceeds the amount that is given to impatient consumers in the interim period, $e_1 > \pi R e_0$. Because early and late consumption are perfect substitutes for patient consumers, impatient consumers could receive some positive payment in $t = 1$, as long as their total amount of consumption remains unchanged. Without loss of generality, we focus on the solution presented in Proposition 1.1 in the following, i.e., impatient consumers only consume late.$^{14}$

1.3.2 Efficient Banking

We now show that the first-best allocation in the model with outside liquidity can be implemented by an institution that is reminiscent of a financial intermediary that signs demand-deposit contracts with consumers in period zero. In contrast to the situation without outside liquidity, a bank now only invests in the long asset and raises liquid funds in the interim period. It raises funds by issuing claims and selling them to investors. We will refer to those claims as debt.$^{15}$

As in the D&D setup, one may think of the banking sector as a contestable market. The assumption of free entry and the resulting perfect competition imply that financial intermediaries implies contracts that maximize the expected utility of consumers.$^{16}$ Again, the law of large numbers is assumed to apply on the bank level, resulting in a gross return of $R$ on the long asset with certainty, and a fraction of early impatient consumers of exactly $\pi$.

Therefore, banks can implement the first-best in the following way (see also Figure 1.2): banks collect the total endowment $e_0$ of all consumers as deposits in period zero against the promise that consumers can withdraw $Re_0$ units at any time. In order to serve their obligations, banks invest all of the economy’s $t = 0$ endowment in the long assets, transforming them in $Re_0$ units in $t = 2$. In the interim period, banks sell claims $d$ on their portfolio of long assets to

$^{14}$Notice that this allocation can be attained by choosing any $d \in [\pi R e_0, e_1]$.

$^{15}$Notice that the bank could likewise issue equity claims. As we frame the problem as one of rollover, however, we refer to the claims as debt claims without giving a specific microeconomic reasoning why debt is preferred over equity.

$^{16}$Alternatively, one may assume that the banking sector is a mechanism or a coalition of consumers that maximizes the consumers’ expected utility.
investors. Because banks are assumed to be able to diversify the liquidity and the return risk, there is no adverse selection in the market for claims in the interim period. Therefore, the investors’ participation constraint implies that banks can sell their claims at par. Banks will sell claims with a total value of $\pi Re_0$, and only impatient consumers withdraw early. The issuance of claims is equivalent to a rollover of debt, as the liability towards depositors is replaced by liabilities towards investors.

\[ t = 0 \quad t = 1 \quad t = 2 \]

**Figure 1.2:** Private Outside Liquidity. The graph illustrates investment and the flow of goods (solid arrows) and claims (dashed arrows). Claims associated with demand-deposit contracts are not depicted. In $t = 0$, consumers deposit their endowment $e_0$ at the banks. Banks invest the endowment in the long asset, transforming $e_0$ units of the good into $Re_0$ units. In the interim period, early consumers are served by selling claims $d$ to investors. In $t = 2$, banks redeem the claims of investors and repay investors and late consumers using the returns of the long asset.

**Proposition 1.2** (Implementation of the first-best). The first-best allocation $c_1(1) = c_2(0) = Re_0$ can be implemented in a demand deposit economy. Banks invest only in the long asset. Banks serve withdrawing consumers in the interim period by issuing claims on future returns and selling these to investors in exchange for outside liquidity.

The implementation of the first-best allocation thus involves privately produced assets. Instead of investing in storage in $t = 0$, financial intermediaries issue claims on their future returns in the interim period. The proceeds from selling these to investors are used to serve withdrawing consumers. This al-
1.4 Private Outside Liquidity and Rollover Risk

In this section, we show that a bank may face a rollover freeze if it relies on raising liquidity by issuing claims in the interim period. It turns out that the efficient private provision of liquidity is inherently fragile. Therefore, banks might refrain from relying on outside liquidity. Instead, they might rely on the inefficient storage technology and offer the D&D contract.

1.4.1 Rollover Freeze

Consider the subgame starting in the interim period, given that banks have invested the complete endowment in the long asset. In this subgame, consumers are endowed with a demand-deposit contract promising $Re_0$ units in either period, and banks issue claims on their future returns in order to serve withdrawing consumers. Consumers have the choice to withdraw early or to wait, and investors have the choice whether to buy claims on bank assets. In the previous section, we saw that there exists an efficient equilibrium of this subgame in which consumers do not run on banks and investors roll over the banks’ debt. However, there are strategic complementarities between agents, giving rise to multiple equilibria. As in the D&D model, there is a strategic complementarity between consumers whether or not to withdraw early. In the model with outside liquidity, an additional strategic complementarity arises...
between investors concerning their decision whether to buy claims on bank assets and thus to roll over the banks’ debt. Furthermore, there is also strategic complementarity across these two groups of agents. In the following, however, we will only focus on the strategic complementarity between investors.

In order to understand the rollover freeze, consider a situation in which no investor is willing to purchase bank claims. Let us first assume that only impatient consumers withdraw early. In this case, banks will need to liquidate a positive fraction \( z = \min[\pi/\ell, 1] \) of their long assets at the inefficient rate \( \ell R \) in order to serve impatient consumers with an amount of \( \pi Re_0 \) units. This liquidation implies that the bank will only be left with \( (1 - z)Re_0 \) in \( t = 2 \). Therefore, the bank will not have sufficient funds at hand in order to serve its patient customers or any investors in \( t = 2 \). Therefore, an individual investor will not provide any liquidity in \( t = 1 \), as banks will be insolvent in \( t = 2 \). This implies that even if consumers behave diligently and do not run on the bank, a rollover freeze always constitutes an equilibrium.

This consideration also leads to the insight that the standard measures to prevent inefficient liquidation and thus financial crises, such as deposit insurance (DI) or suspension of convertibility (SoC), become ineffective. The reason is that these policies are only targeted at breaking the strategic complementarity between depositors – they are concerned with the demand for liquidity, but not with its supply. The DI may keep patient consumers from running on banks, but a bank run is not the only way a bank can become illiquid and insolvent once a bank relies on outside liquidity. Banks may in fact experience a rollover freeze as the deposit insurance does not alter the strategic complementarity between investors. Moreover, SoC is also ineffective. By suspending convertibility, banks can limit the amount they pay out to early consumers, which induces stability in the D&D model because it eliminates the need for liquidation. However, if banks rely on outside liquidity, this measure does not prevent liquidation in case of a rollover freeze, inducing consumers to run.

**Lemma 1.1.** *In the \( t = 1 \) subgame, a rollover freeze by investors constitutes a Nash equilibrium. Moreover, a rollover freeze may occur independently of whether there is a bank run or not, and irrespectively of the existence of a credible deposit insurance or of banks committing to suspend convertibility.*

In the following, we show that the fact that there may be a rollover freeze in \( t = 1 \) makes banks either refrain from providing the efficient level of liquidity or it will expose the economy to the rollover risk. In the latter case, the economy will be fragile despite DI or SoC.
1.4 Private Outside Liquidity and Rollover Risk

1.4.2 Inefficient Liquidity Provision

Because the subgame of the interim period has an efficient as well as an adverse equilibrium, the whole game (starting in period zero) has at least one additional, inefficient subgame-perfect Nash equilibrium. While there might be a continuum of equilibria, we are interested in the generic case where investors coordinate on a rollover freeze. In a subgame-perfect Nash equilibrium, consumers and banks anticipate not being able to raise any outside liquidity in the interim period. Given that outside liquidity is not available in the interim period, banks have to rely on storage again. The constraint efficient allocation is given by the Diamond-Dybvig allocation described in Section 1.2.2.

Proposition 1.3. The model has a subgame-perfect Nash equilibrium in which investors do not roll over bank debt, consumers do not run on banks, and banks implement the Diamond-Dybvig consumption allocation, given by $c_1(1) = c_1^{DD}$ and $c_2(0) = c_2^{DD}$, and $c_1(0) = c_2(1) = 0$.

We have seen that if banks rely on raising outside liquidity by issuing claims on their future return, they are exposed to the risk of investors coordinating on a rollover freeze. The most efficient allocation entails full exposure to the rollover risk resulting from the coordination problem. If banks fear a rollover freeze, they might completely shy away from relying on outside liquidity, rather implementing the less efficient D&D allocation.

The reasoning in Proposition 1.3 is in fact very similar to the argument in the D&D model that if a bank run was expected in $t = 1$, consumers would not be willing to deposit their endowment in the bank in $t = 0$. However, it is important to notice that the adverse equilibrium cannot be eliminated by the standard measures (DI or SoC) in our model. This is due to the key friction of non-contractible private outside liquidity. One may assume that the government offers a credible DI or banks may commit to SoC. In our setup, this will not eliminate the fragility associated with the efficient provision of liquidity. In fact, if there is no credible DI, a third equilibrium may exist in which investors would not roll over bank debt and investors would run on the bank, which is why no bank is founded in the first place. In turn, if there is a credible DI, this equilibrium does not exist. However, the DI is tested in the equilibrium of Proposition 1.3 and may be costly for the institution providing it.

Finally, Proposition 1.3 can be seen as an argument for why liquidity provision by banks may be limited in general. We argue that efficient liquidity provision rests on reliance on outside liquidity. However, privately produced
assets may not be able to ensure the provision of outside liquidity. Due to the rollover risk associated with privately produced assets, financial intermediaries may thus not be able to implement the optimal allocation. This line of argument stands in contrast to models arguing that banks are especially suited to provide banking services because of their fragile capital structure (Calomiris and Kahn, 1991; Diamond and Rajan, 2001). In our setup, the fragility in the interim period can cause banks to refrain from supplying liquidity in an optimal fashion.

1.4.3 Fragility

Until this point, we have tied our hands by assuming that investors cannot coordinate their behavior on something that is not observed or not initially contractible. Formally, this means that investors cannot play a strategy by which they condition their action on a public signal that is only revealed in the interim period. This implies that a rollover freeze will never occur in a subgame-perfect Nash equilibrium in pure strategies. Either banks successfully rely on outside liquidity because they know that a rollover will be successful, or they anticipate a rollover freeze and rely on the storage technology. In equilibrium, the rollover “risk” is degenerate, as it either occurs with a probability of zero, or it occurs with a probability of one, but has no effect.17

We now want to consider a setup where investors can coordinate on a rollover freeze. The notion of coordination problems in the tradition of the D&D model is that depositors decide in the interim period whether to withdraw, thus coordinating on whether to run on the bank only after the investment decision has been made. Formally, the concept of subgame perfection requires agents to choose a strategy in period zero. Therefore, uncertainty about the action in $t = 1$ can only prevail if there exists a public signal upon which agents can condition their action. A popular illustration of such a coordination device is the concept of sunspots.18

We adopt this notion and assume that with some exogenous probability $p \in (0, 1)$ a sunspot occurs, and investors play a strategy that prescribes not

17 There exists no equilibrium in which investors play mixed strategies and banks rely on rollover. While this might seem strange, it is worth mentioning that investors play a weakly dominated strategy in the “rollover equilibrium”. As soon as we introduce marginal net profits for investors, rollover stops being weakly dominated and an equilibrium in mixed strategies arises.

to roll over the banks’ liabilities in case of this sunspot.\textsuperscript{19} We restrict our attention to the two extremes where the probability of a rollover freeze is either close to one or close to zero.

**Proposition 1.4.** As the probability of a rollover freeze converges to one, i.e., \( p \rightarrow 1 \), the optimal investment converges towards the Diamond-Dybvig case, \( I(p) \rightarrow I^{DD} \). Furthermore, there exists a threshold \( p_{\ell} \in (0, 1) \) such that for \( p \leq p_{\ell} \), it is optimal fully to rely on private outside liquidity storage, \( I^{*}(p) = e_{0} \).

For the proof of Proposition 1.4, see the Appendix. The result, however, is very intuitive as it rests on the insight of the following trade-off: On the one hand, efficiency can be attained by choosing high investment in the illiquid but profitable long-term technology. Because banks thereby rely on outside liquidity, this is associated with a high rollover risk. On the other hand, stability can be attained if banks are not exposed to rollover risk. To this end, banks make use of the storage technology and thus rely on inside liquidity. In other words, the trade-off is between strong maturity mismatch and narrow banking.

If the sunspot probability \( p \) is sufficiently high, banks will implement the D&D allocation. Banks and consumers know that each unit of early consumption that is not covered by investment in the storage technology has to be raised by liquidating long assets in case of a rollover freeze. Therefore, banks will finance every unit of early consumption by using inside liquidity and the optimal allocation under this constraint is the D&D allocation. In contrast, if the probability of a rollover freeze is sufficiently small, banks choose full exposure to rollover risk by only investing in the long asset, and implement a consumption level of \( R_{e0} \) for both consumer types. This implies that banks have to engage in substantial liquidation in case of a rollover freeze, but given that this risk is very low, they are willing to accept this risk.

It is worth noticing that even if a rollover freeze occurs with a positive probability, it may still be optimal that banks fully rely on outside liquidity. In this case, the economy is inherently fragile and a financial crisis may unfold in equilibrium if investors coordinate on a rollover freeze.

\textsuperscript{19} We do not model the underlying reason for the occurrence of these sunspots, and if we did, their occurrence would probably depend upon the banks’ behavior.
1.5 PUBLIC OUTSIDE LIQUIDITY

In Section 1.3, we showed that optimal liquidity management does not rely on inside liquidity, but rather on outside liquidity. Building on this, Section 1.4 revealed that the efficient allocation can be implemented by banks issuing private claims and relying on the rollover of debt. However, outside liquidity in exchange for privately produced assets is associated with a rollover risk. The anticipation of a rollover freeze can lead to inefficient investment choices ex-ante. We now analyze how this friction could be overcome. In particular, we ask whether the government can mitigate the problem by providing liquidity.

In general, a government has the ability – unlike private agents – to commit future income via taxation or money creation. This makes claims against a public authority inherently safer than claims produced by the private sector. In the first part of this section, we therefore assume that the government never defaults. In this case, we show that the government can increase welfare by issuing a public claim that can be used by banks to manage liquidity. In the second part of this section, we relax the assumption that the government cannot default and show that the benefits from public provision of liquidity may vanish.

1.5.1 LIQUIDITY MANAGEMENT AND GOVERNMENT BONDS

Let us assume that the government never defaults. Consider the following mechanism in which the government provides liquidity by issuing a government bond (Figure 1.3): In $t = 0$, consumers deposit their endowment $e_0$ at a bank in exchange for a demand-deposit contract allowing the consumer to withdraw $Re_0$ in either period. Banks receive government bonds that promise a payment of $b$ units by the government in $t = 2$. In exchange for the $b$ government bonds, banks write a debt contract with the government, promising to pay $d$ units to the government in $t = 2$. Banks and government will lend and borrow such that $d = b \geq \pi Re_0$. Effectively, banks are expanding their balance sheets by an amount of $b$.

In $t = 1$, banks sell $\pi Re_0$ units of government bonds to the investors and use the resulting liquidity to serve withdrawing consumers. In $t = 2$, the government has due gross liabilities of $b$ units, necessary to redeem the government bonds. An amount of $\pi Re_0$ is paid to investors. The difference of $b - \pi Re_0$ units is a gross liability towards banks, resulting from the government bonds they did not sell to investors in $t = 1$. However, the banks also owe $d = b$ units to the government. Therefore, they have a net liability of $\pi Re_0$ units towards the government. The banks have an overall return of $Re_0$ from the
long assets which is used to pay out \((1 - \pi)Re_0\) units to the patient consumers and \(\pi Re_0\) to the government.

![Figure 1.3: Public Outside Liquidity](image)

**Figure 1.3:** Public Outside Liquidity. The graph illustrates investment and the flow of goods and claims. In this graph, we assume that \(b = d = \pi e_0 R\). The dotted arrow denotes the investment in assets and thus the “transfer” of goods between periods. The solid arrows denote the flow of goods. The dashed arrows denote the flow of net claims. For simplicity, the claims of consumers towards banks (resulting from the demand-deposit contracts) are left out.

**Proposition 1.5.** If the government provides government bonds in \(t = 0\), the banks are able to implement the first-best consumption allocation, given by \(c_1(1) = c_2(0) = Re_0\) and \(c_1(0) = c_2(1) = 0\). Furthermore, the rollover risk is eliminated and the economy has a unique equilibrium.

By expanding their balance sheet, the banks implement the first-best allocation while the rollover risk is completely eliminated. Even if all other investors refused to buy government bonds, this would not influence the incentives of an individual investor. Public outside liquidity eliminates the coordination problem concerning the supply of liquidity by investors. Note though that public liquidity provision as described does not address the coordination problem between consumers concerning their withdrawal decisions. However, given that there is public outside liquidity, a bank run equilibrium can be eliminated by the standard measures, i.e., by introducing a deposit insurance or allowing banks to suspend convertibility. This is important as we saw in the previous section that these measures are ineffective as long as
the rollover problem is not addressed, but in this context they are effective at eliminating the coordination problem.

The central reason for the stability is that by assumption the government’s solvency, unlike that of a bank, does not depend on the behavior of investors. This eliminates any strategic considerations of investors when deciding to purchase government bonds in the interim period. Therefore, multiplicity of equilibria vanishes once government securities are used for liquidity management.\footnote{This is reminiscent of how multiplicity of equilibria is eliminated in the Kiyotaki and Moore (1997) model when government bonds are introduced (see p. 515 in Tirole, 2010).}

Observe that there are alternative implementations of the first-best allocation to the one shown in Figure 1.3. There are two obvious alternatives. First, the government could insure all current and future bank liabilities ex-ante. Second, the government could provide liquidity itself in the interim period. Both mechanisms are equivalent in terms of the results in our setup, as they also eliminate the fragility and thereby enhance efficiency. However, we argue that both alternatives may not be equally desirable as they may be more problematic in a richer setup in which other issues such as agency problems may arise. Insuring bank liabilities may give rise to certain risks on behalf of the bank (e.g., risk-shifting) and creditors (e.g., weak disciplining effects). Moreover, if the government actively manages liquidity by lending directly to banks when they need funds, i.e., in a crisis, this may lead to excessive maturity mismatch as in, e.g., Farhi and Tirole (2012). We discuss these two issues in more depth in Section 6.

1.5.2 **Government Solvency**

So far, we have made the extreme assumption that the government is always able to repay its debt, irrespectively of what investors do and of whether there is a banking crisis. This assumption gives government bonds the important characteristic of being immune against rollover risk. We now relax this assumption in two different ways. First, we allow the government to default with some exogenous probability. We show that, in this case, public liquidity provision may still be optimal. Second, we assume that the government’s ability to repay debt is endogenous and depends on the performance of the banking sector. In this case, the benefits from public liquidity provision vanish.

Assume first the government defaults with some positive probability which is given exogenously. This is not necessarily detrimental to efficiency and
stability. Under the condition that investors are risk-neutral, and that the government’s solvency is only revealed after $t = 1$, the optimal allocation could still be implemented. Under these conditions, the value of government bonds is still independent of the behavior of the investors. Thus, even if there is an exogenous default probability, there is no risk of a coordination failure between investors. Government bonds would be traded at the fair price (under par), and the government debt must be chosen such that its expected repayment equals the banks’ liabilities towards the government. If we require the debt contracts between the banks and the government to be budget-balanced in expectation, the implementation could be as follows: Assume that the government defaults with probability $\rho$ and repays nothing in this case. The government still holds a claim of $d = \pi Re_0$ against the banks, whereas the banks hold claims with a face value of $b = d/\rho = \pi Re_0/\rho$ against banks. When selling these claims to investors, the fair value is given by $d = \pi Re_0$.

Let us now relax the exogeneity assumption and go to the other extreme. Assume that the government can only repay its debt if banks are solvent and thus fully serve their liabilities towards the government $d$. We thus relax the assumption that the government has access to exogenous funds, e.g., via taxation. In this setup, all features of the setup without government bonds reappear. The equilibrium of the $t = 1$ subgame still exists in which investors roll over debt, and one equilibrium where they do not. The whole game thus still has a subgame-perfect equilibrium that implements the first-best. However, this equilibrium is not unique – there also exists an equilibrium of the $t = 1$ subgame where a rollover freeze is accompanied by a government default. This induces banks to refrain from implementing the optimal allocation and the D&D allocation is implemented instead. In this case, the public provision of liquidity cannot help to overcome the coordination problem. The fragility of an economy in which both the solvency of banks and that of the government are endogenous and interdependent is discussed in more detail in Chapter 2.

1.6 Liquidity Regulation in Basel III

Our model shows that the government should use its unique ability to ensure efficient liquidity management by issuing government securities that are held by financial intermediaries. This becomes necessary because the efficient liquidity management relies on the provision of non-contractable liquidity by private agents. By issuing government bonds, the government can prevent private agents from coordinating on an equilibrium in which liquidity supply...
breaks down. This is a simple way to stabilize the financial sector and, at the same time, to circumvent the undesired consequences that might arise if the regulator directly insured bank liabilities or provided emergency liquidity in case of a financial crisis. In our model, financial intermediaries voluntarily hold government bonds to manage liquidity. However, in a richer model, banks may prefer to hold privately produced assets if these assets promise a higher return than government securities. In this case, a regulator might optimally force banks to hold government securities in order to enhance stable liquidity provision.

In the following, we relate our findings to the regulatory treatment of liquidity risk in the context of prudential supervision. Our model can shed light on the economic consequences of some recently proposed regulation. The Third Basel Accord (Basel III) introduces a new assessment and regulation of liquidity risk by defining two minimum standards of funding liquidity (see Basel Committee, 2010). The two central measures are the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR). The LCR requirement aims to ensure that a bank can withstand a “significantly severe liquidity stress scenario” with a horizon of 30 days. It requires a bank to have a sufficient stock of liquid assets in order to cover its liquidity needs during the next month. The objective of the NSFR requirement is to ensure stable funding over a one-year horizon. It requires a bank to have a sufficient amount of equity, long-term debt, and other “stable” funding to finance its stock of illiquid assets during the next year.

