Fiscal News, Uncertainty, and the Business Cycle

Inaugural-Dissertation
zur Erlangung des Grades eines Doktors
der Wirtschafts- und Gesellschaftswissenschaften
durch die
Rechts- und Staatswissenschaftliche Fakultät
der Rheinischen Friedrich-Wilhelms-Universität
Bonn

vorgelegt von
Johannes Pfeifer
aus Trier

Bonn 2012
Dekan: Prof. Dr. Klaus Sandmann
Erstreferent: Prof. Dr. Gernot Müller
Zweitreferent: Prof. Dr. Christian Bayer

Tag der mündlichen Prüfung: 31.01.2012

Diese Dissertation ist auf dem Hochschulschriftenserver der ULB Bonn http://hss.ulb.uni-bonn.de/diss_online elektronisch publiziert.
Acknowledgments

I would like to express my heartfelt gratitude to my Ph.D. supervisor Gernot Müller for rekindling my interest in macroeconomics after first deserting it for empirical finance. Without his guidance, encouragement, and invaluable support the completion of this thesis would have been unthinkable. His valuable criticism and comments, which helped to vastly improve my research and its presentation, were truly appreciated. While some of his directions turned out to be dead ends, I learned a lot wandering on these paths and I am truly grateful for that.

My special gratitude goes to Christian Bayer for becoming my thesis committee member, always lending an open ear to my questions, and providing valuable advice