We acknowledge that both measures can in principle be useful tools to reduce the fragility arising from maturity transformation. Both the LCR and NSFR address the fundamental problem of maturity mismatch, resulting from short-term liabilities and long-term illiquid assets. In the light of our model, however, the regulatory details are not strict enough, as the inherent fragility of privately produced liquidity is not adequately addressed. Banks face two types of self-fulfilling liquidity problems: On the asset side, seemingly liquid, privately produced assets might turn illiquid. On the liability side, seemingly stable wholesale and deposit funding might evaporate in a similar fashion. In particular, the ability to borrow against privately produced assets is limited. As our model showed, government bonds play a unique role as they can eliminate an equilibrium in which private agents coordinate on not supplying liquidity. We argue that the LCR and NSFR underestimate the risk of such adverse equilibria and do not sufficiently distinguish between

\[\text{Chapter 2 provides a more detailed explanation of why private information and limited arbitrage capital can cause liquidity problems in markets for privately produced assets.}\]
1.6 Liquidity Regulation in Basel III

private and public assets. After studying the two measures separately, we will illustrate their similarity and their common problems.

1.6.1 Liquidity Coverage Ratio (LCR)

Basel III requires the LCR, defined as the ratio of High Quality Liquid Assets (HQLA) and the (hypothetical) total net cash outflows over the next 30 calendar days, to be above 1.\textsuperscript{22} Thus, the LCR sets a lower bound for the stock of liquid assets, conditional on a bank’s (expected) cash flows. “Total net cash outflow” is defined as the maximum of “total expected net cash outflow” and “25% of total expected cash outflow”. In this context, “expected” denotes a scenario of a combined idiosyncratic and market-wide shock that entails (among others) a partial run-off of retail deposits and a partial reduction in unsecured wholesale funding and secured short-term financing.

The definition of HQLA is such that privately produced assets can partly be used to satisfy the LCR requirement. HQLA can be divided into two categories: Level 1 assets are cash, central bank reserves, and government bonds with 0% risk-weight. Level 2 assets can again be divided in two sub-categories, Level 2A and Level 2B assets. A minimum haircut of 15% has to be applied to all Level 2 assets, which is supposed to capture their devaluation in a crisis scenario. After applying this haircut, Level 2 assets must not make up more than 40% of the whole stock of HQLA. Level 2A assets include government bonds with risk weights below 20% as well as corporate debt securities (including commercial papers) and covered bonds with a rating of at least AA-. Level 2B assets also include Residential Mortgage Backed Securities (RMBS) with ratings of at least AA, corporate debt securities with ratings of at least BBB-, and common equity shares which are constituent of a major stock index. These assets are subject to a haircut between 25% and 50% and must not make up more than 15% of the stock of HQLA.\textsuperscript{23}

In addition to these requirements, the Basel Committee specifies liquidity requirements of eligible assets in the following way: Level 2 assets must be traded in large, deep and active repo or cash markets characterised by a low level of concentration [and] have a proven record as a reliable source of liquidity in the markets (repo or sale) even during stressed market conditions (ie maximum decline of price not exceeding 10% or increase in haircut not exceeding 10 percentage

\textsuperscript{22}For details on the LCR, see Basel Committee (2013).

\textsuperscript{23}Note that they must also be included of the 40% cap of all Level 2 assets.
Chapter 1 Outside Liquidity, Rollover Risk, and Government Bonds

points over a 30-day period during a relevant period of significant liquidity stress).\textsuperscript{24}

The requirements for Level 2 assets are thus defined in terms of their past and present liquidity. The underlying notion seems to be that an asset’s past and present liquidity predicts its future liquidity. This is particularly evident in the condition that an asset’s value must have been stable in a “period of significant liquidity stress”. However, even if an asset stayed liquid during a past period of liquidity stress, this does not guarantee its future liquidity.

Under the currently proposed regulation, Level 2 assets can constitute up to 40% of the required assets to ensure short-term liquidity. This portfolio may consist exclusively of claims on the private sector, like corporate loans, MBS, and ABS. A stress scenario like the one in our model shows that such assets are not well suited to ensure the liquidity of banks. If banks rely too much on private assets, the economy might experience a run equilibrium in which private agents coordinate on a liquidity freeze. The price of seemingly safe and liquid claims on private institutions might drop substantially and in a fashion that is not predictable by their prior performance. Such an equilibrium can be ruled out if banks hold a sufficient stock of government bonds to cover the collapse of short-term funding. In order to ensure the stable supply of non-contractable liquidity, only Level 1 assets should be allowed to for liquidity management.

1.6.2 Net Stable Funding Ratio (NSFR)

As a second measure of liquidity regulation, Basel III requires the NSFR to be above 1. The NSFR is defined as the ratio of available stable funding (AFS) and required stable funding (RSF), both with a horizon of one year.\textsuperscript{25} Thus, the NSFR sets a lower bound for the amount of stable funding, conditional on a bank’s portfolio of illiquid assets and off-balance sheet exposures. ASF comprises capital, preferred stock, and liabilities with maturities of at least one year, but also deposits and wholesale funding with short or no maturity that are “expected to stay with the institution for an extended period in an idiosyncratic stress event.” Liabilities of the latter categories have to be multiplied by an ASF factor of less than one. ASF aims to exclude unstable short-term funding, i.e., funding that might quickly be withdrawn or not rolled over. It excludes short-term wholesale funding, such as interbank lending, but

\textsuperscript{24}Basel Committee (2013).
\textsuperscript{25}For details on the NSFR, see Basel Committee (2010).
includes customer deposits, because deposit insurance is supposed to make this funding appear stable.

RSF is a measure of a bank’s illiquid asset portfolio. It is defined as the sum of the value of a bank’s assets, multiplied by a specific RSF factor that is supposed to capture an asset’s liquidity risk, plus a similarly weighted sum of the bank’s off-balance sheet activities or potential liquidity exposures. An asset’s RSF factor is lower the more liquid this asset is. Cash and securities with a maturity below one year have an RSF factor of 0%; other securities and corporate bonds with good ratings have low, but positive RSF factors; other bonds, mortgages and loans have higher RSF factors, and other assets (particularly encumbered assets) have RSF factors of 100%.

The notion behind the NSFR requirement is that the ASF serves a bank to finance its illiquid asset contained in the RSF in times of a liquidity crisis. Those assets not contained in the RSF are liquid and can thus be sold in order to compensate the “unstable” funding that might disappear in a crisis. However, the criticism concerning the specification of the LCR applies in a similar way to the specification of the NSFR, as it does not sufficiently consider the problem of self-fulfilling liquidity dry-ups. The definition of RSF excludes several types of (private) assets that might turn illiquid in a crisis and thus also require stable funding. Moreover, the definition of ASF relies on a crisis scenario in which short-term funding is partly assumed to be stable. Our model shows that non-contractable short-term funding by private agents is needed for efficient intermediation, but these agents can coordinate on a liquidity dry-up. While insured demand-deposits could be considered as stable funding, this does not apply to wholesale funding because investors could coordinate on a freeze of if banks do not hold a sufficient stock of government bonds.

1.6.3 COMPARISON

Although the definitions of the LCR and NSFR appear quite different on first sight, their time horizon is the only distinct difference. To illustrate this point, let us consider a stylized bank balance sheet. Assume that the bank’s assets can be divided into a portfolio of liquid and a portfolio of illiquid assets, and that the liability side consists of short-term debt, long-term debt, and equity, see Figure 1.4.

For a moment, let us ignore the different time horizons of the two liquidity requirements and pretend that all quantities have been defined for the same horizon. In this case, the left side of the balance sheet consists of HQLA
and RSF, because the illiquid assets are exactly those that require stable funding. Which asset is considered to be liquid or illiquid is determined by the scenario of stress that is specified by the regulator. On the liability side, the scenario specifies which kind of funding is expected to disappear and which is expected to stay during a crisis. The expected net cash outflow is defined by the difference between the inflow and outflow of short-term debt in the specified scenario; it thus measures the expected change in the bank’s short-term liabilities. The part of short-term funding which (in the relevant scenario) is not assumed to disappear, together with long-term debt and equity, forms the ASF. Because total liabilities are necessarily equal to total assets, the LCR and the NSFR requirements are equivalent: HQLA exceed expected net cash outflow if and only if ASF exceeds RSF. It follows that the two measures only vary in their time horizon.

The two main criticisms thus apply similarly to both liquidity measures: First, the definition of HQLA is too broad, and the definition of RSF is too restrictive. Second, the definition of the stress scenario appears quite ad-hoc and underestimates the severity of self-fulfilling liquidity crises. These two aspects are strongly interconnected: In our model, we showed that banks optimally rely on the future provision of liquidity, but since this is not contractible initially, investors might coordinate on not providing this liquidity. Such adverse equilibria can only be ruled out if the bank’s balance sheet is structured such that disappearance of non-contractible funding can be compensated by selling liquid assets. We postulate that banks should cover this unstable funding by holding a sufficient stock of government bonds, and that the government should issue a sufficient amount of bonds for this purpose.

In our model, banks optimally expand their balance sheet by holding gov-
ernment bonds, and taking on liabilities with the same maturity towards the government. This increases the ASF as well as the stock of HQLA. Using the notion of Brunnermeier et al. (2013), this eliminates the banks’ the liquidity mismatch. We argue that holding government bonds is the only way of eliminating the liquidity mismatch which is compatible with efficient liquidity provision. Because private assets are subject to self-fulfilling liquidity dry-ups, private assets cannot eliminate the liquidity mismatch.

The 2007-09 financial crisis painfully revealed the fragility associated with private liquidity production. Neither the regulator nor market participants suspected that the ABCP market or the repo market, which were both backed by collateral such as ABS, could turn illiquid. As a consequence, the funding of institutions that were exposed to subprime mortgage risk broke down completely, until public liquidity support was provided. The same applied to institutions that held these assets off-balance sheet, but granted liquidity guarantees to their off-balance sheet vehicles.

However, it would be wrong to conclude that the regulator only needs to tighten the regulation on those assets that turned out to be problematic in the recent crisis. Any privately produced asset might turn illiquid, and any privately supplied liquidity may evaporate. Whenever liquidity management relies on the provision of non-contractible resources in future, liquidity regulation should be restrictive.

\section*{1.7 Conclusion}

This chapter has two main results: First, liquidity management with privately produced assets is either inefficient or associated with rollover risk, which makes an economy inherently fragile. Second, financial intermediaries can implement the optimal allocation by using government bonds. In the absence of public liquidity, financial intermediaries face a trade-off between high investment, which goes along with high rollover risk (high level of illiquid, but profitable long-term investments and low level of storage), and low investment, which comes with low rollover risk (low levels of profitable long-term investments and high level of storage). For the case of the 2007-09 financial crisis, our model suggests that intermediaries chose high investment levels that created a strong maturity mismatch. In the run-up to the crisis, financial intermediaries transformed long-term real investments into short-term securities, thereby aiming at making them liquid. E.g., illiquid assets like ABS and MBS (which are securitized long-term real investments) were transformed into short-term securities such as ABCP. In the crisis, however,
these short-term securities stopped being liquid and adverse consequences of the large-scale maturity mismatch realized.

Importantly, the rollover risk in our model is different from the traditional bank run problem in the style of Diamond and Dybvig (1983). In D&D, the bank run problem can be addressed by contracting in the initial period, e.g., by implementing a deposit insurance or allowing for a suspension of convertibility. In contrast, the rollover risk in our model cannot be eliminated in such a way because this would require contracting with a party that is not available initially. The problem originates from the friction that investors cannot commit initially to provide liquidity later. Their endowment does not realize before the time at which financial intermediaries need liquid funds. This makes liquidity management with privately produced assets inherently fragile.

In the second part, we demonstrate how liquidity management with government securities can improve the efficiency and stability of an economy. The government has the unique ability to commit future resources via taxation. Therefore, government securities are – in comparison to privately produced assets – less prone to coordination failures, i.e., less exposed to rollover risk. This property makes public outside liquidity superior to private outside liquidity.

This chapter thereby also contributes to the following basic but yet unresolved question: How should a public authority deal with liquidity provision? The traditional view since Bagehot (1873) is that a government should lend to illiquid but solvent institutions at high rates, while refusing to lend to insolvent institutions. The implementation of this principle might not be straightforward. That is, it may generally be problematic to identify whether an institution is illiquid or insolvent (see, e.g., Rochet and Vives, 2004). Moreover, Farhi and Tirole (2012) point out that if a government commits to intervening in case of liquidity needs, a collective moral hazard may give rise to an overall excessive maturity mismatch in an economy.

This chapter discusses a different approach to this problem. We find that the government should ensure efficient liquidity management by issuing government securities that should be held by financial intermediaries. This is a simple way of circumventing the undesired consequences that may arise when the regulator insures bank liabilities or provides emergency liquidity in case of a financial crisis. In our model, financial intermediaries are in fact always willing to hold government bonds to manage liquidity. However, in a richer model, banks may prefer to hold privately produced assets if these assets promise a higher return than government securities. In this case, a regulator
might optimally force banks to hold government securities in order to enhance stable liquidity provision.

We thereby reach out to the political debate about which types of assets should be allowed for liquidity management from a regulatory perspective. Basel III introduces the Liquidity Coverage Ratio (LCR) and the Net Stable Funding Ratio (NSFR) as measures and requirements to ensure a bank’s short-term and medium-term liquidity in a stress scenario. However, banks can partly use assets originated by the private sector in order to satisfy this requirement. We argue that this may be a severe source of systemic risk: once liquidity evaporates from the financial sector, these types of assets may cease to be liquid as well. Therefore, these assets might not be helpful to cover short-term liabilities and thus to prevent self-fulfilling crises.
Appendix 1.A Sub-Optimality of Financial Markets under Outside Liquidity

Let us analyze the competitive equilibrium of an economy where consumers hold assets directly and trade on financial markets. In the D&D model, the first-best cannot be achieved via financial markets because, in equilibrium, the competitive market prices give consumers investment incentives that do not induce the investment profile that optimally trades off early liquidity needs and the returns of the long asset. This inefficiency arises because consumers do not take into account the pecuniary externalities of their investment. This is induced by the friction of unobservable liquidity needs (unobservable types).

In our setup, pecuniary externality do not necessarily arise. The mere unobservability of the liquidity type itself does not impede the implementation of the first-best. In the absence of return risk, each consumer could privately invest his whole endowment in the long asset and sell claims on the asset in the interim period. The key frictions in our case are that, on the one hand, the return risk cannot be diversified on the individual level and, on the other hand, the unobservability of the risky return induces adverse selection, leading to inefficient liquidation. The liquidity type friction only intensifies the return risk friction.

The first-best can only be implemented if all resources are initially invested in the long asset. Furthermore, consumers must be perfectly insured against return risk. However, this insurance is only implementable with commitment in period zero. In a pure financial market economy with spot markets for claims on future returns, contingent contracts are not feasible. Therefore, the financial markets cannot implement an allocation that is efficient ex-ante. The non-diversifiable idiosyncratic return risk might also imply that consumers invest a positive fraction of their endowment in storage.

Moreover, the financial-market allocation might not even be efficient ex post, i.e., given the initial private investment. If all consumers in fact invest in the long asset privately, all impatient consumers have to either sell or liquidate their asset in the interim period. Since the return is unobservable, all assets have to be sold at the same price $R^*$. All impatient consumers with a return $R_i > R^*/\ell$ have an incentive to liquidate instead of selling claims, reducing the average quality in the market. The liquidation of projects is a form of adverse selection and constitutes an inefficiency ex post. Moreover, patient consumers with a return $R_i < R^*$ have an incentive to sell at price $R^*$.
instead of waiting, thus exacerbating the adverse selection and inefficiency.

Figure 1.5: Financial Market. The graph illustrates investment and trade under the assumption that full investment in the long asset is chosen. In the interim period, the graph denotes the flow of goods and claims for a consumer who chooses to sell claims on the market. Notice that there are also agents who liquidate their assets or hold on to them until $t = 2$ and thus do not interact with investors.

**APPENDIX 1.B  SUNSPOTS: OPTIMAL CONTRACTS UNDER PRIVATE OUTSIDE LIQUIDITY**

Assume that there is a public signal in $t = 1$ which we might call sunspot, following a Bernoulli distribution with success probability $p$. Assume further that investors play a pure strategy by which they base their rollover decision on this public signal. With probability $p$, all investors refrain from rolling over, and with probability $1 - p$, all investors engage in rollover. We are now looking for the optimal consumption profiles of consumers, i.e., the optimal investment behavior of banks, and abstract from bank runs. The optimal contract between banks and consumers will be state-contingent, i.e., contingent on the sunspot, or equivalently, on the behavior of investors.

Define the investment threshold $I_\ell$ such that

$$\ell u' \left( \frac{e_0 - I_\ell}{\pi} \right) = u' \left( \frac{R I_\ell}{1 - \pi} \right). \quad (1.11)$$

This threshold has the following interpretation: It is ex-post optimal to liq-
外面的流动性，滚存风险和政府债券

在情况的滚动冻结中，投资超过这个阈值后，捋出一个正的分数，因此只在投资超过这个阈值时，捋出一个正的分数。因为 \( \ell < \frac{1}{R} \)，它就持有 \( I_\ell \in (I^{DD}, e_0) \)，也就是说，在一个没有流动性（D&D）的设置中，捋出是不效率的后验的，但是它在银行不投资存储并完全依赖滚存时，有捋出，但投资者协商滚动冻结。

对于滚动的 contingency，最优合约包括完美的 consumption smoothing 通过外面的流动性并且没有捋出，\( c_1 = c_2 = e_0 - I + RI \)。对于滚动冻结的 contingency，捋出是正的如果 \( I > I_\ell \)，捋出是零的如果 \( I \leq I_\ell \)。最优投资 level \( I \) 由滚动冻结的概率来确定。

我们首先推导滚动冻结情况下的最优消费分配，对于给定的投资 level \( I \)。最优问题被给定为

\[
\max_{c_1, c_2, z \in [0,1]} \pi u(c_1) + (1 - \pi)u(c_2),
\]

s.t. \( \pi c_1 \leq e_0 - I + z\ell RI \), and \( \pi c_1 + (1 - \pi)c_2 \leq e_0 - I + z\ell RI + (1 - z)RI \).

(1.12)

我们现在已经（隐含地）给出了最优的条件消费分配给定一个投资 level \( I \)。我们现在已经最大化了这个投资 level。我们把问题分为两个间隔来考虑：\( [0, I_0] \) 和 \( (I_0, e_0] \)。我们可以忽略间隔 \( [0, I_0) \) 因为它被 \( I_0 \) 从支配。

存在两个阈值 \( p_0 \) 和 \( p_\ell \)，使得 \( 0 < p_0 < p_\ell < 1 \)，使得 \( I(p) = e_0 \) 时，\( p \leq p_0 \)，\( I^*(p_\ell) = I_\ell \)。

如果 \( p \in (p_0, p_\ell) \)，最优 \( I^*(p) \in (I_\ell, e_0) \)，最优和汇率是确定的为

\[
0 = p [(z\ell R - 1)u' \left( \frac{e_0 - I^* + z\ell RI}{\pi} \right) + (1 - z)Ru' \left( \frac{(1 - z)RI^*}{1 - \pi} \right)].
\]

(1.15)
1.B Sunspots: Optimal Contracts under Private Outside Liquidity

\[ + (1 - p)(R - 1)u'(e_0 + (R - 1)I^*), \quad \text{and} \quad (1.16) \]

\[ \ell u' \left( \frac{e_0 - I + z\ell RI}{\pi} \right) = u' \left( \frac{(1 - z)RI}{1 - \pi} \right). \quad (1.17) \]

If \( p > p_\ell \), the optimal \( I^*(p) \in [I^{DD}, I_\ell] \) and its level is determined by

\[ p \left[ -u' \left( \frac{e_0 - I^*}{\pi} \right) + Ru' \left( \frac{RI^*}{1 - \pi} \right) \right] + (1-p)(R-1)u'(1+(R-1)I^*) = 0. \quad (1.18) \]

For any \( p \in (p_\ell, 1) \), it holds that \( I^*(p) \in (I^{DD}, I_\ell) \). As \( p \to 1 \), it holds that \( I^*(p) \to I^{DD} \).
Sovereign Defaults, Bank Runs, and Contagion

2.1 Introduction

In this chapter, we provide a model that unifies the notion of self-fulfilling banking crises and sovereign debt crises. We show how these crises can be contagious, i.e., how a bank run can trigger a sovereign default, and vice versa (first type of contagion). We discuss under which conditions a government is unable to eliminate self-fulfilling banking crises by implementing a deposit insurance scheme. Moreover, we illustrate how crises can be contagious across countries (second type of contagion), and how contagious crises can be prevented. This allows us to evaluate the efficacy of recent policy proposals for the implementation of banking union in the euro area. We show under which conditions a supranational Deposit Guarantee Scheme can eliminate self-fulfilling crises at not cost.

The sovereign debt crisis in the euro area which has accompanied and followed the recent financial crisis since early 2009 has made the interdependence of sovereign and financial stability a prominent topic in the academic and political debate. Farhi and Tirole (2014) state that danger of the feedback loop between banking crises and sovereign debt crises is an exceptionally uncontentious economic idea. Several terrifying terms have been invented to describe this phenomenon, like “vicious cycle”, “doom loop”, “diabolic loop”,

41
or “deadly embrace”.

However, this phenomenon is anything but new. Historically, sovereign defaults and banking crises have often preceded and accompanied each other (see, e.g., Reinhart and Rogoff, 2009, 2011), but most existing data concerns emerging economies. Furthermore, there have been surprisingly few formal models that help to guide our theoretical understanding of how sovereign defaults and banking crises are interrelated, in particular for the case of developed and highly leveraged economies. Only recent, theoretical models on this topic were provided, e.g., by Acharya et al. (2014), Farhi and Tirole (2014), Leonello (2013), Cooper and Nikolov (2013), and König et al. (2013).

Banking crises and sovereign debt crises have the common feature that they may result from coordination on a bad equilibrium. In a self-fulfilling bank run, depositors desire to withdraw all at once. This is an equilibrium because if all depositors desire to withdraw at once, it forces an otherwise solvent bank to engage in inefficient liquidation, leading to insolvency (see, e.g., Diamond and Dybvig, 1983; Goldstein and Pauzner, 2005). In a self-fulfilling sovereign debt crisis, investors roll over a sovereign’s debt only at a high risk premium, or even refuse to do so. This constitutes an equilibrium as the high sovereign risk premium increases the government’s debt burden and thereby the likelihood of a default (see, e.g., Calvo, 1988; Cole and Kehoe, 2000).

We present a simple banking model of maturity transformation in the tradition of Diamond and Dybvig (1983). In the first part of this chapter, we consider the case of a closed economy. The model is reduced to a two-period version (we do not model the investment stage) and features consumers, banks, investors, and a government. We make two key assumptions: First, banks hold government bonds that they can sell in a secondary market in order to manage the liquidity needs of consumers. Second, the government’s tax base is correlated with the real economic activity which in turn depends on the performance of the financial sector. The model features a strategic complementarity within the consumers’ withdrawal decision, within the investors’ decision to purchase government bonds, as well as across the decisions of the two types of agents. There exist two types of self-fulfilling equilibria in our model: The first one is a no-crisis equilibrium, in which government bonds trade at face value, and the government as well as the banks fulfill their obligations. The second one is a crisis equilibrium. In the crisis equilibrium, all consumers withdraw early, causing a bank run. Depending on the fiscal soundness of the government, a bank run can be accompanied by a rollover freeze and a sovereign default. If the government is fiscally weak, a banking
crisis and a sovereign default aggravate and reinforce each other in a “vicious circle”. Only if a government is fiscally strong, it can eliminate the crisis equilibrium by providing a deposit insurance.

In the second part of this chapter, we extend our model to a multiple country setup where countries are interdependent, and we analyze cross-country effects of banking crises and sovereign debt crises. We assume that countries are interdependent due to banks diversifying their government bond holdings. If countries are sufficiently interdependent, self-fulfilling twin crises are contagious across borders. We show that if one country is fiscally weak while the other country is fiscally sound, it may be beneficial for both countries to pool their funds. The crisis equilibrium and its adverse consequences can be ruled out ex-ante by the following policy: Both countries form a banking union that implements a supranational deposit insurance scheme, and potentially also a fiscal union. By committing to repay the sovereign debt and to provide deposit insurance jointly, their joint promise will never be tested in equilibrium and is thus costless. A crucial insight is that forming such a union is not only beneficial for the fiscally weak country, but also for the fiscally strong country.

Guided by the insights of the model, we discuss two policy implications. The first policy implication concerns the design of the European Banking Union, with a special focus on the deposit insurance. Our model features cross-border costs of banking crises and sovereign defaults and points out channels through which a crisis in one country can trigger a crisis in another country. This in turn allows rationalizing policy responses by countries that are affected by foreign banking crises or sovereign defaults. The model allows us to give conditions under which a banking union (i.e., a joint deposit insurance) or the combination of a banking and a fiscal union can prevent contagious self-fulfilling banking crises and sovereign defaults. The model hence sheds light on the policy debates following the European debt crisis and allows us to investigate the efficacy of recent policy proposals (European Commission, 2013a). These proposals for a banking union focus on the Single Supervisory Mechanism (SSM) and the Single Resolution Mechanism (SRM). A supranational Deposit Guarantee Scheme (DGS) which would take the current national deposit insurance to a supranational level seems to be politically infeasible so far. By considering the self-fulfilling nature of banking crises, we show to what extent a banking union in its current form is ineffective at preventing such crises. Given that there are differences in the fiscal soundness of its member states, we argue that a banking union might only be effective if it comes with a joint deposit insurance.

The second policy implication concerns the regulatory treatment of banks
Chapter 2 Sovereign Defaults, Bank Runs, and Contagion

holding government bonds. While there may be good reasons for banks to use government bonds as an instrument to manage liquidity needs,\(^1\) we show that this may also be a considerable source of fragility once there is a prospect of a government default. Fragility arises in our setup whenever the government’s ability to repay its debt depends on the performance of the financial sector. This condition may be satisfied in developed economies that have highly leveraged financial systems. This chapter can therefore also be understood as a contribution to the debate concerning the liquidity regulation of banks. Regulatory frameworks typically facilitate the holding of government debt by intermediaries. The Basel Committee on Banking Supervision initially refrained from imposing any capital requirement for government bond holdings (see, e.g., Goodhart, 2011). Positive risk weights for poorly rated government bonds have been put on the agenda only recently, and were introduced in Basel III (Basel Committee, 2011). Our model provides an argument for why the exposure of banks to sovereign debt is a severe problem that is not adequately dealt with under both the current and the currently planned bank regulation.

This chapter is structured as follows: Section 2.2 presents a model of a closed economy, derives the equilibria, and discusses the effect of a deposit insurance. In Section 2.3, the model is extended to a two-country setting with international integration. We analyze contagion across countries and discuss optimal crisis prevention policies. Section 2.4 relates our findings to the current debate about the European Banking Union.

Related Literature
This chapter reaches out to the large literature on self-fulfilling banking crises (see, e.g., Diamond and Dybvig, 1983; Rochet and Vives, 2004; Goldstein and Pauzner, 2005) and self-fulfilling sovereign debt crises (see, e.g., Calvo, 1988; Alesina et al., 1990; Cole and Kehoe, 2000), and attempts to unify some aspects of the two strands.

The first part of this chapter is very closely related to a series of recent papers that model banking crises and sovereign debt crises in unified frameworks (Cooper and Nikolov, 2013; König et al., 2013; Leonello, 2013). Cooper and Nikolov also provide a model with multiple equilibria where the adverse equilibrium is characterized by a vicious cycle in which a government debt crisis and a banking crisis aggravate and reinforce each other. However, their focus is on the pricing of government debt, while emphasize the strategic complementarity of agents. The papers by König et al. and Leonello provide

\(^1\)See, e.g., Holmström and Tirole, 1998, Gorton and Ordoñez, 2013, and Chapter 1.
models featuring unique equilibria – reminiscent of Goldstein’ (2005) twin crisis model – and they analyze how government guarantees affect financial stability and the government’s ability to fulfill its obligation. All three papers have in common that the contagion from a banking crisis to a sovereign default originates from the increased public liabilities that arise from a safety net. In contrast, contagion in our setup arises because a financial crisis reduces the government’s tax base and thus decreases its funding instead of increasing its expenditure. The channel from sovereign debt to banking crisis is similar, however, it results from banks hold government bonds.

With Acharya et al. (2014), we share the notion that the government’s tax base is limited by a Laffer-curve property. Unlike our approach, they focus on the optimal redistribution (bailout) between a financial sector with debt overhang and a corporate sector. They find that a bailout can lose its bite if it lowers the value of government bonds that are held by the financial sector.

In the second part of this chapter, we analyze how crises can be contagious across countries. This part is related to the literature on financial contagion and the spreading of banking panics (see, e.g., Allen and Gale, 2000; Dasgupta, 2004). In particular, the second part of this chapter relates to Bolton and Jeanne (2011) who analyze the cross-border effects of sovereign defaults in financially integrated areas. In their model, government debt is used as collateral in interbank markets. Economic integration is beneficial as banks can diversify their government bond holdings, which fosters welfare-increasing interbank trade. However, this comes with possible contagion of a sovereign default ex-post, and fiscally strong countries might suffer from fiscal integration. This chapter is concerned with maturity transformation by banks and its inherent fragility, and not with the banks’ role in allocating capital. Moreover, government defaults are endogenous in our setup and directly linked to the performance of the banking sector. In contrast to the results of Bolton and Jeanne, we find that fiscally strong countries might actually benefit from fiscal integration if this prevents self-fulfilling crises.

Farhi and Tirole (2014) consider a model featuring fundamental financial and fiscal shocks in which banks hold domestic and foreign government bonds. Banks have an incentive to engage in excessive risk taking, particularly in collective moral hazard because the national government cannot commit to refrain from bailouts. This provides a new argument in favor of a banking union because the government is better off by delegating regulation to a supranational supervisor who takes a tough ex-post regulatory stance.


2.2 Single-Country Model

2.2.1 Setup

Consider an economy that goes through a sequence of two dates, \( t \in \{1, 2\} \). The economy is populated by a continuum of consumers of mass one and a continuum of investors of mass one. Moreover, there is a banking sector and a government. There exists a single good that can be used for both consumption and investment, and all units are denoted in terms of this good.

**Consumers**

Each consumer \( i \) is endowed with a demand deposit contract \((c_1^*, c_2^*)\) that allows her either to withdraw \( c_1^* \) units from her bank account in \( t = 1 \) or \( c_2^* \) units in \( t = 2 \). Consumers have preferences as proposed by Diamond and Dybvig (1983). There are two types: a fraction \( \pi \in [0, 1] \) of consumers is impatient, while the remaining fraction \( (1 - \pi) \) is patient. Impatient consumers only derive utility from consuming early; their utility is given by \( u(c_1) \). Patient consumers are indifferent between consuming early and late; their utility is given by \( u(c_1 + c_2) \). Types are private information of each consumer. Consumers face the decision to withdraw and to consume in \( t = 1 \) or to withdraw and consume in \( t = 2 \). Notice that the attributes “patient / impatient” characterize the consumer’s exogenous types. In contrast, the attributes “late / early” will characterize the endogenous decision of consumers: an “early consumer” withdraws and consumes in \( t = 1 \), while a “late consumer” withdraws and consumes in \( t = 2 \). We denote the decision of each consumer \( i \) to withdraw as well as to consume early with \( \omega_i \in \{0, 1\} \), where \( \omega_i \) takes the value one if consumer \( i \) withdraws in \( t = 1 \). Let \( \omega = \int_0^1 \omega_i \, di \) be the aggregate mass of early consumers.

**Banking Sector**

There is a banking sector that has the demand deposit contracts – which are the assets of consumers – as liabilities. It owns two types of assets: it holds government bonds as well as an illiquid portfolio of loans, both maturing at \( t = 2 \).

Banks are assumed to hold government bonds for the purpose of liquidity management. While we are not giving a micro-foundation for why banks are holding government bonds, we refer to various arguments for why financial intermediaries use government securities for liquidity management.

Government bonds are valuable as a medium of transfer across time (see, e.g., Gale, 1990; Woodford, 1990), and private agents may not be able to provide sufficient pledgable income (Holmström and Tirole, 1998). Furthermore,
government securities – unlike private assets – are not subject to adverse selection (Gorton and Ordoñez, 2013), and government securities are simply less exposed to rollover risk than privately produced assets (compare Chapter 1).

In our model, banks are not considered to be agents. They behave mechanically in that they serve early-withdrawing consumers by selling government bonds to investors and by liquidating the illiquid assets if necessary. Having the demand deposit contracts as liabilities, banks need to serve a mass \( \omega \) of consumers with \( c_1^* \) units in \( t = 1 \) each, and a mass \( 1 - \omega \) of consumers with \( c_2^* \) units in \( t = 2 \). Banks own a stock of government bonds which mature in \( t = 2 \). Bonds are liquid in the sense that they may be sold to investors in \( t = 1 \). Selling these government bonds allow banks to fulfill their short-term liability, i.e., to serve early consumers. The total amount of government debt in the economy is given by \( B \), and banks own a fraction \( \alpha \) of them, i.e., they own \( \alpha B < B \) units of government bonds. One unit of the government bond is a promise of the government to repay one unit of the good in \( t = 2 \). Details of the government bonds will be further specified below.

Moreover, banks also own \( I \) units of an illiquid asset to serve their long-term liabilities. The illiquid asset has an after-tax return of \( r = (1 - \tau)R > 1 \) in period two. The asset can be liquidated in \( t = 1 \), yielding a return per unit of \( \ell < 1 \). The fraction of illiquid assets which banks liquidate is denoted by \( z \). The total return of liquidation is thus given by \( z\ell I \). As indicated, the illiquid asset can be interpreted as a loan portfolio which pays off in the long run. In the short run, it can be liquidated at a substantial discount. The liquidation value \( \ell \) can be interpreted as the price in the secondary market for the bank’s loan portfolios and the discount may result from various frictions we do not model.\(^2\)

**Government**

There is a government that has an outstanding amount of debt \( B \), maturing in \( t = 2 \). Like banks, the government is assumed to behave mechanically. The government always repays its debt if possible and defaults otherwise. In \( t = 2 \), the government has an overall tax revenue of \( T(z) = E + \tau(1 - z)RI \) at its disposal. It consists of an exogenous tax revenue of \( E \geq 0 \), and an endogenous tax revenue \( \tau(1 - z)RI \) from taxation of the illiquid technology of the banking sector, where \( \tau \in (0, 1) \) is fixed. The tax revenue is used for

\(^2\)The assumption of low liquidation values is standard in the banking literature and may result from moral hazard (Holmström and Tirole, 1997), limited commitment of future cash-flows (Hart and Moore, 1994), adverse selection (Flannery, 1996), or uncertainty-averse investors (Uhlig, 2010).
the repayment of the government’s debt.\footnote{The remaining government budget can be used for other purposes. It could be used to provide a public good, or it could be transferred to the consumers. The exact use of remaining funds is not relevant in our model.}

We interpret the exogenous tax revenue $E$ as the tax revenue that the government generates irrespective of the performance of the banking sector. In turn, the endogenous tax revenue displays the fiscal revenue that depends on the performance of the banking sector and thus decreases in the level of liquidation $z$. It should thus be interpreted as the taxable economic activity that is generated through successful intermediation by banks. We assume that the government cannot raise any taxes in $t = 1$. This clearly displays an extreme simplification. However, we argue that a government’s ability to raise taxes at any point in time has natural limits,\footnote{See, e.g., the Laffer-curve property in Acharya et al. (2014).} and we make the simplifying assumption that it is zero in the short run.

Importantly, we assume that the government repays its debt whenever $B \leq T(z)$. For simplicity we assume that it fully defaults otherwise. With this assumption, we deviate from large parts of the sovereign risk literature and completely abstract from willingness to pay considerations.\footnote{The literature on sovereign debt and risk has been shaped by the willingness to pay view, which argues that governments repay their debt when the costs of repayment are lower than the penalty expected for default. In the literature, default penalties have been argued to be, e.g., exclusion from capital markets or trade sanctions (see, e.g., Eaton and Gersovitz, 1981; Bulow and Rogoff, 1989).} However, we refer to recent contributions arguing that ability-to-pay constraints dominate willingness-to-pay considerations, especially in advanced economies with a high degree of leverage where defaults may trigger severe financial sector turmoil (Gennaioli et al., 2014; Acharya and Rajan, 2013). If the government cannot default selectively (Guembel and Sussman, 2009; Broner et al., 2010), its incentives to default are generally very weak whenever the costs of defaulting are very high for domestic creditors. Thus, a sovereign default in a leveraged economy is likely to result from a binding ability to pay constraint.

**Investors**

There is a continuum of investors of mass 1. Each investor $j$ is equipped with one unit of the good in $t = 1$. Investors are risk-neutral and do not discount. Investors buy government bonds from banks whenever their return is non-negative. Formally, the decision of an outside investor $j$ to purchase government bonds from banks at face value or not is denoted $\eta_j \in \{0, 1\}$. It takes the value one if she is willing to buy a government bond at a price of one. Let $\eta = \min[\alpha B, \int_0^1 \eta_j dj]$ be the aggregate mass of outside investors that
buy government bonds at face value from banks.

In the following, we will refer to the purchase of government bonds by investors as *rollover*. Note that, in our setup, it does not matter whether the government needs to borrow \( t = 1 \) in order to repay banks that hold bonds that mature in \( t = 1 \) or whether banks need to sell government bonds that mature in \( t = 2 \) in a secondary market in \( t = 1 \). The first scenario clearly looks like a classical rollover problem. As both scenarios are equivalent, we use the expression rollover in order to simplify the wording.

**Parameters**

In the following, we make some restrictions on the model’s parameters in order to ensure outcomes and effects in a relevant domain. The first three assumptions guarantee the existence of a no crises equilibrium (also referred to as type I equilibrium), while the last assumption ensures the existence of a crisis equilibrium (type II equilibrium).

**Assumption 2.1.** \( c_2^* \geq c_1^* \)

Assumption 2.1 guarantees that it is incentive-compatible for patient consumers to withdraw late and to consume in \( t = 2 \) conditional on banks being able to pay out their promised payment, i.e., conditional on no liquidation.

**Assumption 2.2.** \( \pi c_1^* = \alpha B \leq 1 \) and \( rI = (1 - \pi)c_2^* \)

The first equation of Assumption 2.2 ensures that banks can serve all impatient consumers by selling their government bond holdings at face value. The second equation ensures that all patient consumers can be served by the long-term return of the loan portfolio if they withdraw late. Moreover, \( \alpha B \leq 1 \) implies that investors have enough funds to purchase all government bonds from banks at face value.

**Assumption 2.3.** \( T(0) = E + \tau RI \geq B \)

Assumption 2.3 ensures that the government’s tax revenue is sufficient to repay the government’s debt given that there is no liquidation by banks.

**Assumption 2.4.** \( (1 - \pi)c_1^* > \ell I \)

Assumption 2.4 implies that the banks will be insolvent and illiquid in \( t = 1 \) in case all consumers withdraw early, irrespective of the government’s solvency. The reason is that liquidation is sufficiently inefficient for a panic-based bank run to exist. While the patient consumers’ claims might be met by selling the government bonds, Assumption 2.4 implies that if all patient
consumers withdraw early, their claims equal to \((1 - \pi)c_1^*\) cannot be met by proceeds of complete liquidation, \(\ell I\). That is, the banking sector will be illiquid and insolvent in \(t = 1\) whenever there is complete withdrawal and liquidation.

### 2.2.2 Outcomes

In the following section, we show that the economy described above has two equilibria in pure strategies: a no crisis (type I) equilibrium and a crisis (type II) equilibrium. In the no-crisis equilibrium, only impatient consumers withdraw early and outside investors roll over the government’s debt. In the crisis equilibrium, all consumers withdraw early, causing a bank run. Depending on the fiscal soundness of the government, a bank run can be accompanied by a sovereign default and a rollover freeze.

In order to derive the equilibrium outcomes, we first analyze the banks’ liquidation of the loan portfolio for any given level of aggregate withdrawal and any rollover decision. We can then calculate the value of the demand deposit contract, as well as the value of government bonds in \(t = 2\), as functions of aggregate withdrawal and rollover. This in turn will pin down the optimal individual withdrawal and rollover decisions in \(t = 1\).

#### Liquidation

Banks have to fulfill their obligations in \(t = 1\) whenever possible. Recall that \(\omega\) denotes the mass of consumers that withdraw early, and \(\eta\) the mass of investors purchasing government bonds at face value. Banks need liquid funds of \(\omega c_1^*\) in \(t = 1\), since they have to pay \(c_1^*\) units of the good to a mass \(\omega\) of consumers. Banks sell \(\eta\) units of the governments bonds to investors. Given \(\omega\) and \(\eta\), banks must liquidate a fraction \(z\) such that their liquid funds equal the demand for early consumption or engage in complete liquidation, \(z = 1\), otherwise. Liquidation \(z\) is implicitly given by the budget equation \(\omega c_1^* = \eta + z\ell I\) whenever feasible, or explicitly by

\[
z(\omega, \eta) = \min \left[1, \frac{[\omega c_1^* - \eta]^+}{\ell I}\right]. \tag{2.1}
\]

If banks can serve all withdrawing consumers by selling government bonds, liquidation is unnecessary. However, if the proceeds from selling government bonds are not sufficient to serve all withdrawing consumers, banks will have to engage in inefficient liquidation of the loan portfolio.
2.2 Single-Country Model

Withdrawal and Rollover

The individual decision of patient consumers to withdraw depends on the funds that banks have available in \( t = 2 \). Similarly, the decision of investors to purchase government bonds depends on the funds that the government has available in \( t = 2 \). Whenever there is liquidation, the amount left for late consumers and the tax revenue of the government decrease.

The deposit contract \((c_1^*, c_2^*)\) is characterized by promised payments. If there is liquidation, actual repayments \((c_1, c_2)\) may fall short of the promised levels. In period one, banks have to serve any withdrawing consumer with \( c_1^* \) whenever possible. If banks engage in liquidation, this reduces the level of late consumption \( c_2 \), and if consumers in addition start to run on the banks, this also reduces \( c_1 \). For impatient consumers, it is dominant strategy to withdraw early, implying \( \omega \in [\pi, 1] \). Given liquidation \( z(\eta, \omega) \), the payments made to each patient consumer who is withdrawing late is given by

\[
  c_2(z(\omega, \eta), \omega) = (1 - z(\omega, \eta)) \frac{1 - \pi}{1 - \omega} c_2^*.
\]  

(2.2)

A patient consumer only withdraws early if \( c_2 < c_1^* \).\(^6\) The optimal withdrawal decision of a patient consumer is therefore given by

\[
  \omega_i^*(\omega, \eta) = \begin{cases} 
    0 & \text{if } c_2(z(\omega, \eta)) \geq c_1^* \\
    1 & \text{if } c_2(z(\omega, \eta)) < c_1^*.
  \end{cases}
\]  

(2.3)

We can derive the optimal rollover decision in a similar fashion. Given \( z(\eta, \omega) \), the government has a tax revenue of

\[
  T(z(\omega, \eta)) = E + \tau(1 - z(\omega, \eta))RI.
\]  

(2.4)

The government repays its debt whenever the tax revenue \( T(z(\eta, \omega)) \) exceeds the government’s outstanding debt \( B \), and defaults otherwise. Investors purchase government debt at face value if the government will be able to repay its debt, and do not purchase if the government is expected to default. An investor’s rollover decision is thus given by

\[
  \eta_i^*(\omega, \eta) = \begin{cases} 
    1 & \text{if } B \leq T(z(\omega, \eta)) \\
    0 & \text{if } B > T(z(\omega, \eta)).
  \end{cases}
\]  

(2.5)

\(^6\)We define \( c_2(1, 1) := 0 \), i.e., the potential late consumption is zero in case of complete liquidation.
The interrelation of the model’s key variables is summarized in Figure 2.1. The left cycle is the well-known cycle that lies at the heart of a self-fulfilling bank run, as in the classic bank-run model by Diamond and Dybvig (1983): Increased liquidation lowers the level of funds available for late consumption. This in turn increases the incentive to withdraw early. High early withdrawal, however, further increases liquidation.\(^7\) The right cycle shows how an anticipated sovereign default can be self-fulfilling: The inability of banks to sell government bonds forces them to liquidate some of the loan portfolio. Liquidation reduces the tax base and thus future tax revenue. This in turn may reduce the amount the government can repay. Consequentially, investors may become unwilling to purchase government bonds, forcing banks to liquidate even more.

The two cycles are connected through the liquidation of the illiquid loan portfolio. This allows a banking crisis to be contagious by triggering a sovereign debt crisis, and vice versa (1st type of contagion).

**Definition 2.1.** A Nash equilibrium in pure strategies is given by a set of consumers’ withdrawal decisions \(\{\omega_i\}\) and outside investors’ rollover decisions \(\{\eta_j\}\), such that these decisions are best responses, i.e., \(\omega_i = \omega_i^*(\omega, \eta) \ \forall i\) and \(\eta_j = \eta_j^*(\omega, \eta) \ \forall j\), where \(\omega = \int_0^1 \omega_i di\), and \(\eta = \min[\alpha B, \int_0^1 \eta_j dj]\).

We are now equipped in order to formulate the first result:

**Proposition 2.1.** *The model has two Nash equilibria in pure strategies.*

\(^7\)For the sake of completeness, the dotted arrow represents a positive feedback effect of early withdrawal on late consumption: more consumers withdrawing early implies that the remaining available funds are distributed among a smaller mass of late consumers. This channel represents the same effect through which a bank run is welfare-increasing in Allen and Gale (1998).
2.2 Single-Country Model

a) **Type I equilibrium** \((\omega, \eta) = (\pi, \alpha B)\):

Only impatient consumers withdraw early, banks do not liquidate, the tax revenue is sufficient to repay creditors, and investors are willing to buy government bonds.

b) **Type II equilibrium**

\(E < B\) **Sovereign default and bank run** \((\omega, \eta) = (1, 0)\): All consumers withdraw early and there is no rollover, inducing full liquidation. This results in illiquidity and insolvency of both the government and the banking sector.

\(E \geq B\) **Bank run** \((\omega, \eta) = (1, \alpha B)\): Investors roll over government debt, but all consumers withdraw early. Although there is a bank run and full liquidation, the government is still able to fully serve its debt.

For the proof of Proposition 2.1, see the Appendix. The multiplicity of equilibria arises from the strategic complementarity between agents. There are three different components of strategic complementarity in the model. First, there is a strategic complementarity between consumers in their decision to withdraw: the more consumers withdraw, the higher the incentive for an individual consumer to withdraw as well. Second, there is strategic complementarity between the investors in their decision to purchase government bonds: more investors purchasing government bonds increases the individual incentive to purchase government bonds as well. Third, there is strategic complementarity across the two types of agents: higher levels of withdrawal decrease the incentive to roll over and vice versa.

Note that in the above setup both types of equilibria always exist. The type I equilibrium is always characterized by successful debt rollover and the absence of a panic-based bank run. The type II equilibrium is characterized by either a twin crisis where a sovereign default and a panic-based bank run accompany each other, or by a panic-based bank run without sovereign default. The type II equilibrium is a twin crisis whenever the exogenous tax base \(E\) is less than the government’s outstanding debt \(B\), or if \(E/B < 1\). In this case, banking crises and sovereign debt crises are contagious in the sense that they aggravate and reinforce each other. Whenever \(E\) exceeds \(B\), i.e., \(E/B \geq 1\), the government will be able to repay its debt irrespective of the occurrence of a banking crisis. In this case, a sovereign default will never occur, but a bank run still constitutes an equilibrium.

The parameter \(E\) (or the ratio \(E/B\)) can be interpreted as a measure of the government’s fiscal stability. If \(E/B \geq 1\), the government can raise taxes
irrespective of the performance of banks which will suffice to repay the outstanding debt. The taxable economic activity thus does not depend too much on the provision of financial services. If \( E/B < 1 \), the government’s ability to tax and to repay is closely linked to the banking sector, i.e., the taxable economic activity depends strongly on the performance of the banking sector. Therefore, whenever \( E/B < 1 \), the crisis equilibrium is not only characterized by a banking crisis, but also by a sovereign default.

Generally, \( E/B < 1 \) is reminiscent of the \textit{crisis zone} in Cole and Kehoe (2000): when the exogenous tax base that is available irrespective of the performance of the banking sector is low, runs become possible. In the following, we will show that this may be true irrespective of the existence of a deposit insurance.

\section*{2.2.3 Deposit Insurance Scheme (DIS)}

We now analyze the effect of a deposit insurance. We define deposit insurance to be a guarantee by the government that each consumer receives \( c^*_1 \) units at a period of his choice. If the deposit insurance is credible, it prevents patient consumers from withdrawing early because in any contingency consumers get at least as much in period two as in period one. In the next paragraph, we will analyze under which conditions a deposit insurance is credible in our setup. We assume that the government uses its tax revenue to repay its bonds first, and only uses its remaining funds to fulfill the DIS afterwards if possible. This ordering might seem odd at first sight because government bonds only get repaid in period two, but the deposit insurance might already be needed in period one. However, since the government does not have funds in period one - recall that we assumed that the short-term tax base is zero - it will have to borrow in order to provide a DIS. The government will only be able to borrow and actually fulfill a deposit insurance if its outstanding debt is not already exceeding its available funds. Therefore, the government bonds are effectively senior to the deposit insurance.

The deposit insurance is credible if the government is able to repay its debt and to pay for the deposit insurance in any contingency. The most adverse contingency is the case in which all consumers withdraw early, and the banks thus have to engage in full liquidation. A sufficient condition for the deposit insurance to be credible is that the government can repay its debt \( B \). Therefore, banks can sell their \( \pi c^*_1 \) bonds at face value. The complete liquidation of the illiquid loan portfolio provides the banks with an additional amount of \( \ell I \) units. Thus, the deposit insurance has to cover the missing funds.
2.2 Single-Country Model

in order to serve the each consumer with $c^*_1$ units. Therefore, the maximal amount a deposit insurance might have to cover is given by $DI = c^*_1 - I - \pi c^*_1$.

Whenever $E \geq B + DI$, the deposit insurance scheme is credible, because the government can actually provide this amount in any contingency.

**Proposition 2.2.** *By providing a deposit insurance scheme, the government can eliminate the crisis equilibrium iff $E \geq B + DI$.*

The government is able to eliminate the crisis equilibrium whenever its exogenous tax revenue exceeds the sum of the outstanding debt $B$ and the maximum cost of a deposit insurance $DI$. In this case, it can repay its debt and credibly insure deposits of all consumers. The deposit insurance is never tested and therefore eliminates the adverse equilibrium at no costs. For $B \leq E < B + DI$, there are multiple equilibria. The government cannot prevent a bank run because the deposit insurance scheme is not credible, but since it can serve its debt, a rollover freeze does not occur in equilibrium. For $E < B$, the government can neither prevent a bank run nor a sovereign default. Figure 2.2 shows which type of equilibria exist for different levels of $E$ under the deposit insurance scheme.

![Figure 2.2: Existence of Equilibria under the Deposit Insurance Scheme. The type I equilibrium always exist. The type II equilibrium only exists if the deposit insurance is not credible, i.e., if $E < B + DI$. It is characterized by a bank run for $E \geq B$, and by a twin crisis for $E < B$.](image)

Finally, notice that the deposit insurance does not bail out banks; it only steps in after banks have already defaulted on their liabilities. In fact, it would be more efficient in our model to bail out banks in order to prevent them from engaging in inefficient liquidation. However, in terms of preventing the crisis equilibrium, a bailout mechanism would have exactly the same effects as a DIS. The government could announce that it would bail out the banks in case of a crisis and thereby eliminate the crisis equilibrium if the announcement is credible. For this to be true, the government would need exactly the same budget, i.e., $B + DI$. 

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2.3 TWO-COUNTRY MODEL

We now consider an extended, two-country setting of the model. This allows us to analyze under which conditions a crisis in one country may be contagious, triggering a crisis in another. We will use the setup to investigate which policies eliminate the adverse crisis equilibrium and ensure financial stability. In our model, a country consists of domestic consumers who hold demand deposit contracts with domestic banks. Furthermore, there is a government that taxes domestic economic activity. In our model, investors are not associated with countries.

Assume that there are two countries that are labeled home $H$ and foreign $F$. Without loss of generality, we take the view of the home country to facilitate the verbal interpretation of our analysis. Both countries are as described in the single-country case and identical to each other, except for some international financial interdependence. Furthermore, we vary the amount of exogenous tax revenue $E^H$ and $E^F$. A country $k$ is called fiscally sound whenever $E^k$ is very high, and fiscally weak whenever $E^k$ is low. We assume throughout most of this section that each country implements a deposit insurance scheme targeting domestic depositors whenever feasible. We analyze a policy setup where both countries can form a banking union or a fiscal and banking union. The banking union is a supranational policy tool that implements a joint deposit insurance for both countries. When tested, the costs are borne by the two countries jointly. We contrast these policies with a situation of political autarky where there is no supranational policy. Throughout the analysis, we maintain Assumptions 2.1 to 2.4 for both countries.

Importantly, we assume that countries are interdependent. We introduce interdependency by assuming that banks of both countries hold government bonds of both countries. While we assume this interdependence, we refer to empirical evidence as well as to theoretical explanations why government bond holdings are diversified.\(^8\)

In a nutshell, we will present two main results: First, crises can be contagious across countries once there is interdependence. A sovereign debt crisis in the foreign country is always costly for the home country, and also triggers a crisis in the home country if the interdependence is sufficiently strong. Second, a fiscal and banking union may eliminate the adverse equilibrium at no

\(^8\)For empirical evidence, see Bolton and Jeanne (2011) and Cooper and Nikolov (2013), who describe the cross-country holdings of government bonds in the euro area by using the European Banking Authority Stress Test data. Moreover, cross-country holdings of government bonds can result, e.g., from international activities of banks, or from diversification considerations (see, e.g., Bolton and Jeanne, 2011).
costs if joint exogenous tax revenue is sufficiently high.

2.3.1 Setup

Assume that banks in both countries still hold a portfolio of government bonds. However, now this portfolio not only contains bonds of the domestic country, but also bonds of the other government. In both countries, banks hold an amount \((1 - \lambda)\alpha_B\) of the domestic and \(\lambda \alpha_B\) of the non-domestic government bonds, where \(\lambda \in (0, 1)\). The mass of investors who are willing to buy bonds of the respective government is denoted by \(\eta^H\) and \(\eta^F\). When buying government bonds, investors do not discriminate based on the nationality of banks selling the bonds.

As before, banks in the home country need to serve each early consumer with \(c^*_1\) units in \(t = 1\), potentially forcing them to liquidate a fraction \(z^H(\omega^H, \eta^H, \eta^F)\) of its loan portfolio. The budget equation of home banks in \(t = 1\) is therefore given by

\[
\omega^H c^*_1 = (1 - \lambda)\eta^H + \lambda \eta^F + z^H \ell I \quad (2.6)
\]

whenever possible. In analogy to the single-country case we can express liquidation as

\[
z^H(\omega^H, \eta^H, \eta^F) = \min \left[ 1, \frac{[\omega^H c^*_1 - (1 - \lambda)\eta^H - \lambda \eta^F]^+}{\ell I} \right]. \quad (2.7)
\]

Observe that, in contrast to the single country case, home banks’ liquidation is now not only a function of aggregate withdrawal and aggregate rollover in the home country, but also a function of aggregate rollover of the foreign country’s sovereign debt. Late consumption and tax revenue are given as above: they are functions of the liquidation fraction, \(c^H_2(z^H, \omega^H)\) and \(T^H(z^H)\). Therefore, if the foreign country defaults, which goes along a rollover freeze of foreign debt, the consumption and the tax revenue in the home country decreases because the countries are interdependent. We focus on a case where there is a high degree of interdependence between the countries.

Assumption 2.5. \(\lambda \geq \frac{c^*_2 - c^*_1}{\ell I / \pi c^*_1}\)

The assumption implies that interdependence, measured by \(\lambda\), is so strong that whenever there is a sovereign default abroad, there also is a bank run at home – unless home depositors are kept from running by a deposit insurance scheme. A high \(\lambda\) implies that once the foreign government defaults, losses of banks at home on the foreign government bonds are also high. Assumption 2.5
implies a default abroad in fact induces a liquidation that would lower the late consumption to a level below the promised amount of early consumption, \( c_2 < c_1^* \), giving patient consumers an incentive to withdraw early. For a formal analysis, see the proof of Proposition 2.3 in the Appendix.

2.3.2 INTERNATIONAL CONTAGION

Let us first assume that countries do not intervene abroad, but only provide a deposit insurance scheme to domestic depositors. As mentioned above, we refer to this as a political autarky. We analyze how a sovereign default abroad (i.e., \( \eta^F = 0 \), possible whenever \( E^F < B \)) may be contagious and affect outcomes in the home country. In doing so, we implicitly characterize the crisis equilibrium of the two-country economy.

Whenever there is a sovereign default abroad, the amount required to make a deposit insurance at home credible is given by \( \tilde{D}I = DI + \lambda \alpha B \). This amount is larger than in the single-country case. In order to make the deposit insurance scheme credible in the two-country case, the home country’s government has to be able to cover the losses on foreign government bonds in addition to the cost of the deposit insurance, as specified in the single-country setup.

**Proposition 2.3.** In a Nash equilibrium in which there is a sovereign default in the foreign country, the following outcomes prevail in the home country:

- **Sovereign default and bank run** \((\omega^H, \eta^H) = (1,0)\): All consumers withdraw early and there is no rollover, inducing full liquidation and thus resulting in illiquidity and insolvency of both the government and the banking sector.

- **Bank run** \((\omega^H, \eta^H) = (1,\alpha B)\): Investors purchase government debt, but all consumers withdraw early. Although there is full liquidation, the government is still able fully to serve its debt.

- **No bank run, but costly deposit insurance** \((\omega^H, \eta^H) = (\pi,\alpha B)\): Investors purchase government debt, and only impatient consumers withdraw early. However, the deposit insurance scheme becomes costly.

The proof of Proposition 2.3 can be found in the Appendix. Let us discuss these results in some more depth. In the first case, the home country has weak
fiscal fundamentals; a sovereign debt crisis abroad will always trigger a twin crisis in the home country as well. In the second case, \( E^H \) is in an intermediate range and the home country can repay its debt for sure, but it cannot provide a credible deposit insurance. In this case, banks in the home country make a loss of \( \lambda \pi c^*_t = \lambda \alpha B \), forcing them to liquidate a share of their loan portfolio, which triggers a bank run. Finally, in the third case, the fiscal fundamentals are strong and the home country can credibly promise to repay its debt and insure its deposits. Therefore, the home country can rule out a bank run at home once the foreign country defaults. However, the crisis abroad remains contagious in that banks incur a loss of \( \lambda \alpha B \). Because the remaining funds of banks in \( t = 2 \) are smaller than \( (1 - \pi)c^*_t \) by Assumption 2.5, the deposit insurance scheme has to step in. The results of Proposition 2.3 are depicted in the lower area of Figure 2.3, for \( E^F < B \). The three different scenarios are represented by the areas I to III.

### 2.3.3 Optimal Policies: Supranational Institutions

We have seen that a crisis abroad causes real losses for home banks and is thus contagious under political autarky even if the home government is able to provide a credible deposit insurance. However, it might be possible to prevent the crisis abroad through the implementation of adequate supranational institutions. We are looking for institutions that constitute a Pareto improvement compared to the situation of political autarky, in the sense that both countries weakly benefit from this policy. We focus on two different institutional setups: first, the implementation of a banking union, and second, the joint implementation of a banking union and a fiscal union. In our model, a banking union describes a supranational institution that provides a deposit insurance scheme for both countries and is financed by both countries. Similarly, in a fiscal union, both countries mutualize sovereign debt and promise to repay the debt of both countries together.

**Proposition 2.4.** Assume \( E^H + E^F \geq 2(B + DI) \) and \( E^H > E^F \).

- \( E^F < \tilde{E} \) **A banking union** is Pareto-efficient only if it is complemented with a **fiscal union**.
- \( E^F \in [\tilde{E}, B + DI) \) **A banking union** is required for Pareto efficiency, but a fiscal union is not necessary.
- \( E^F \geq B + DI \) Remaining in **political autarky** is Pareto-efficient, there is no need for a banking or fiscal union.

The threshold \( \tilde{E} \) is defined as \( \tilde{E} = B - \frac{[\ell -(1 - \lambda)\pi c^*_t]^{+}}{\ell} - \tau RI \).
Chapter 2 Sovereign Defaults, Bank Runs, and Contagion

Figure 2.3: Equilibria under Political Autarky. This figure depicts the types of crisis equilibria in the case of political autarky (each government only provides a DIS for domestic depositors) for different values of external tax revenues $E^H$ and $E^F$. In region I, the crisis equilibrium is a twin crisis (sovereign default and bank run) in both countries. In region II, the fiscally weak country defaults and experiences a bank run, while the fiscally sound country does not default, but experiences a banking crisis. In region III, there is a twin crisis in the fiscally weak country and no crisis in the fiscally sound country, but the DIS is costly. In region IV, one country or both countries experience a banking crisis, but sovereigns do not default and there is no contagion. The banking crises can occur independently of each other. In region V, there is a banking crisis in the fiscally weak country, but no contagion, and no crisis in the fiscally sound country. In region VI, no crisis equilibrium exists.
The assumption of $E^H + E^F \geq 2(B + DI)$ implies that the pooled exogenous tax revenues of both countries suffice to repay the government debt and credibly to insure the depositors of both countries. Let us go backwards to illustrate the results of Proposition 2.4. If $E^F \geq B + DI$, the foreign government is fiscally sound and can prevent a crisis by providing a deposit insurance scheme on its own, so Pareto efficiency is already attained under political autarky. As soon as $E^F < B + DI$, the foreign country cannot provide a credible deposit insurance any more and a bank run can occur. Therefore, a joint deposit insurance is needed. Based on the level of $E^F$, we have to make one further case distinction. Notice that even though the banking union prevents a bank run, banks might have to liquidate because of a rollover freeze. The rollover freeze can only occur if $E^F < \tilde{E}$. As long as $E^F$ is above this threshold, the remaining tax revenue after liquidation suffices to repay the government bonds $B$. Because the rollover freeze is ruled out, the banking union is a sufficient measure. However, if the exogenous tax revenue falls below this threshold, the rollover freeze can only be ruled out by the additional implementation of a fiscal union through which the home government guarantees the repayment of foreign government debt.

The results are depicted in Figure 2.4. Proposition 2.4 is concerned with the area above the dashed line, where $E^H + E^F \geq 2(B + DI)$. In region (i) no union is required. A banking union is strict Pareto improvement in regions (ii) and (iii), whereas in region (iv) the implementation of both a banking and a fiscal union is required.

We conclude that if the countries are sufficiently different with respect to their exogenous tax revenue, it may be beneficial for both countries to form a banking union as this eliminates the adverse crisis equilibrium at no costs.

Discussion

Notice that there is no uncertainty regarding fiscal soundness in our model, i.e., it is clear which country is fiscally weak and which country is fiscally strong. However, both countries have an incentive to form a banking union or even a fiscal union. The type of unions discussed can therefore be understood as something that is different from typical insurance against potential adverse states in the future. A typical insurance would be a contract between agents which is signed before relevant states are realized and which aims at insuring at least one of the contracting parties. Typically, there is ex post one party that makes losses on the contract because it has to transfer net funds to the other party.

In our case, however, the insurance contract can be signed after the values of external tax revenues, $E^H$ and $E^F$, are realized because there is no transfer of
Figure 2.4: Efficient Policy Measures. This figure depicts regions in which the crisis equilibrium can be eliminated by either a banking union or the joint implementation of a Banking and fiscal union for different values of external tax revenues $E^H$ and $E^F$. In region (i), a crisis equilibrium does not exist even under political autarky, thus a union is not needed. In region (ii), a banking union stabilizes the weaker country by ruling out a bank run. While it does not benefit the stronger country, it does not cost anything either. In region (iii), the banking union rules out a bank run and a sovereign default of the weaker country, thus benefiting both countries. Finally, in region (iv), the banking union is not effective anymore. Here, only the joint implementation of banking and fiscal union can eliminate the crisis, and it is costless for both countries. If the sum of exogenous tax revenues is too small, it is not possible to rule out crisis equilibria by forming a union (region (v)). While the fiscally stronger country might experience neither a sovereign default nor a bank run, it suffers whenever the weaker country experiences a sovereign default.
funds from the strong to the weak country. In contrast, both countries benefit from this atypical insurance even ex post, even though the union might be valued more by the fiscally weak than by the fiscally strong country. Because it is effective in preventing self-fulfilling crises, the unions are costless for both countries. This consideration implies that if there was initial uncertainty about which of the two countries is the strong one and which is the weak one, both countries would have an incentive to form the union.

2.4 The European Banking Union

We now use the insights of our model to investigate the efficacy of recent policy proposals. The proclaimed goal of the proposal for the implementation of a banking union in the euro area is to ensure financial stability and to break the “potentially vicious circle between banks and sovereigns” (European Commission, 2013a).

The current proposals for the formation of a banking union consist of three components. First, the Single Supervisory Mechanism (SSM), which is supposed to be complemented by a single rulebook of the European Banking Authority (Council of the European Union, 2013). Second, the Single Resolution Mechanism (SRM) for the centralization of competencies and resources for managing the failure of banks (European Commission, 2013b). Third, a supranational Deposit Guarantee Scheme (DGS).

Currently, the first two components are already implemented (SSM) or close to being implemented (SRM), but a supranational deposit insurance scheme so far seems to be politically infeasible and is currently off the table (European Commission, 2013a). Hellwig (2014) points out several doubts about the effectiveness of SSM and SRM at dealing with cross-border externalities, especially for the case of banks that operate in several countries. Furthermore, he points out that national authorities may be unable or unwilling to provide funding in case of a crisis, calling for a fiscal backstop at the European level. Our model points to a further problem: We show that the lack of a supranational DGS may be a serious shortcoming of the European Banking Union, and may undermine the overall efficacy of the proposed reforms in ensuring financial stability.

Note that due to the stylized nature of our model there is no role for supervision and resolution of banks. Thus, our model remains silent on the efficacy of the components of the banking union that have already been or are about to be implemented (supervision and resolution). Clearly, both components are crucial for harmonizing banking regulation on the European level and
may well be considered as a key achievement.

In turn, our model can actually say something on the supranational DGS, the component policy makers currently seem to refuse to implement. Our model states that if there is sufficient interdependence between countries and a high degree of heterogeneity in the countries’ fiscal soundness, a banking union as well as a fiscal union may eliminate the self-fulfilling crisis equilibrium. Observe that in fact banks are highly interconnected within the euro area. Moreover, observe that there are countries that may be considered fiscally sound (e.g., Germany and France), and others that may be considered fiscally weak (e.g., Spain and Italy).9 If one is willing to believe in the self-fulfilling nature of financial crises, a deposit insurance scheme, potentially complemented by a fiscal union, may implement financial stability at no costs. This also implies that the refusal to implement a deposit insurance scheme may lead to potentially costly contagion across countries, which could be avoided.

A deposit insurance scheme works best if it is credible and never tested and thus eliminates the possibility of self-fulfilling crises at no costs. In order to understand the importance of this insight in the context of the European situation, consider the following: Assume that there is a fiscally sound country that would never experience a self-fulfilling crisis if it was in autarky. However, its interdependence with another country implies its banks will realize losses once there is a crisis in the foreign country. Thus, ensuring domestic financial stability through, e.g., a deposit insurance scheme will become costly for the government once its banks have realized losses. A crisis abroad may therefore cause real costs at home once there is sufficient interdependence. Given the self-fulfilling nature of the crisis abroad, it may be optimal for the home country to participate in a mechanism that prevents the crisis abroad at low (or even at zero) costs. Preventing the crisis abroad eliminates contagion and thus ensures financial stability at home in this setting. Our model shows that this is possible by implementing a banking union (equivalent to a joint deposit insurance scheme in our model) which is complemented with a fiscal union if necessary.

In order to apply this insight to the European situation, one needs to appreciate the fact that a deposit insurance in fiscally weak countries may not be credible. It may therefore not be able to prevent a banking crisis in the

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9We do not consider our model to fit the case of Greece. It is more than questionable whether Greece could have repaid its debt even if its debt had been a rolled over. The crisis in Greece does not appear to be only self-fulfilling, but rather due to fundamental problems. Greece and Portugal rather had “old-fashioned sovereign debt crises” (Hellwig, 2014).
2.5 Conclusion

respective country, a crisis that can be contagious and thus costly for fiscally sound countries as well. A banking union with a joint deposit insurance scheme may increase the credibility of the deposit insurance. In fact, the deposit insurance scheme may become fully credible once it is backed by fiscally sound governments, eliminating the crisis equilibrium altogether. In fact, in our very simple setup, such a mechanism can eliminate the crisis equilibrium at no cost.

One may hypothesize that politicians in fiscally sound countries currently seem to be scared of implementing a joint deposit insurance scheme. The rationale is that it could appeal to voters as another form of mutualization of national debt, with a clear disadvantage for taxpayers in fiscally sound countries. E.g., German politicians may fear to scare their voters as a banking union may imply that German taxpayer can potentially be liable for losses of, e.g., Spanish banks. Our model indicates that this may turn out to be bitter irony: exactly the refusal of implementing a full-fledged banking union with a joint deposit insurance scheme may make future crises more costly for the respective tax payers.

2.5 Conclusion

Our model has two main contributions. First, we discuss how banking crises and sovereign defaults can be contagious across countries. The setup allows us to rationalize supranational policies that aim at preventing sovereign and financial crises. Our specific setup gives conditions under which a fiscal and a banking union are effective measures to eliminate an adverse run equilibrium. We use these results to comment on the policy debates on the making of a banking union in the euro area. Importantly, our model indicates that a banking union with a joint deposit insurance scheme may be a mechanism to prevent contagious self-fulfilling banking crises. It possibly has to be complemented by a fiscal union to be entirely effective. We argue that the current proposal for a banking union, consisting only of supranational supervision and resolution mechanisms, is insufficient to break the vicious cycle between sovereigns and banks.

Second, the model illustrates the risks associated with banks holding government bonds. In our model, fragility arises whenever the fate of the government and the financial sector are closely connected. This condition is likely met in developed and highly leveraged financial systems where banks hold government bonds and where economic activity depends on the performance of the financial sector. This chapter thus sheds light on the debate regarding
the regulation of government bonds holding by intermediaries. More specifically, it gives a rationale for why exposure of banks to sovereign risk may be problematic.

The stylized nature of our model implies that our insights and policy implications have to be taken with a grain of salt and cannot be translated one-to-one for every institutional arrangement. In our model, we abstract from fundamental uncertainty (i.e., macroeconomic shocks) as a source of a crisis, and from potential moral hazard resulting from an established banking and fiscal union. Both elements may be of importance in reality. For the case of negative macroeconomic shocks in a foreign country, a supranational deposit insurance may moderate a crisis, but this might come with real costs for the home country. In addition, the presence of an international insurance may induce a country’s institutions (government, supervision, and banks) to gamble. Both aspects might induce fiscally strong countries to refrain from a fiscal and an extensive banking union. This is not an argument against such unions, though. It rather calls for detailed contractual definitions of the union’s scope, and for strict regulation and supervision that is located at level of the union. The SSM and SRM can mitigate such moral hazard on the country level, and thus build the foundation which is necessary for implementing a supranational DGS.
2.A Proofs

Appendix 2.A Proofs

Proof of Proposition 2.1. We first analyze proof the existence of the Type I equilibrium where \((\omega, \eta) = (\pi, \alpha B)\): By Equation (2.1), banks do not engage in liquidation, \(z(\omega, \eta) = 0\), yielding a late consumption of \(c_2 = c_2^*\) and a tax revenue of \(T = E + \tau R I\). Assumptions 2.1 to 2.3, and Equations (2.3) and (2.5) imply that patient consumers do not withdraw early, \(\omega_i^*(\omega, \eta) = 0 \quad \forall i\), and outside investors roll over the debt \(\eta_j^*(\omega, \eta) = 1 \quad \forall j\). Therefore \((\omega, \eta) = (\pi, \pi c_1^*)\) constitutes a Nash Equilibrium.

We now proof the existence of the Type II equilibrium. We distinguish two cases.

\(E < B\) Sovereign default and bank run \((\omega, \eta) = (1, 0)\):
The liquidation is given by \(z(\omega, \eta) = 1\), yielding \(c_2 = 0\) and \(T = E\). We get \(\omega_i^*(\omega, \eta) = 1 \quad \forall i\) and \(\eta_j^*(\omega, \eta) = 0 \quad \forall j\). Therefore \((\omega, \eta) = (1, 0)\) constitutes a Nash Equilibrium.

\(E \geq B\) Bank run \((\omega, \eta) = (1, \alpha B)\):
The liquidation is given by \(z(\omega, \eta) = 1\), yielding \(c_2 = 0\) and \(T = E\). We get \(\omega_i^*(\omega, \eta) = 1 \quad \forall i\) and \(\eta_j^*(\omega, \eta) = 1 \quad \forall j\). Therefore \((\omega, \eta) = (1, \alpha B)\) constitutes a Nash Equilibrium.

Proof of Proposition 2.3. A sovereign default in the foreign country implies that \(\eta_F^* = 0\), implying that domestic banks make a loss of \((1 - h)\alpha B\). Assumption 2.5 implies that this loss induces a liquidation which necessarily triggers a bank run in the home country in the absence of a deposit insurance. To prove this fact, we show that even if there was rollover of sovereign debt and no run, depositors would still prefer to run, i.e., \(c_2^H < c_1^*\). In this case, the liquidation would be \(z^H(\pi, \alpha B, 0) = (\pi c_1^* - (1 - \lambda)\pi c_1^*)/\ell I = \lambda \pi c_1^*/\ell I\). By Assumption 2.5, it follows that \(z^H > c_2^H - c_1^* - c_2^H c_1^*/c_2^* = 1 - c_1^*/c_2^*\). Late consumption is given by \(c_2^H(z^H, \pi) = (1 - z^H) c_2^*\). It follows that \(c_2^H(z^H, \pi) < (c_1^*/c_2^*) c_2^* = c_1^*\). Therefore, the bank run is inevitable in the absence of a deposit insurance.

\(E^H < B:\) Sovereign default and bank run, \((\omega^H, \eta^H) = (1, 0)\)
Because the government cannot provide a deposit insurance, a bank run is triggered. This leads to full liquidation and reduces the tax revenue to \(T^H = E^H < B\), inducing a sovereign default and a rollover freeze.

\(E^H \in [B, B + \bar{D}I)\): Bank run, \((\omega^H, \eta^H) = (1, \alpha B)\)
Because the government cannot provide a deposit insurance, a bank run is triggered. This leads to full liquidation and reduces the tax revenue to \(T^H = E^H > B\). The sovereign can repay its debt, and rollover is ensured.
\( E^H \geq B + D I \): No Bank run but costly deposit insurance, 
\((\omega^H, \eta^H) = (\pi, \alpha B)\)

The government can provide a deposit insurance scheme and thus prevent a bank run, and it can also repay its debt, ensuring the rollover of debt. However, the deposit insurance is costly.

Proof of Proposition 2.4. Given that \( E^F \geq B + D I \), it is immediately clear that both countries are stable under political autarky, therefore a union is not needed.

In the presence of a banking union, the foreign government cannot experience a sovereign debt crisis if \( E^F \geq \bar{E} \). Because joint funds suffice to make a banking union credible, it prevents a run in the foreign country. If there was a rollover freeze in the foreign country, banks would have to liquidate \( z^F(\pi, 0, \alpha B) = \min[1, (1 - \lambda)\pi_1^*/(\ell I)] \). This induces a tax revenue of \( T^F(z^F) \geq \bar{E} + [\ell I - (1 - \lambda)\pi_1^*/(\ell I)]^{-1} \tau RI = B \). The foreign government can thus repay its debt, and a rollover freeze cannot occur in equilibrium. Therefore, the banking union is sufficient to eliminate any crisis altogether if \( E^F \geq \bar{E} \).

If however \( E^F < \bar{E} \), a rollover freeze constitutes an equilibrium even in the presence of a banking union which prevents a bank run. In case of a rollover freeze, the tax revenue is given by \( T^F(z^F) < \bar{E} + [\ell I - (1 - \lambda)\pi_1^*/(\ell I)]^{-1} \tau RI = B \). Therefore, the joint implementation of the banking and the fiscal union is required. This policy measure is costless for the home country because by providing the deposit insurance and guaranteeing to repay all government debt, it rules out a bank run and ensures rollover of foreign government debt. The deposit insurance will not be tested, and because foreign banks do not engage in liquidation, the foreign government has sufficient tax revenue to repay its debt by itself.

Returning to the case of \( E^F \in [\bar{E}, B - D I] \), we can distinguish two different scenarios. If \( E^F \in [\bar{E}, B] \), both countries strictly benefit from the implementation of the banking union. The foreign country does not experience any crisis, and because the default of the foreign sovereign is ruled out, losses of home banks on foreign government bonds are eliminated. In contrast, if \( E^F \in [B, B + D I] \), the foreign country will always be able to repay its debt. Therefore, the home country cannot be affected by a crisis at all. Even if there was a bank run in the foreign country, the home country would not suffer because the exposure is only through foreign debt which is unaffected. Thus, only the foreign country benefits from the banking union, but the home country does not suffer. This distinction is illustrated by the regions (ii) and (iii) in Figure 2.4. \( \Box \)
3

Banks, Shadow Banking, and Fragility

3.1 Introduction

This chapter contributes to the theoretical understanding of how shadow banking activities can set the stage for a financial crisis. Maturity and liquidity mismatch of unregulated financial intermediators – often described as shadow banking – were a key ingredient to the 2007-09 financial crisis (Brunnermeier, 2009; FCIC, 2011). Prior to the crisis, the shadow banking sector was largely involved in financing long-term real investments such as housing. With an increase in delinquency rates of subprime mortgages, uncertainty about the performance of returns of those investments emerged. This led to various kinds of run-like events in the shadow banking sector, including the collapse of the market for asset-backed commercial papers (ABCP) (Kacperczyk and Schnabl, 2009; Covitz et al., 2013), the counterparty runs on Bear Stearns and Lehman Brothers in tri-party repo (Copeland et al., 2011; Krishnamurthy et al., 2014), and the large-scale run on the money market fund industry in the aftermath of the Lehman failure (Kacperczyk and Schnabl, 2013; Schmidt et al., 2014). The turmoil in the shadow banking sector ultimately translated into broader financial-sector turmoil in which several commercial banks were on the brink of failure. Ultimately, governments and central banks had to intervene on a large scale.

We develop a model in which shadow banking emerges alongside commercial banking in order to circumvent financial regulation. We show that if
the shadow banking sector grows too large, fragility arises in the sense that panic-based runs may occur. The size of the shadow banking sector is crucial because it determines the volume of assets being sold on the secondary market in case of a run. We assume that arbitrage capital in this market is limited. Therefore, if the shadow banking sector is too large relative to available arbitrage capital, fire-sale prices are depressed due to cash-in-the-market pricing, and self-fulfilling runs become possible. Moreover, if shadow banking activities are intertwined with activities of commercial banks, a crisis in the shadow banking sector may also trigger a crisis in the regulated banking sector. Eventually, the efficacy of existing safety nets for regulated banks may be undermined. By considering regulatory arbitrage, our model challenges the view that a credible deposit insurance may eliminate adverse run equilibria in model with maturity transformation at no cost.

The term “shadow banking” was coined during the 2007-09 financial crisis in order to describe financial intermediation activities that were unknown to a broader public prior to the crisis.\(^1\) However, the term is imprecise and ambiguous, and its definition varies substantially even within the academic debate. Among the most prominent definitions are the ones by Pozzar et al. (2013), by the Financial Stability Board (FSB, 2013), and by Claessens and Ratnovski (2014).\(^2\) Building on these definitions, we use the term “shadow banking” to describe banking activities (risk, maturity, and liquidity transformation) that take place outside the regulatory perimeter of banking and do not have direct access to public backstops, but may require backstops to operate.

Prior to the crisis, shadow banking had evolved as a popular alternative to commercial banking in order to finance long-term real investments via short-term borrowing. E.g., asset-backed securities (ABS) were financed through asset-backed commercial papers (ABCP). While the shadow banking sector

\(^{1}\)The expression was first used by Paul McCulley at the Jackson Hole Symposium in Wyoming, who described shadow banking as “the whole alphabet soup of levered up non-bank investment conduits, vehicles, and structures”(McCulley, 2007).

\(^{2}\)Pozsar et al. (2013) define shadow banking as “credit, maturity, and liquidity transformation without direct and explicit access to public sources of liquidity or credit backstops”. The FSB (2013) describes shadow banking as “credit intermediation involving entities and activities (fully or partially) outside the regular banking system”. Finally, Claessens and Ratnovski (2014) propose the label “shadow banking” for “all financial activities, except traditional banking, which require a private or public backstop to operate”. All approaches describe shadow banking as financial activities that are similar to those of traditional banks. While the FSB emphasizes regulatory aspects, the other two address the access to or the need for backstops. Our definition borrows from all three and tries to combine the different aspects.
3.1 Introduction

had a stable record prior to the crisis, its activities expanded rapidly in the years up to the crisis (see, e.g., FCIC, 2011; FSB, 2013; Claessens et al., 2012).³

The 2007-09 financial crisis began when an increase in delinquency rates of subprime mortgages induced uncertainty about the performance of ABS. In August 2007, BNP Paribas suspended convertibility of three of its funds that were exposed to risk of subprime mortgages bundled in ABS, and there was a sharp contraction of short-term funding of off-balance sheet conduits such as ABCP conduits and structured investment vehicles (SIVs) that financed their ABS holdings by issuing ABCP and medium-term notes (Kacperczyk and Schnabl, 2009). The empirical evidence suggests that this contraction resembled the essential features of a run-like event or a rollover freeze in the ABCP market (see Covitz et al., 2013). Due to the breakdown of the ABCP market and due to continuing bad news from the housing market, the institutions that produced ABS got into trouble. This culminated in the counterparty runs on Bear Stearns in March 2008 in tri-party repo, and finally in the collapse of Lehman Brothers in September 2008.⁴ The failure of Lehman Brothers caused further turmoil, including “Reserve Primary Fund” breaking the buck, thus finally triggering a full-blown run on the money market fund industry (Kacperczyk and Schnabl, 2013; Schmidt et al., 2014). Our model contributes to the theoretical understanding for why sharp contraction in short-term funding may occur in the shadow banking sector. In particular, our model offers a rationale of how regulatory arbitrage may set the stage for panic-based runs such as the run on ABCP conduits and MMFs in the 2007-09 crises, and how such runs can also adversely affect the regulated banking sector – even in the absence of runs on commercial banks.

We discuss a simple banking model of maturity transformation in the tradition of Diamond and Dybvig (1983) and Qi (1994) in order to illustrate how shadow banking can sow the seeds of a financial crisis. In our model, com-

³The only MMF that ever broke the buck before Lehman’s default was the “Community Bankers Money Fund” (see Kacperczyk and Schnabl (2013)). Shadow banking activities had been evolving since the 1970s and experienced a growth boost in the 1990s when MMFs expanded their investments from government and corporate bonds towards ABS also.

⁴In the direct aftermath of the crisis, the academic debate had – due to the availability of data – largely focused on the run on repo (Gorton and Metrick, 2012). Copeland et al. (2011) as well as Krishnamurthy et al. (2014) point out that the repo market experienced a margin spiral in the sense of Brunnermeier and Pedersen (2009), but did not necessarily experience a run. The counterparty runs on Bear Stearns in tri-party repo programs in March 2008 and the run on Lehman Brother in September 2008 are exceptions.
cial banks’ liabilities are covered by a deposit insurance. Because this might induce moral hazard on the part of the banks, they are subject to regulation, which induces regulatory costs for the banks. The shadow banking sector competes with commercial banks in offering maturity transformation services to investors. In contrast to commercial banks, shadow banking activities are neither covered by the safety net nor burdened with regulatory costs.

Our first key result is that the relative size of the shadow banking sector determines its stability. If the short-term financing of shadow banks breaks down, they are forced to sell their securitized assets on a secondary market. The liquidity in this market is limited by the budget of arbitrageurs. If the size of the shadow banking sector is small relative to the capacity of this secondary market, shadow banks can sell their assets at face value in case of a run. Because they can raise a sufficient amount of liquidity in this way, a run does not constitute an equilibrium. However, if the shadow banking sector is too large, the arbitrageurs’ budget does not suffice to buy all assets at face value. Instead, cash-in-the-market pricing à la Allen and Gale (1994) leads to depressed fire-sale prices in case of a run. Because shadow banks cannot raise a sufficient amount of liquidity, self-fulfilling runs constitute an equilibrium. Depressed fire-sale prices are reminiscent of theories on the limits to arbitrage (see, e.g., Shleifer and Vishny, 1997, 2011) and give rise to multiple equilibria in our model.

As a second key result we find that if commercial banks themselves operate shadow banks, a larger size of the shadow banking sector is sustainable. In this case, the shadow banking sector indirectly benefits from the safety net for commercial banks. Because of this safety net, bank depositors never panic and banks thus have additional liquid funds to support their shadow banks. This enlarges the parameter space for which shadow banking is stable. However, once the threat of a crisis reappears, a crisis in the shadow banking sector also harms the sector of regulated commercial banking.

Finally, the third important result is that a safety net for banks may not only be unable to prevent a banking crisis in the presence of regulatory arbitrage. In fact, it may become tested and costly for the regulator (or taxpayer). If banks and shadow banking are separated, runs only occur in the shadow banking sector, while the regulated commercial banking sector is unaffected. If they are intertwined, a crisis in the shadow banking sector translates into a system-wide crisis and ultimately the safety net becomes tested, and eventually costly, for its provider. This is at odds with the view that safety nets such as a deposit insurance are an effective measure to prevent panic-based banking crises. In traditional banking models of maturity transformation, such as
Diamond and Dybvig (1983) and Qi (1994), credible deposit insurance can break the strategic complementarity of investors and eliminate adverse run equilibria at no costs, as it is never tested. The efficacy of such safety nets was widely agreed upon until recently; see, e.g., Gorton (2012) on “creating the quiet period”. We show that this may not be the case when regulatory arbitrage is possible. Regulatory arbitrage may undermine the efficacy of safety nets.

For most parts of this chapter we treat the shadow banking sector as consisting of one vertically integrated institution. However, we show that our model can be extended such that its structure is closer to the actual shadow banking sector in the US prior to the crisis, see also Figure 3.5. We mostly follow and simplify the descriptions by (Pozsar et al., 2013) and show that all results hold when we consider a shadow banking sector that consists of investment banks (broker dealers), ABCP conduits such as special investment vehicles (SIVs) and money market mutual funds (MMFs) instead of single shadow banks. This also allows us to derive separate conditions for runs from investors on MMFs and for runs from MMFs on ABCP conduits.

The main contribution of this chapter is to show how regulatory arbitrage-induced shadow banking can contribute to the evolution of financial crises. We illustrate how shadow banking activities undermine the effectiveness of a safety net that is installed to prevent self-fulfilling bank runs. Moreover, we show how shadow banking may make the safety net costly for the regulator in case of a crisis. We argue that the understanding of how shadow banking activities contribute to the evolution of systemic risk is not only key to understanding the recent financial crisis. Our results indicate that circumvention of regulation can generally have severe adverse consequences on financial stability. We argue that it is an essential part of any analysis of the efficacy of regulatory interventions to consider the extent of possible regulatory arbitrage. Thus, this chapter is not only concerned with the 07-09 crisis but attempts to make more general point on the dangers associated with regulatory arbitrage. This may be of importance for those economies in which shadow banking is booming such as currently in China (see Awrey, 2015; Dang et al., 2014).

While the simple nature of our model keeps the analysis tractable, we exclude certain features that might be considered relevant. In our view, the most important ones are the following two: First, in our model, a financial crisis is a purely self-fulfilling phenomenon. We do not claim that the turmoils in summer 2007 were a pure liquidity problem. Clearly, ABCP conduits had severe solvency problems as a consequence of increased delinquency rates. However,
this chapter is an attempt to demonstrate how the structure of the financial system can set the stage for a severe fragility: because of maturity mismatch in a large shadow banking sector without an explicit safety net, small shocks can lead to large repercussions. Second, by focusing on regulatory arbitrage as the sole reason for the existence of shadow banking, we ignore potential positive welfare effects of shadow banking and securitization. There are several other rationales for why shadow banking exists: securitization can be an effective instrument to share macroeconomic interest rate risk (Hellwig, 1994) or to cater to the demand for safe debt (Gennaioli et al., 2013); it can make assets marketable by overcoming adverse selection problems (Gorton and Pennacchi, 1990, 1995; Dang et al., 2013a); and it can increase the efficiency of bankruptcy processes (Gorton and Souleles, 2006). In contrast, we focus on the regulatory arbitrage hypothesis which has received considerable support by the empirical findings of Acharya et al. (2013). Therefore, it is important to keep in mind that whenever we speak of shadow banking and its consequences for financial stability, we mainly address shadow banking that originates from regulatory arbitrage. However, the fragility that we find in our model may arguably also exists in a different context.

There is a fast-growing literature on theoretical aspects of shadow banking. Our modeling approach is related to the paper by Martin et al. (2014). However, their focus lies on the run on repo and on the differences between bilateral and tri-party repo in determining the stability of single financial institutions. In turn, we focus on ABCP and system-wide crises. The paper by Bolton et al. (2011) is the first contribution to provide an origination and distribution model of banking with multiple equilibria in which adverse selection is contagious over time. Gennaioli et al. (2013) provide a model in which the demand for safe debt drives securitization. In their framework, fragility in the shadow banking sector arises when tail-risk is neglected.

Other contributions that deal with shadow banking are Ordoñez (2013), Goodhart et al. (2012, 2013), and Plantin (2014). Ordoñez focuses on potential moral hazard on the part of banks. In his model, shadow banking is potentially welfare-enhancing as it allows to circumvent imperfect regulation. However, it is only stable if shadow banks value their reputation and thus behave diligently; it becomes fragile otherwise. The emphasis of Goodhart et al. lies on incorporating shadow banking into a general equilibrium model. Plantin studies the optimal prudential capital regulation when regulatory arbitrage is possible. In contrast to all three, we focus on the destabilizing effects of shadow banking in the sense that it gives rise to run equilibria.

This chapter proceeds as follows: In Section 2, we illustrate the baseline
3.2 A Model of Intergenerational Banking

Our baseline model is an overlapping-generation version of the model of maturity transformation by Diamond and Dybvig (1983) which was first introduced by Qi (1994).

There is an economy that goes through an infinite number of time periods \( t \in \mathbb{Z} \). There exists a single good that can be used for consumption as well as investment. In each period \( t \), a new generation of investors is born, consisting of a unit mass of agents. Each investor is born with an endowment of one unit of the good, and her lifetime is three periods: \( (t, t+1, t+2) \). Upon birth, all investors are identical, but in period \( t+1 \), their type is privately revealed: With a probability of \( \pi \), an investor is impatient and her utility is given by \( u(c_{t+1}) \). With a probability of \( 1-\pi \), the investor is patient and her utility is given by \( u(c_{t+2}) \). Assume that the function \( u(\cdot) \) is strictly increasing, strictly concave, twice continuously differentiable, and satisfies the following Inada conditions: \( u'(0) = \infty \), and \( u'(\infty) = 0 \).

In each period \( t \), there are two different assets (investment technologies): a short asset (storage technology), and a long asset (production technology). The short asset transforms one unit of the good at time \( t \) into one unit of the good at \( t+1 \), thus effectively storing the good. The long asset is represented by a continuum of investment projects. An investment project is a metaphor for an agent who is endowed with a project (e.g., an entrepreneur with a production technology or a consumer who desires to finance a house), but has no funds she can invest.

There is no aggregate, but only idiosyncratic return risk: each investment project requires one unit of investment in \( t \) and yields a stochastic return of \( R_i \) units in \( t+2 \). The return \( R_i \) is the realization of an independently and identically distributed random variable \( \tilde{R} \), characterized by a probability distribution \( F \). \( F \) is continuous and strictly increasing on some interval \([R, \tilde{R}] \subset \mathbb{R}^+\), with \( \mathbb{E}[R_i] = R > 1 \). We assume that the realization of an investment project’s long-term return, \( R_i \), is privately revealed to whoever finances the project.

The idiosyncratic return risk of the long asset implies that financial inter-
mediaries dominate a financial markets solution in terms of welfare because of adverse selection in the financial market. In turn, unlike participants of a financial market, a financial intermediary will not be subject to these problems as he is able to diversify and create assets that are not subject to asymmetric information.

Finally, an investment project may be physically liquidated prematurely in $t+1$, yielding a liquidation return of $\ell R_i/R$, where $\ell \in (0, 1/R)$. The liquidation return of a project thus depends on the project’s stochastic long-term return. The average liquidation return of a project is equal to $\ell$.

### Intergenerational Banking

In the following, we describe the mechanics of intergenerational banking and derive steady state equilibria, closely following Qi (1994). We assume that there is a banking sector operating in the economy, consisting of identical infinitely lived banks that take deposits and make investments. It is assumed that the law of large numbers applies at the bank level, i.e., a bank neither faces uncertainty regarding the fraction of impatient investors nor regarding the aggregate return of the long asset.

In each period $t \in \mathbb{Z}$, banks receive new deposits $D_t$. They sign a demand-deposit contract with investors which specifies a short and a long interest rate. Per unit of deposit, an investor is allowed either to withdraw $r_{t,1}$ units after one period, or $r_{t,2}$ units after two periods. In period $t$, banks yield the returns from the last period’s investment in storage, $S_{t-1}$, and the returns from investment in the production technology in the second but last period, $I_{t-2}$. They can use these funds to pay out withdrawing investors and to make new investment in the production and in the storage technology.

We are interested in steady states of this intergenerational banking. A steady state is given by a collection of payoffs, i.e., a short and a long interest rate, $(r_1, r_2)$, a deposit decision $D$, and an investment decisions $I$ and $S$. We are only interested in those steady states in which investors deposit all their funds in the banks, $D = 1$, and the total investment in the storage and production technology does not exceed new deposits, i.e., $S + I \leq D$. This

---

5Because asset quality is not observable, there is only one market price. Impatient consumers with high-return assets have an incentive to liquidate them instead of selling them, and patient consumers with low-return assets have an incentive to sell. This drives the market price below average return and inhibits the implementation of the first-best.

6Steady states with $S + I > D$ also exists, but in those equilibria, banks have some wealth which is kept constant over time, the net returns of which are payed out to investors each period. This scenario does not appear particularly plausible or interesting.
3.2 A Model of Intergenerational Banking

yields the investment constraint

\[ S + I \leq 1. \]  \hfill (3.1)

Moreover, we restrict attention to those steady states in which only impatient consumers withdraw early. We will show later that these withdrawal decisions as well as the deposit decision are actually optimal choices in a steady state equilibrium. In such a steady state, banks have to pay \( \pi r_1 \) units to impatient investors and \( (1 - \pi) r_2 \) units to patient consumers in every period. Since payoffs and investments are limited by returns and new deposits, the following resource constraint must hold:

\[ \pi r_1 + (1 - \pi) r_2 + S + I \leq RI + S + 1. \]  \hfill (3.2)

This constraint can be simplified to obtain a simple feasibility condition for steady-state payoffs:

**Definition 3.1 (Steady-state Payoff).** A steady-state payoff \((r_1, r_2)\) is budget feasible if

\[ \pi r_1 + (1 - \pi) r_2 \leq (R - 1) I + 1. \]  \hfill (3.3)

In a next step, we want to select the optimal steady state among the set of budget feasible steady states. Our objective is to choose the steady state that maximizes the welfare of a representative generation of investors, or equivalently, the expected utility of one representative investor. We can partition this analysis by deriving the optimal investment behavior of banks in a first step, and then addressing the optimal interest rates. We see that the budget constraint (3.3) is not influenced by \( S \). Thus, the banks’ optimal investment behavior follows directly:

**Lemma 3.1 (Optimal Investment).** The optimal investment behavior of banks is given by \( I = 1 \) and \( S = 0 \), i.e., there is no investment in storage. The budget constraint reduces to

\[ \pi r_1 + (1 - \pi) r_2 \leq R. \]  \hfill (3.4)

The intergenerational feature of banking implies that storage is not needed for the optimal provision of liquidity. Any investment in storage would be inefficient and would hence imply a deterioration.

We can now derive the optimal steady-state payoffs \((r_1, r_2)\), i.e., the optimal division between long and short interest rate. It is straightforward to see that the first-best steady-state payoff is given by perfect consumption smoothing, \((r_1^{FB}, r_2^{FB}) = (R, R)\). However, the first-best cannot be implemented as it...
is not incentive compatible. The incentive-compatibility and participation constraints are given by
\[
\begin{align*}
  r_1 &\leq r_2, \quad (3.5) \\
  r_1^2 &\leq r_2, \quad (3.6) \\
  \text{and} \quad r_2 &\geq R. \quad (3.7)
\end{align*}
\]

Constraint (3.5) ensures that patient investors wait until the last period of their lifetime instead of withdrawing early and storing their funds. Constraint (3.6) ensures that patient investors do not withdraw early and re-deposit their funds. By this type of re-investment, investors can earn the short interest rate twice. As long as net returns are positive, the latter condition is stronger, implying that the yield curve must not be decreasing. Finally, constraint (3.7) ensures that investors do not engage in private investment and side-trading. In fact, this condition is the upper bound to the side-trading constraint. The adverse selection problem induced by the idiosyncratic return risk relaxed this constraint, but the constraint will turn out not to be binding anyhow.

Obviously, constraint (3.6) is violated in the first-best, inducing patient investors to withdraw early and to deposit their funds in the banks a second time. In the second-best, constraints (3.4) and (3.6) are binding, resulting in a flat yield curve, \( r_2 = r_1^2 \). Following Equation (3.4), the interest rate is such that
\[
\pi r_1 + (1 - \pi)r_1^2 = R. \quad (3.8)
\]

**Proposition 3.1** (Qi 1994). In the second-best steady state, the intergenerational banking sector collects the complete endowment, \( D = 1 \), and exclusively invests in the long-asset, \( I = 1 \). In exchange, banks offer demand-deposit contracts with a one-period interest rate given by
\[
r_1^* = \frac{\sqrt{\pi^2 + 4(1 - \pi)R} - \pi}{2(1 - \pi)}, \quad (3.9)
\]
and a two-period interest rate given by
\[
r_2^* = r_1^{*2}. \quad (3.10)
\]

It holds that \( r_2^* > R > r_1^* > 1 \). Unlike in the Diamond and Dybvig (1983) model, the first-best and the second-best do not coincide. The intergenerational structure introduces the new IC constraint that the long interest rate
must be sufficiently larger than the short one in order to keep patient investors from withdrawal and reinvestment.\footnote{However, the intergenerational structure also relaxes the feasibility constraint. Although the yield curve is allowed to be decreasing in the model of Diamond and Dybvig (1983), the second-best of intergenerational banking dominates the first-best of Diamond and Dybvig (1983) for a large set of utility functions because banks do not have to rely on inefficient storage.}

**Steady-State Equilibrium**

Until now, we have not formally specified the game in a game-theoretic sense. Consider the infinite game where in each period \( t \in \mathbb{Z} \), investors born in period \( t \) decide whether to deposit, and investors born in \( t - 1 \) decide whether to withdraw or to wait for one more period. We do not engage in a full game-theoretic analysis. In particular, we do not characterize all equilibria of this game, but only focus on the equilibrium characterized by the above steady state, and analyze potential deviations. Banks are assumed to behave mechanically according to this steady state.

**Lemma 3.2.** The second-best steady state constitutes an equilibrium of the infinite game.

If all investors deposit their funds in the banks, and if only impatient consumers withdraw early, it is in fact individually optimal for each investor to do the same. The second-best problem already incorporates the incentive compatibility constraints as well as the participation constraint. Patient investors have no incentive to withdraw early, given that all other patient investors behave in the same way and given that new investors deposit in the bank. Nor do investors have an incentive to invest privately in the production or storage technology, as the bank offers a weakly higher long-run return than \( R \).

**Fragility**

We will now study the stability of intergenerational banking in the absence of a deposit insurance. Models of maturity transformation such as Diamond and Dybvig (1983) and Qi (1994) may exhibit multiple equilibria in their subgames. Strategic complementarity between the investors may give rise to equilibria in which all investors withdraw early, i.e., bank run equilibria.

In the following, we analyze the subgame starting in period \( t \) under the assumption that behavior until date \( t - 1 \) is as in the second-best steady-state equilibrium. We derive the condition under which banks might experience a run by investors, i.e., the condition for the existence of a run equilibrium in the period-\( t \) subgame. In the case of intergenerational banking, we consider a “run” in period \( t \) to be an event in which all investors born in \( t - 1 \) withdraw.
their funds, and none of the newly-born investors deposit their endowment. In case of such a run, the bank has to liquidate funds in order to serve withdrawing investors. In addition to the expected withdrawal of impatient consumers, the bank now also has to serve one additional generation of patient investors withdrawing early.

**Lemma 3.3.** Assume that the economy is in the second-best steady state. In case of a run, the banks’ liquidity shortfall is \((1 - \pi)r_1^*\).

**Proof.** In case of such a run in period \(t\), banks have to repay what they have invested on behalf of the mass of \((1 - \pi)\) patient investors in \(t - 2\) who have claims worth \(r_1^{*2}\). Moreover, they have to pay all funds that they invested on behalf of those investors from \(t - 1\) who have claims worth \(r_1^*\). Banks thus need a total amount of \((1 - \pi)r_1^{*2} + r_1^*\) in case of a run.

However, banks only have an amount \(R\) of liquid funds available in \(t\) from the investment they made in \(t - 2\). Recall from Proposition 3.1 that \(\pi r_1^* + (1 - \pi)r_1^{*2} = R\). The banks’ liquidity shortfall in case of a run by investors is thus given by

\[
(1 - \pi)r_1^{*2} + r_1^* - R = (1 - \pi)r_1^*.
\] (3.11)

Let us assume that the liquidation rate is sufficiently small relative to the potential liabilities of banks in case of a run:

**Assumption 3.1.** \(\ell < (1 - \pi)r_1^*\).

Assumption 3.1 implies that, if in some period \(t\) all depositors withdraw their funds and newborn investors do not deposit their endowment, the liquidation return that the bank can realize does not suffice to serve all withdrawing consumers. Therefore, the bank is illiquid and insolvent.

**Proposition 3.2.** Assume that the economy is in the second-best steady state. In the subgame starting in period \(t\), a run of investors on banks constitutes an equilibrium.

This proposition states that the steady state is fragile in the sense that there is scope for a run. Assumption 3.1 implies that it is optimal for a patient investor to withdraw early if all other patient investors do so and if new investors do not deposit. Note that Proposition 3.2 only states that a run is an equilibrium of a subgame, but does not say anything about equilibria of the whole game. However, our emphasis lies on the stability/fragility of the steady-state equilibrium.
3.3 Banks and Shadow Banks

An important insight from Diamond and Dybvig (1983) and Qi (1994) is that a credible deposit insurance may actually eliminate the adverse equilibrium at no cost. If the insurance is credible, it eliminates the strategic complementarity and is thus never tested. In fact, this is also true in the setup described above. Assume that there is a regulator that can cover the liquidity shortfall in any contingency, including a full-blown bank run. In the context of our model, this amounts to assuming that the regulator has funds of \((1 - \pi) r_1^* - \ell\) at its disposal in any period. Whenever patient investors are guaranteed an amount \(r_1^*\) by the regulator, they do not have an incentive to withdraw early.\(^8\) In contrast, this does not hold in the presence of regulatory arbitrage, as we will show in the following sections.

3.3 Banks and Shadow Banks

We now extend the model described above by three elements: First, we make the assumption that commercial banks are covered by a safety net, but are also subject to regulation and therefore have to bear regulatory costs. Second, there are unregulated shadow banks that compete with banks by also offering maturity transformation services. Investors can choose whether to deposit their funds in a bank or in the shadow bank. Depositing in the shadow bank is associated with some opportunity cost that varies across investors. Third, there is a secondary market in which shadow banks can sell their assets to arbitrageurs. The amount of liquidity in this market is assumed to be exogenous.

In the following, we describe the extended setup in detail and derive the steady-state equilibrium, before analyzing whether the economy is stable or whether it features multiple equilibria and panic-based runs may occur.

Commercial Banking and Regulatory Costs

From now on, we assume that commercial banks are covered by a safety net that is provided by some unspecified regulator, ruling out runs in the commercial banking sector.\(^9\) Because of this safety net, banks are not disciplined by their depositors, such that – in a richer model – moral hazard could arise.

\(^8\)We ignore the possibility for suspension of convertibility. Diamond and Dybvig (1983) already indicate that suspension of convertibility is critical if there is uncertainty about the fraction of early and late consumers. Moreover, as Qi (1994) shows, suspension of convertibility is also ineffective if withdrawing depositors are paid out by new depositors.

\(^9\)The regulator is assumed to have sufficient funds to provide a safety net. Moreover, he can commit to actually applying the safety net in case it is necessary, i.e., in case of a run.
We therefore assume that banks are regulated (e.g., they are subject to a minimum capital requirement). This is assumed to be costly for the bank. In what follows, we will not model the moral hazard explicitly and assume that regulatory costs are exogenous. However, in Appendix 3.A we provide an extension of our model in which we illustrate how moral hazard may arise from the existence of the safety net, and why costly regulation is necessary to prevent moral hazard. The presence of a credible deposit insurance implies that depositors have no incentive to monitor their bank. Because banks have limited liability, this gives bankers an incentive to engage in excessive risk-taking or to invest in assets with private benefits. This in turn calls for regulatory interventions, e.g., in the form of minimal capital requirements which are costly for bank managers.

We assume that banks have to pay a regulatory cost \( \gamma \) per unit invested in the long asset, resulting in a gross return of \( R - \gamma \). We further assume that regulatory costs are not too high, i.e., even after subtracting the regulatory costs, the long asset is still more attractive than storage.

\textbf{Assumption 3.2.} \( R > 1 + \gamma \).

Because of the lower gross return, banks can now only offer a per-period interest rate \( r_b \) such that

\[ \pi r_b + (1 - \pi) r_b^2 = R - \gamma. \]  

(3.12)

Under this regulation, the interest rate on bank deposits is explicitly given by

\[ r_b = \frac{\sqrt{\pi^2 + 4(1 - \pi)(R - \gamma)} - \pi}{2(1 - \pi)}. \]  

(3.13)

The banking sector thus functions like the banking sector in the previous section. The only difference is that banks cannot transfer the gross return \( R \) to investors, but only the return net of regulatory cost, \( R - \gamma \).

\textbf{Shadow Banks}

We now introduce a shadow banking sector that also offers credit, liquidity, and maturity transformation to investors. We start out with very simple structure of the financial system, see Figure 3.1. Shadow banks, like regular banks, invest in long assets and transform these investments into short-term claims. In this section, we do not distinguish between different actors in the shadow banking sector, but assume that there is one representative, vertically integrated institution that we call shadow bank. This shadow bank is essen-
3.3 Banks and Shadow Banks

![Diagram of the Financial System]

**Figure 3.1:** Structure of the Financial System. Shadow banks, like regular banks, invest in long assets and transform these investments into short-term claims. There is a secondary market in which shadow banks can sell their assets to arbitrageurs.

...tially identical to a commercial bank, with the exception that its deposits are not insured, and that it is not subject to the same regulation. While by legal standards shadow banks do not offer demand deposits in reality, they do issue claims that are essentially equivalent to demand deposits, such as equity shares with a stable net assets value (stable NAV). For tractability, we will assume that shadow banks are literally taking demand deposits.

We assume that shadow banks are subject to some shadow-banking cost. We assume that shadow banks face some cost of managing their loan portfolio, of securitizing loans, and of reporting to their investors. Since shadow banking is not completely unregulated, they might also incur some cost of regulation which is substantially smaller than that of regular banks. Finally, shadow banking cost may also include the cost of finding regulatory loopholes that allow to conduct shadow banking in the first place.

Shadow banks invest in a continuum of long assets with idiosyncratic returns $R_i$. As the law of large numbers is assumed to apply, the return of their
Chapter 3 Banks, Shadow Banking, and Fragility

portfolio is $R$. Similar to regulatory costs, shadow banking is assumed to come with a per-unit cost of $\rho$. Therefore, the per-unit return of assets in the shadow banking sector is $R - \rho$. Similar to the regulatory cost $\gamma$, we also assume that the shadow-banking cost is not too high, i.e., even after subtracting the shadow-banking cost, the long asset is still more attractive than storage:

**Assumption 3.3.** $R > 1 + \rho$.

Given this shadow-banking cost, shadow banks offer a per-period interest rate $r_{sb}$ to investors such that

$$\pi r_{sb} + (1 - \pi) r_{sb}^2 = R - \rho,$$

implying a return of

$$r_{sb} = \frac{\sqrt{\pi^2 + 4(1 - \pi)(R - \rho) - \pi}}{2(1 - \pi)}.$$

**Secondary Markets and Arbitrageurs**

There exists a secondary market for the shadow banks’ assets. The potential buyers on this market are arbitrageurs who have an outside option with a risk-free return of $\hat{r}$, i.e., they are willing to buy the shadow banks’ assets at a price that offers them a safe return of at least $\hat{r}$. Arbitrageurs can be thought of as experts (pension funds, hedge funds) that do not necessarily hold such assets in normal times, but purchase them if they are available at some discounts and thus promise gains from arbitrage. Moreover, we assume that arbitrageurs do not want to deposit their funds in shadow banks because their outside option is more attractive:

**Assumption 3.4.** $\hat{r} > r_{sb}$.

This reservation interest rate implies that arbitrageurs’ reservation price for an asset with a return of $R - \rho$ is given by $p_a = (R - \rho)/\hat{r}$.

Assume that there is no market power on any side of the secondary market. Moreover, there is a fixed amount of cash in this market. We assume that arbitrageurs have a total budget of $A$, implying that cash-in-the-market pricing can occur. The equilibrium supply and price of shadow banks’ assets on the secondary market will be derived below.

The idea behind this assumption is that not every individual or institution has the expertise to purchase these financial products. Moreover, the equity
and collateral of these arbitrageurs is limited, so they cannot borrow and invest infinite amounts.\footnote{See Shleifer and Vishny (1997) for a theory on the limits to arbitrage.}

Upon birth, investors can choose whether to deposit their endowment in a regulated bank or in a shadow bank. Depositing at shadow bank comes at some opportunity cost. We assume that investors are initially located at a regulated bank. Switching to a shadow bank comes at a cost of $s_i$, where $s_i$ is independently and identically distributed according to the distribution function $G$. We assume that $G$ is a continuous function that is strictly increasing on its support $R^+$, and that $G(0) = 0$. The switching cost is assumed to enter into the investors' utility additively separable from the consumption utility.

This switching cost should not be taken literally. One can think of these costs as monitoring or screening costs for investors that become necessary when choosing a shadow bank (e.g., an MMF) as these are not protected by a deposit insurance (see Appendix 3.A for more details). For simplicity, we have assumed that all depositors have the same size. However, we could alternatively come up with a model where investors have different endowments (see Appendix 3.B). It is very plausible that the ratio of switching costs to the endowment is lower for larger investors (e.g., for corporations that need to store liquid funds of several millions for a few days). Another interpretation is the forgone service benefits that depositors lose when leaving commercial banks, such as payment services and ATMs.

**Investors’ Behavior**

Given the interest rates of commercial banks, $r_b$, of shadow banks, $r_{sb}$, and given the switching cost distribution $G$, we can pin down the size of the shadow banking sector.

**Lemma 3.4.** Assume that banks offer an interest rate $r_b$ and shadow banks offer an interest rate of $r_{sb}$, as specified above. Then there exists a unique threshold $s^*$ such that an investor switches to a shadow bank if and only if $s_i \leq s^*$. The mass of investors depositing in the shadow banking sector is given by $G(s^*)$. It holds that $s^* = f(\gamma, \rho)$, where $f_\gamma > 0$ and $f_\rho < 0$.

**Proof.** Take $r_b$ and $r_{sb}$ as described above. We know $r_b$ decreases in $\gamma$, and $r_{sb}$ decreases $\rho$. Staying at a commercial bank provides an investor with an expected consumption utility of $EU_b = \pi u(r_b) + (1 - \pi)u(r_b^2)$. Switching to a shadow bank is associated with an expected consumption utility of $EU_{sb} = \pi u(r_{sb}) + (1 - \pi)u(r_{sb}^2)$. Observe that $EU_b$ decreases in $\gamma$ and $EU_{sb}$ decreases in $\rho$. 
Chapter 3 Banks, Shadow Banking, and Fragility

An investor with switching cost \( s_i \) switches to the shadow banking sector if \( EU_b < EU_{sb} - s_i \). This implies that all investors with \( s_i \leq EU_{sb} - EU_b \) switch to shadow banks. We define \( s^* \equiv f(\gamma, \rho) = EU_{sb}(\rho) - EU_b(\gamma) \). A mass \( G(s^*) \) of each generation’s investors switches to shadow banks, and a mass \( 1 - G(s^*) \) stays at commercial banks. Because \( u \) is twice continuously differentiable, it holds that \( \partial EU_b / \partial \gamma < 0 \) and \( \partial EU_{sb} / \partial \rho > 0 \). Thus, \( f \) is a continuously differentiable function with \( f_\gamma > 0 \) and \( f_\rho < 0 \).

An investor with \( s_i = s^* \) is indifferent between depositing at a bank or a shadow bank. All investors with lower switching costs choose a shadow; their mass is given by \( G(s^*) \). The size of the shadow banking sector increases in the regulatory cost \( \gamma \) and decreases in the shadow-banking cost \( \rho \). For the case of logarithmic utility, the switching point is given by \( s^* = (2 - \pi)\gamma - \rho \).

We are now equipped to characterize the economy’s steady state equilibrium:

**Proposition 3.3.** In the second-best steady-state equilibrium, the intergenerational banking sector collects an amount of deposits \( D_b = 1 - G(s^*) \) in each period, and invests all funds in the long-asset, \( I_b = 1 - G(s^*) \). They offer demand-deposit contracts with a per-period interest rate of

\[
r_b = \frac{\sqrt{\pi^2 + 4(1 - \pi)(R - \gamma)} - \pi}{2(1 - \pi)}.
\] (3.16)

Shadow banks collect an amount of deposits \( D_{sb} = G(s^*) \) and exclusively invest in ABS, \( I_{sb} = 1 - G(s^*) \). They offer a demand-deposit contracts with a per-period interest rate of

\[
r_{sb} = \frac{\sqrt{\pi^2 + 4(1 - \pi)(R - \rho)} - \pi}{2(1 - \pi)}.
\] (3.17)

It holds that \( s^* = f(\gamma, \rho) \), where \( f_\gamma > 0 \) and \( f_\rho < 0 \). There are no assets traded in the secondary market.

Proposition 3.3 described the steady state in which regulated commercial banks and shadow banks coexist. The interest rates are given by \( r_b \) and \( r_{sb} \) and depend on \( \gamma \) and \( \rho \), which determines the size of the shadow banking sector as described by Lemma 3.4. It is important to notice that, in this steady-state equilibrium, no assets are being sold to arbitrageurs on the secondary market, as there are no gains from trade.
3.3 Banks and Shadow Banks

3.3.1 Fragility of Shadow Banks

As in the previous section, we now study the stability of shadow banks. We analyze the subgame starting in period $t$ under the assumption that behavior until date $t-1$ is as in the steady-state equilibrium specified in Proposition 3.3. We derive the condition under which shadow banks might experience a run by investors, i.e., the condition for the existence of a run equilibrium in the period-$t$ subgame.

Because deposits in the shadow banking sector are not insured, a run on shadow banks is not excluded per se. However, as will become clear below, runs only occur if the shadow banking sector is too large. Generally, there are two types of runs that could potentially take place in the adverse equilibrium of the $t=1$ subgame.

In our model, a run is the event where all old investors withdraw their funds from shadow banks, and new investors do not deposit any new funds. Whether a run on shadow banks constitutes an equilibrium depends on whether shadow banks can raise enough liquidity in the secondary market to serve all their obligations.

**Lemma 3.5.** Assume that the economy is in the second-best steady state. In case of a run on shadow banks, their liquidity shortfall is given by $G(s^*)(1 - \pi)r_{sb}$.

**Proof.** See proof of Lemma 3.3. $(1 - \pi)r_{sb}$ is the relative liquidity shortfall, the amount of missing liquidity per unit of investment in the shadow banking sector.

In order to cover this shortfall, shadow banks can either sell their investment of period $t-1$ to the arbitrageurs, or they can liquidate these assets. We assume that liquidation of assets will never be enough to cover the shortfall:

**Assumption 3.5.** $\ell < (1 - \pi)r_{sb}$. 

This assumption implies that the liquidation value is sufficiently smaller than the long-run net return $R - \rho$. Similar to Assumption 3.1, also in the case of shadow banks, which pay a per-period interest of $r_{sb}$, liquidation cannot be used to serve investors.

In the last section, the assumption of a low liquidation value implied that (in the absence of a deposit insurance) a run on banks can always occur. The presence of arbitrageurs who are willing to buy shadow banks’ assets in

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11 Liquidation is not essential to our model. It also goes through in case liquidation is not possible; we can just set $\ell = 0$. 

87
a secondary market implies that the threat of a run is not necessarily omnipresent. We will show that the arbitrageurs’ valuation of these assets and, more importantly, the size of their budget determines whether run equilibria exist.

**Lemma 3.6.** Assume that the economy is in the second-best steady state. A run on shadow banks constitutes an equilibrium of the period-$t$ subgame if

$$
\frac{R - \rho}{\hat{r}} < (1 - \pi)r_{sb}.
$$

(3.18)

If the arbitrageurs’ outside option is very profitable, they are only willing to pay a low price. If this price is lower than the relative liquidity shortfall, a run is always self-fulfilling. Shadow banks have to sell their assets, and the resulting revenue does not suffice to serve all investors because the arbitrageurs’ valuation and thus the market price are too low.

From now on, we will assume that the arbitrageurs’ outside option is not too profitable, i.e., their reservation price is sufficiently large:

**Assumption 3.6.** $(R - \rho)/\hat{r} \geq (1 - \pi)r_{sb}$.

Observe that, in case of a run shadow banks’ supply is partially inelastic as they have to cover their complete liquidity shortfall. There are two cases to be considered: In the first case, the arbitrageurs’ funds are sufficient to purchase all funds the shadow banks sell at face value, while in the second case, the arbitrageurs’ budget is not sufficient and the price is determined by cash-in-the-market pricing. Runs on shadow banks constitutes an equilibrium only in this second case.

**Proposition 3.4.** Assume that the economy is in the second-best steady state. A run on shadow banks constitutes an equilibrium of the period-$t$ subgame if and only if

$$
G(s^*) > \frac{A}{(1 - \pi)r_{sb}} \equiv \xi.
$$

(3.19)

**Proof of Proposition 3.4.** Assume that investors collectively withdraw funds from shadow banks and deposit no new funds in period $t$. It will be optimal for a single investor also to withdraw if the shadow banks become illiquid and insolvent in $t$.

Recall from Lemma 3.5 that the liquidity shortfall of shadow banks in case of a run is given by $LS = G(s^*)(1 - \pi)r_{sb}$. Liquidating all assets would yield
3.3 Banks and Shadow Banks

$G(s^*)\ell$. According to Assumption 3.5, this will always be less than the liquidity shortfall. Shadow banks will never be able to cover the liquidity shortfall by liquidating their assets.

The relevant question is whether shadow banks can raise sufficient funds by selling their assets to arbitrageurs. There are two conditions which are jointly necessary and sufficient: First, the funds of arbitrageurs have to exceed the liquidity shortfall, and second, the arbitrageurs’ valuation of shadow banks’ assets has to exceed the liquidity shortfall. The second condition is met by assumption.

Regarding the first condition, there are two cases to be considered: In the first case, the arbitrageurs’ funds exceed the liquidity shortfall. Because the shortfall by assumption is lower than the valuation of arbitrageurs, their funds are sufficient to purchase all funds the shadow banks sell at the reservation price. In the second case, the arbitrageurs’ budget is not sufficient, and the price is determined by cash-in-the-market pricing.

The first case is given by $A \geq (1 - \pi)r_{sb}G(s^*)$. All assets held by shadow banks can be sold at the arbitrageurs’ reservation price $p_a = (R - \rho)/\hat{r}$. Because the arbitrageurs’ valuation of shadow banks’ assets as well as the amount of cash in the market exceeds the shadow banks’ potential liquidity needs, all investors could be served in case of a run. It thus is a strictly dominant strategy for each old patient investor not to withdraw, and for new investors to deposit new funds, and a run does not constitute an equilibrium.

The second case is given by $A < (1 - \pi)r_{sb}G(s^*)$. In this case, shadow banks cannot sell all their assets at the arbitrageurs’ reservation price. If all existing investors withdraw and no new investors deposit, shadow banks cannot raise the required funds to fulfill their obligations by selling their assets because the amount of assets on the secondary market exceeds the budget of arbitrageurs. The asset price drops below the reservation price and shadow banks are forced to sell their entire portfolio. Still, shadow banks can only raise a total amount $A$ of liquidity, which is insufficient to serve withdrawing investors. It follows that, in this case, it is optimal for old and new investors to withdraw and not to deposit, respectively. A run thus constitutes an equilibrium if and only if the stated condition is satisfied.

The key mechanism giving rise to multiple equilibria is cash-in-the-market pricing (see, e.g., Allen and Gale, 1994) in the secondary market for long assets. It results from limited arbitrage capital and is related to the notion of limits to arbitrage (see, e.g., Shleifer and Vishny (1997)). The fact that there are not enough arbitrageurs (and that these arbitrageurs cannot raise enough funds) to purchase all assets of the shadow banks can induce the price
Figure 3.2: Fire-sale Prices. This graph depicts the potential fire-sale prices of shadow banks’ securities. Whenever $G(s^*) \leq \xi = A/(1 - \pi)r_{sb}$, arbitrageurs’ funds suffice to purchase all assets of shadow banks at face value. In the unique equilibrium of the period-$t$ subgame, there are no panic-based withdrawals from shadow banks. In turn, if $G(s^*) > \xi$, the funds of arbitrageurs are insufficient, and the period-$t$ subgame has multiple equilibria. If all investors withdraw from shadow banks, the price in the secondary market drops to the red line.

If the assets to fall short of $p_a$. This implies that shadow banks may in fact be unable to serve their obligations once they sell all their long-term securities prematurely. This, in turn, makes it optimal for investors to run on shadow banks once all other investors run.

In order to illustrate the role of limited availability of arbitrage capital we examine the hypothetical fire-sale price. Cash-in-the-market pricing describes a situation where the buyers’ budget constraint is binding and the supply is fixed. The price adjusts such that demand balances the fixed supply. If the arbitrageurs’ budget is a binding resource constraint, the price $p$ is such that

$$pG(s^*) = A. \quad (3.20)$$

The fire-sale price is a function of the amount of assets that are on the market in case of a run on the shadow banking sector, which is given by the size of the shadow banking sector $G(s^*)$. The price is given by
3.3 Banks and Shadow Banks

The equilibrium fire-sale prices as a function of the size of the shadow banking sector is illustrated in Figure 3.2.

\[
p(s^*) = \begin{cases} 
  \frac{(R - \rho)}{\hat{r}} & \text{if } G(s^*) \leq \xi, \\
  A/G(s^*) & \text{if } G(s^*) \in (\xi, A/\ell], \\
  \ell & \text{if } G(s^*) > A/\ell. 
\end{cases} \tag{3.21}
\]

The equilibrium fire-sale prices as a function of the size of the shadow banking sector is illustrated in Figure 3.2.

\[\gamma \quad R - 1\]

\[\begin{array}{c}
\text{Fragility} \\
G(s^*) > \xi \\
\text{Stable} \\
\text{Shadow Banking} \\
0 < G(s^*) < \xi \\
\text{No Shadow Banking} \\
G(s^*) = 0 \\
\end{array}\]

\[\begin{array}{c}
0 \\
0 \\
R - 1 \\
\rho \\
\end{array}\]

**Figure 3.3:** Existence of Equilibria. This figure visualizes the equilibrium characteristics of the financial system for different values of $\gamma$ and $\rho$. For $\gamma < \rho$, shadow banking is not made use of in equilibrium, as it is dominated by commercial banking. If $\gamma > \rho$, the shadow banking sector has positive size. As long as the difference $\gamma - \rho$ is small, shadow banking is stable. If the difference increases, the size of the shadow banking sector also increases and finally introduces fragility into the financial system.

Whether the period $t$ subgame has multiple equilibria ultimately depends on the parameters $\rho$ and $\gamma$, as they determine the size of the shadow banking sector. This is depicted in Figure 3.3. Whenever the regulatory costs $\gamma$ exceed the shadow-banking costs $\rho$ (i.e., above the 45 degree line), the shadow banking sector has positive size in equilibrium, i.e. $G(s^*) > 0$. However, as long as the shadow banking sector is small relative to the arbitrageurs’ budget, it is stable. Only when regulatory costs $\gamma$ are sufficiently larger than shadow-banking cost $\rho$ does the size $G(s^*)$ of the shadow banking sector exceed the
critical threshold $\xi$, and shadow banking becomes fragile.

### 3.3.2 Liquidity Guarantees

So far, there has been no connection between the regulated commercial banking sector and the shadow banking sector; both sectors compete for the investors’ funds. We now assume that commercial banks themselves actively engage in shadow banking: They engage in shadow banking through off-balance sheet subsidiaries, i.e., they operate shadow banks as documented in Acharya et al. (2013). Our model provides a positive analysis, the fact that banks engage in shadow banking themselves does not result from optimal behavior in our setup. We assume that commercial banks explicitly or implicitly provide their shadow banks with liquidity guarantees. They may have strong incentives to support their conduits in case of distress, e.g., in order to protect their reputation, see Segura (2014). Moreover, we assume that commercial banks can sell their assets on the same secondary market as shadow banks.\(^\text{12}\)

As above, we assume that the commercial banks’ demand-deposit liabilities are covered by a credible safety net. This safety net being credible implies that commercial banks do not experience runs by investors. Patient investors who are located at a commercial bank will thus never withdraw their funds early.

Liquidity guarantees imply that in case of a run on shadow banks, commercial banks supply them with liquid funds. This increases the critical size up to which the shadow banking sector is stable. However, this comes with an unfavorable side effect: Once this critical size is exceeded and shadow banks experience a run, the crisis spreads to the commercial banking sector and makes the safety net costly.

**Proposition 3.5.** Assume that the economy is in the second-best steady state described in Proposition 3.3 and all shadow banks are granted liquidity guarantees by commercial banks. A run of investors on shadow banks constitutes an equilibrium of the subgame starting in period $t$ if and only if

$$G(s^*) > \max[A, \ell] + 1 - \pi r_{sb} + 1 = \vartheta,$$

where $s^* = f(\gamma, \rho)$. It holds that $\vartheta > \xi$.

\(^{12}\text{It is not straightforward that banks can sell their loans on the ABS market. In case of a crisis, however, banks might try to securitize their loan portfolio in order to sell it.}\)
3.3 Banks and Shadow Banks

Proof. In case of a run, the shadow banks’ need for liquidity is given as above by

\[ G(s^*)(1 - \pi) r_{sb}. \]

Banks can sell their loans on the same secondary market in case of a crisis. Still, the total endowment of arbitrageurs in this market is given by \( A \). Therefore, either banks and shadow banks sell their assets in the secondary market, or both types of institutions liquidate their assets. They jointly still only raise an amount \( A \) from selling long-term securities on the secondary market or \( \ell \) units from liquidating all long assets. The maximum amount they can raise is thus \( \max[A, \ell] \). On top, commercial banks also have an additional amount \( 1 - G(s^*) \) of liquid funds available since new investors still deposit their endowment at commercial banks because of the safety net for commercial banks.

The liquidity guarantees by commercial banks can satisfy the shadow banks’ liquidity needs in case of a run if

\[ \max[A, \ell] + (1 - G(s^*)) \geq G(s^*)(1 - \pi)r_{sb}, \] (3.23)

which is equivalent to

\[ G(s^*) \leq \frac{\max[A, \ell] + 1}{(1 - \pi)r_{sb} + 1} = \vartheta. \] (3.24)

If \( G(s^*) \leq \vartheta \), the liquidity guarantees suffice to satisfy the liquidity needs in case of a run, so a run does not constitute an equilibrium. If \( G(s^*) > \vartheta \), the liquidity guarantees do suffice to satisfy the liquidity needs in case of a run, and a run does constitute an equilibrium.

If commercial banks themselves operate shadow banks and provide them with liquidity guarantees, the parameter space in which shadow banking is stable is enlarged compared to a situation without liquidity guarantees, i.e., the critical threshold for the size of the shadow banking sector \( \vartheta \) is now larger than \( \xi \), the threshold in the absence of liquidity guarantees. This shift is also depicted in Figure 3.4. The reason for this result is that banks have additional liquid funds, even in case of a crisis: Because of the deposit insurance, they always receive funds from new depositors, and their patient depositors never have an incentive to withdraw early.

In traditional banking models, policy tools like a deposit insurance eliminate self-fulfilling adverse equilibria at no cost. This is not necessarily true in our model: once the shadow banking sector exceeds the size \( \vartheta \), a run in the
shadow banking sector constitutes an equilibrium despite the safety net for commercial banks, and despite the liquidity guarantees of banks. Shadow banks – by circumventing the existing regulation – place themselves outside the safety net and are thus prone to runs. If the regulated commercial banks offer liquidity guarantees, a crisis in the shadow banking sector also spreads to the regulated banking sector. Ultimately, self-fulfilling adverse equilibria are not necessarily eliminated by the safety net and may become costly.

**Corollary 3.1.** Assume that \( G(s^*) > \vartheta \), and assume banks provide liquidity guarantees to shadow banks. In case of a run in the shadow banking sector, the safety net for regulated commercial banks is tested and the regulator must inject an amount

\[
G(s^*)(1 - \pi)r_{sb} - \max[A, \ell] > 0. \tag{3.25}
\]

**Proof.** This corollary follows directly from the proof of Proposition 3.5. \( G(s^*)(1 - \pi)r_{sb} \) denotes the liquidity need of shadow banks in case of a run, and \( \max[A, \ell] \) denotes the amount that shadow banks can raise by selling...
or liquidating their assets. While commercial banks may cover part of the shadow banks’ liquidity short-fall by fulfilling their liquidity guarantees, this amount also has to be compensated by the regulator because otherwise banks cannot serve their depositors in the future.

If the regulated commercial banking and the shadow banking sector are intertwined, a crisis may not be limited to the shadow banking sector, but also spread to the commercial banks, thus testing the safety net. Ultimately, the regulator has to step in and cover the commercial banks’ liabilities. Therefore, the model challenges the view that policy measures like a deposit insurance necessarily are an efficient mechanism for preventing self-fulfilling crises. Historically, safety nets such as a deposit insurance schemes were perceived as an effective measure to prevent panic-based banking crises. The view is supported by traditional banking models of maturity transformation such as Diamond and Dybvig (1983) and Qi (1994). In the classic models of self-fulfilling bank runs, a credible deposit insurance can break the strategic complementarity in the withdrawal decision of bank customers at no cost. We show that this may not be the case when regulatory arbitrage is possible and regulated and unregulated banking activities are intertwined.

3.4 MMFs, ABCP Conduits, and SPV

In the last section we presented a model in which the shadow banking sector consisted of one vertically integrated, representative shadow bank. We will now consider a shadow banking sector that offers credit, liquidity, and maturity transformation to investors through vertically separated institutions. This structure of the shadow banking sector (compare Figure 3.5) is exogenous in our model. It is empirically motivated; we selectively follow and simplify the descriptions by Pozsar et al. (2013). Altogether, the actors of the shadow banking system invest in long assets and transform these investments into short-term claims. However, we distinguish between different actors in the shadow banking sector.

In our setup, shadow banking consists of special purpose vehicles (SPVs), ABCP conduits, and money market mutual funds (MMFs). Investment banks securitize assets such as loans (i.e., the long assets in our model) via SPVs, thereby transforming them into asset-backed securities (ABS). Through diversified investments, they eliminate the idiosyncratic risk of loans and conduct risk transformation. Note that SPVs typically do not lend to firms or consumers directly, but rather purchase loans from loan originators such as
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Investors / Ultimate Lenders

- Insured Deposits
- Commercial Banks
  - Loans
  - Secondary Market for Securities
    - Liquidity Guarantees
    - SPV
    - ABCP conduit
      - ABS
      - ABCP
      - MMF
    - Quasi-deposits

Projects / Ultimate Borrowers

- Loans

Service division in the shadow banking sector

- Liquidity transformation
- Maturity transformation
- Risk transformation (securitization)
- Loan origination by commercial banks or mortgage brokers

**Figure 3.5**: Detailed Structure of Shadow Banking. The structure of the shadow banking sector is mostly exogenous in our model; we selectively follow and simplify the descriptions by Pozsar et al. (2013). Shadow banking consists of special purpose vehicle (SPVs), ABCP conduits such as structured investment vehicles (SIVs), and money market mutual funds (MMFs). SPVs transform assets into asset-backed securities (ABS) in order to make them tradable, i.e., they conduct risk and liquidity transformation (securitization). ABS have a long maturity; they are bought by ABCP conduits. ABCP conduits, in turn, use short-term debt to finance these long-term assets; they sell asset-backed commercial papers (ABCP) to MMFs, i.e., they conduct maturity transformation. MMFs are the door to the shadow banking sector: They offer deposit-like claims to investors, such as shares with a stable net assets value (NAV), thus conducting another form of liquidity transformation. Finally, there is a secondary market in which ABS can be sold to arbitrageurs.
mortgage agencies or commercial banks.

SPVs buy long assets with idiosyncratic returns, transform them into ABS, and sell them to ABCP conduits. The empirically motivated narrative is that investment banks use SPVs to purchase loans from loan originators such as mortgage brokers or commercial banks. These SPVs bundle the claims into securitized loans (ABS), successfully diversifying the idiosyncratic return risk. Securitization makes the long assets tradable by eliminating the adverse selection problem that is associated with idiosyncratic return risk. For simplicity, we assume that the shadow-banking cost $\rho$ occurs at this stage. Thus, the ABS that are sold by SPVs have a return of $R - \rho$.

ABCP conduits purchase these securitized assets with long maturities (ABS) and finance their business by issuing short-term claims that they sell to MMFs. To put it more technically, ABCP conduits (such as structured investment vehicles (SIVs)) purchase ABS and finance themselves through asset-backed commercial papers (ABCPs), which they sell to MMFs. ABCP conduits hence conduct maturity transformation. Maturity transformation is the central element and the key service of banking in our model, and it is the main source of fragility.

For investors, MMFs are the door to the shadow banking sector as they transform short-term debt (such as ABCP) into claims that are essentially equivalent to demand deposits, such as equity shares with a stable net assets value (stable NAV). MMFs thus conduct liquidity transformation. Again, we will assume that MMFs are literally taking demand deposits.

At the heart of the shadow banking sector is the maturity transformation by ABCP conduits. ABCP conduits purchase securitized assets (ABS) from investment banks’ SPVs. As described above, these assets have a return of $R - \rho$ and a maturity of two periods. ABCP conduits can finance themselves by borrowing from MMFs via ABCPs. Moreover, they can also sell ABS to arbitrageurs in the secondary market as specified above. ABCP conduits may be legally independent entities, but they are largely founded and run by regulated banks in order to engage in unregulated off-balance sheet maturity transformation (Acharya et al., 2013). Because such maturity transformation is very fragile, banks stabilize their conduits by providing them with liquidity guarantees. In this section, we assume that ABCP conduits are fully insured through such liquidity guarantees.

Investors can access the services of the shadow banking sector via MMFs.

\[ \text{13} \text{ ABCP conduits also use other securities to finance their activities, such as medium term notes. For simplicity, we focus on ABCPs.} \]
which are assumed to intermediate between investors and ABCP conduits.\footnote{This is equivalent to assuming that investors face large transaction costs or do not have the expertise to buy ABCP directly.} MMFs offer demand-deposit contracts to investors while purchasing short-term claims on ABCP conduits.\footnote{A MMF typically sells shares to investors, and the fund’s sponsor guarantees a stable NAV, i.e., it guarantees to buy back shares at a price of one at any time. As mentioned above, the stable NAV implies that an MMF share is a claim that is equivalent to a demand-deposit contract.} MMFs offer a per-period interest rate $r_{mmf}$ to investors and purchase ABCP (short-term debt) with a per-period return $r_{abcp}$. Competition among MMFs and among ABCP conduits implies that $r_{mmf} = r_{abcp} = r_{sb}$. Investors face the same situation as described in the last section, and the size of the shadow banking sector is again given by $G(s^*)$.

In the following, we will analyze the fragility of the different institutions within the shadow banking sector. First, we will assume that MMFs have perfect support by a sponsor and analyze under which condition a run of MMFs on ABCP conduits constitutes an equilibrium. Second, we will relax the assumption of sponsor support and analyze under which conditions investors might run on MMFs.

### 3.4.1 Runs on ABCP Conduits

As in the previous section, we now study the stability of the shadow banking sector. We analyze the subgame starting in period $t$ under the assumption that behavior until date $t - 1$ is as in the steady-state equilibrium specified in Proposition 3.3. We again analyze the case in which banks grant liquidity guarantees to the shadow banking sector, in this case to ABCP conduits. Moreover, we assume for the moment that MMFs receive absolutely credible sponsor support, implying that investors will never run on MMFs. We derive the condition under which ABCP conduits might experience a run by MMFs, i.e., the condition for the existence of a run equilibrium in the period-$t$ subgame.

**Proposition 3.6.** Assume that the economy is in the second-best steady state described in Proposition 3.3 and all ABCP conduits are granted liquidity guarantees by commercial banks. A run of MMFs on ABCP conduits constitutes an equilibrium of the subgame starting in period $t$ if and only if

$$G(s^*) > \max\{A, \ell\} + 1 + (1 - \pi) r_{sb} + 1 \equiv \vartheta,$$

\label{eq:3.26}
where \( s^* = f(\gamma, \rho) \). It holds that \( \vartheta > \xi \).

**Proof.** See the proof of Proposition 3.5. MMFs have the same type of claims that investors had in the last section, and the ABCP conduits have the same liabilities and assets, i.e., the same maturity structure, that shadow banks had before. \( \square \)

Proposition 3.6 is the equivalent to Proposition 3.4. If commercial banks engage in off-balance sheet activities, i.e., if banks themselves operate ABCP conduits and provide them with liquidity guarantees, the critical threshold for the size of the shadow banking sector is \( \vartheta \). Again, deposit insurance does not eliminate self-fulfilling adverse equilibria at no cost. If the shadow banking sector exceeds the size \( \vartheta \), a run in the shadow banking sector constitutes an equilibrium despite the safety net for commercial banks, and despite the liquidity guarantees of banks. By running off-balance sheet conduits and providing them with liquidity guarantees, banks circumvent the existing regulation and abuse their safety net.

The equivalent to Corollary 3.1 also holds in this setup:

**Corollary 3.2.** Assume that \( G(s^*) > \vartheta \), and assume banks provide liquidity guarantees to ABCP conduits. In case of a run in the shadow banking sector, the safety net for regulated commercial banks is tested and the regulator must inject an amount

\[
G(s^*)(1 - \pi)r_{sb} - \max[A, \ell] > 0.
\] (3.27)

Because the liquidity guarantees that commercial banks provide to ABCP conduits induces contagion of the regulated banking sector, a crisis becomes costly for the regulator.

### 3.4.2 Runs on MMFs

In the previous sections, we ruled out runs on MMFs by assuming that they have credible support by a sponsor. Credible sponsor support means that even if all investors withdraw their funds from an MMF, the sponsor is able to provide sufficient liquidity to the MMF such that it can serve all investors. Recall that we use the narrative that MMFs are literally offering demand-deposit contracts. In practice, an MMF issues equity shares, and its sponsor guarantees stable NAV for theses shares, i.e., it promises to buy these shares at face value in case of liquidity problems.
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We now relax the assumption that the guarantee is always credible. We explicitly model the credibility of the guarantee by assuming that the sponsors have $m$ units of liquidity per unit of investment in the MMF that they can provide in case of a crisis. Moreover, we keep the assumption of existing liquidity guarantees. We show that providing $mG(s^*)$ units only credibly prevents a run on MMFs if this amount is sufficient to fill the liquidity shortfall in case of a run of investors on MMFs, which in turn triggers a run of MMFs on ABCP conduits.

**Proposition 3.7.** Assume that the economy is in the second-best steady state equilibrium described in Proposition 3.3. Assume further that all ABCP conduits are granted liquidity guarantees by commercial banks, and that per unit of investment, MMFs receive $m$ units of liquidity support from their sponsor. A run of investors on MMFs may occur whenever

$$G(s^*) > \frac{\max[A, \ell] + 1}{(1 - \pi)r_{sb} + 1 - m} = \nu > \vartheta.$$  

(3.28)

If the $\nu > G(s^*) > \vartheta$, investors never run on MMFs. However, MMFs might run on ABCP conduits, which then draw on the sponsor support.

**Proof.** Observe that once an MMF needs liquid funds because investors withdraw unexpectedly, it will stop rolling over ABCP. Now, whenever the shadow banking sector exceeds the critical threshold $\vartheta$, a run of MMFs on ABCP conduits is self-fulfilling, as ABCP conduits will make losses only in this case. This is therefore a necessary condition for a run by investors on MMFs. If it is not satisfied, MMFs are always able to fulfill their obligations by stopping the rollover of ABCP, making it a weakly dominant strategy for patient investors not to withdraw early. However, it is not a sufficient condition.

Observe that the resulting liquidity shortfall for the MMFs is given by

$$[(1 - \pi)r_{sb} + 1]G(s^*) - \max[A, \ell] - 1.$$  

(3.29)

Therefore, a run of investors on MMFs constitutes an equilibrium only if

$$mG(s^*) < [(1 - \pi)r_{sb} + 1]G(s^*) - \max[A, \ell] - 1.$$  

(3.30)

The result builds on the fact that sponsor support is like a liquidity backstop. If there is a run by MMFs on ABCP conduits, MMFs will make losses. This additionally triggers a run of investors on MMFs if the sponsor is not
able to cover these losses. Again, losses depend on the fire-sale price. The fire-sale price, in turn, depends on the amount of assets sold in case of a run by MMFs on ABCP conduits, which is determined by the size of the shadow banking sector. If the shadow banking sector is so large that runs by MMFs on ABCP conduits occur, but not so large that losses cannot be covered by the sponsors, investors do not run. This is the case for \( \nu > G(s^*) > \vartheta \). In turn, if the shadow banking sector size exceeds \( \nu \), a run by MMFs on ABCP conduits will always be accompanied by a run of investors on MMFs because sponsor support is insufficient to cover losses in case of a run.

### 3.5 Conclusion

The main contribution of this chapter is to show how regulatory arbitrage-induced shadow banking can sow the seeds of a financial crisis. We illustrate how shadow banking activities undermine the effectiveness of a safety net that is installed to prevent a financial crisis. Moreover, we show how regulatory arbitrage may even induce the safety net to be costly for the regulator (or taxpayer) in case of a crisis.

Our model features multiple equilibria. The key mechanism giving rise to multiple equilibria is cash-in-the-market pricing in the secondary market for shadow banks’ long-term securities which results from limited availability of arbitrage capital. Cash-in-the-market pricing leads to depressed fire-sale prices if there are too many assets on the market. The amount of assets is thus crucial in determining whether shadow banking is fragile or not. In turn, the amount of assets sold in case of a run on shadow banks is determined by the size of the shadow banking sector. Therefore, multiple equilibria only exist if the shadow banking sector is large.

As indicated earlier, our model lacks certain features that might be considered relevant that should be considered in future research. First, a financial crisis is a purely self-fulfilling phenomena in our model, while fundamental values do play a role in reality. However, this chapter is an attempt to demonstrate how the structure of the financial system can set the stage for severe fragility: Because of maturity mismatch in a large shadow banking sector without access to a safety net, small shocks can lead to large repercussions. Second, by focusing on regulatory arbitrage as the sole reason for the existence of a shadow banking, we ignore potential positive welfare effects of shadow banking and securitization, such as catering to the demand for liquid assets or improving risk allocation. However, the fragility that arises in the context of regulatory arbitrage arguably also exists for other types of banking activities.
activities outside the regulatory perimeter.

Despite the simple nature of our model, we can still draw some conclusions. Our key finding is that the size of the shadow banking sector plays a crucial role for the stability of the financial system. However, the actual quantities of shadow banking activities are not completely clear to academics and regulators. Therefore, a first important implication of our model is that the size of the shadow banking sector (or, more precisely, the magnitude of maturity mismatch in the shadow banking sector) and the interconnectedness of banking and shadow banking should be variables that regulating authorities keep track of. The *Global Shadow Banking Monitoring Report 2013*\(^{16}\) displays a very valuable step in the right direction. Still, the report calls for devoting even more resources to tackling concrete data issues. Our model can be taken as an argument in support of this view.

We make a strong case for why regulatory arbitrage poses a severe risk to financial stability. However, it would be wrong to conclude that regulation should thus be reduced.\(^{17}\) One needs to keep in mind that – under the presumption that regulation is in place for a good reason – it is not regulation itself that poses a problem, but the circumvention of regulation. If the regulator insures depositors in order to eliminate self-fulfilling runs of depositors, she may need to impose some regulation on banks in order to prevent moral hazard. Regulatory arbitrage may eventually reintroduce the possibility of runs. However, this does not alter the fact that it is a good idea to aim at preventing runs in the first place.

Under the premise that regulatory arbitrage cannot be prevented at all, our model indicates that financial stability may not always be reached by providing a safety net and regulating banks. One may consider a richer set of policy interventions that go beyond safety nets and regulation. E.g., the government or the central bank may have the ability to intervene on the secondary market in case of a crisis. However, such interventions are likely to give rise to different problems as they may change incentives ex-ante, e.g., they may give rise to excessive collective maturity mismatch as in Farhi and Tirole (2012). A richer model than ours would be needed to analyze such effects consistently.

In turn, under the premise that regulatory arbitrage can be prevented or can be made more difficult, we argue that it should be prevented or at least reduced. Given that regulatory arbitrage can be very costly in terms of creat-

\(^{16}\)FSB (2013).

\(^{17}\)There are also argument against strict regulation, building on reputation concerns or charter value effects, see, e.g., Ordoñez (2013).
ing systemic risk, it should be made very costly to those who are conducting it. While this may sound self-evident at first, a glimpse at the history of bank regulation and its loopholes should be a reminder that regulatory arbitrage and the associated risks have not always been a major concern.\footnote{See, e.g., Jones (2000) for an early analysis of how the Basel requirements were circumvented.}

The regulatory response to the 2007-09 financial crisis has tried to deal with many of the aspects in which shadow banking has contributed to the crisis by circumventing regulation. However, it is less clear what arbitrage of current regulation may look like, particularly because shadow baking activities are of great and still growing importance, especially in emerging countries such as China (Awrey, 2015; Dang et al., 2014). We argue that in a dynamic world with constant financial innovation, regulatory arbitrage is not adequately dealt with by focusing on regulatory loopholes of the past only. In contrast, prudential supervision calls for strong awareness and constant monitoring of newly developing forms of regulatory arbitrage.
APPENDIX 3.A  MORAL HAZARD AND REGULATORY COSTS

In the model described above, regulatory costs enter as an exogenous parameter $\gamma$. In this section, we extend the model by a few aspects to provide a foundation for this assumption. We show that once a bank is covered by a safety net that is in place to prevent self-fulfilling runs (e.g., a deposit insurance), the bank will not be disciplined by investors and will have incentives to invest in a riskier project with private benefits. The regulator thus needs to impose a minimum capital requirement in order to ensure that the bank behaves diligently. As raising capital is assumed to be costly for the bank (e.g., due to dilution costs), the overall return a bank will make will be reduced by the regulation. We recommend reading this part only after having finished reading Section 3.4.

Let us assume that commercial banks as well as shadow banks are run by owner-managers. Assume that bank managers receive some constant private benefit $w$ (per unit of deposits) as long as their (shadow) bank is operating. If the bank goes bankrupt, the manager loses his bank and his income. The manager discounts the future at rate $\delta < 1$, his discounted income over his (infinite) lifetime is given by $w/(1-\delta)$. Now assume that next to the short asset and the long asset described in the beginning of Section 3.2, bank managers also have access to an additional production technology that we call “private asset”. This private asset is similar to the long asset, but it has the property that, with some probability $\alpha$, the asset defaults completely. In addition, this asset produces some private benefit $b$ (per unit) for the bank manager. We assume that the long asset associated with a private benefit is never socially optimal, i.e.,

$$R > (1 - \alpha)R + b. \quad (3.31)$$

This structure is reminiscent of how moral hazard is introduced by e.g. Holmström and Tirole (1997).

If the manager invests in the private asset instead of the long asset, the bank still offers the same demand deposit contract as in the standard case. The bank can serve its depositors with probability $1 - \alpha$, but with probability $\alpha$ it defaults. We assume that investors can observe what the manager is doing. However, this monitoring is associated with private costs for the investors. We assume that these monitoring costs vary across investors and each investor $i$ has some monitoring costs $s_i$ which are drawn from $G(s)$. These monitoring costs...
Moral Hazard and Regulatory Costs

There are three different environments that a (shadow) bank can operate in: In the first environment, the manager holds no equity and his depositors are not protected by a deposit insurance. In the second environment, the manager does not hold (inside) equity either, but his depositors are protected by a deposit insurance. In the third environment, the manager does hold an equity position $e$.

Let us consider the case without equity and without deposit insurance first. The absence of deposit insurance induces investors to monitor the manager and to withdraw (or not deposit) their funds if the manager misbehaves. Therefore, the manager will behave diligently.\footnote{For tractability, we abstract from the strategic interaction between investors which arises because monitoring has positive externalities.}

In contrast, in the presence of a deposit insurance scheme, investors do not care about what the manager is doing. If the manager has no “skin in the game” (i.e., if he has no inside equity), he chooses the private asset iff

$$b > [1 - (1 - \alpha)\delta]\frac{w}{(1 + \delta)}. \tag{3.32}$$

If this inequality is satisfied, the deposit insurance becomes tested, i.e. has to cover claims, with probability $\alpha$. The regulator therefore has an incentive to ensure diligence of the manager by regulating him. While there are multiple ways to regulate a bank manager, we assume that the regulation requires the bank to hold a minimal amount of equity $e$ per unit of deposits.

This changes the manager’s incentives. Because he now has “skin in the game”, he will behave diligently whenever

$$e > b - [1 - (1 - \alpha)\delta]\frac{w}{(1 + \delta)}. \tag{3.33}$$

By choosing an equity requirement $\bar{e} \equiv b - [1 - (1 - \alpha)\delta]w/(1 + \delta)$ per unit of deposit, the regulator can ensure diligence.

Formally, incorporating this moral hazard and the resulting regulation into the framework of banking and shadow banking in Section 3.4 works in the following way: There exists a sector of commercial banking which is covered by a safety net and regulated to prevent moral hazard. Bank managers have to raise equity, and this is costly, e.g., due to dilution costs. We define the cost of raising $e$ units of equity to be the regulatory cost $\gamma$.

There also exists an unregulated shadow banking sector which is not subject...
to this capital requirement. However, there is a shadow-banking cost of $\rho$. In addition, investors who choose to deposit their funds with the shadow banks have to spend the monitoring cost $s_i$. As shown in Section 3.4, only investors with costs $s_i < s^*$ will choose to invest in the shadow banking sector.

**Appendix 3.B Heterogeneous Investors**

In the main part of the paper, we assumed that all investors have the same size (endowment of one unit). The heterogeneity among investors consists in the switching costs $s_i$ that are distributed according to some distribution function $G$ in the population. We argued that there are several reasons why investors have heterogeneous switching costs, and that the only necessary feature of the model is that switching costs relative to the investors’ budget is heterogeneous.

In this appendix we want to show that we obtain qualitatively similar results if we assume that all investors have identical switching costs, but different endowments. For simplicity, let us assume that the investors’ endowment is either high, $x_i = x_h$, or low, $x_i = x_l$. The fraction of “large investors”, i.e., with a high endowment, is given by $p$. The switching cost is assumed to be identical across investors, $s_i = s$. For convenience, we assume that switching costs are monetary, i.e., the utility from receiving $c$ units from a shadow bank is given by $u(c - s)$.

For an investor with endowment $x_i$, the expected utility of depositing at a commercial bank is given by

$$EU_b(x_i) = \pi u(x_i r_b) + (1 - \pi) u(x_i r_b^2), \quad (3.34)$$

while the utility from depositing at a shadow bank is given by

$$EU_{sb}(x_i) = \pi u(x_i r_{sb} - s) + (1 - \pi) u(x_i r_{sb}^2 - s). \quad (3.35)$$

Again, an investor chooses the shadow bank if $EU_{sb}(x_i) > EU_b(x_i)$. If the endowments and the switching costs are such that $EU_{sb}(x_h) > EU_b(x_h)$ and $EU_{sb}(x_l) < EU_b(x_l)$, all large investors choose the shadow banking sector, while all small investors stay with the commercial banks. The size of the shadow banking sector is thus given by the fraction of “large investors”, $p$. And the size of the commercial banking sector is thus given by the fraction of “small investors”, $1 - p$. 106
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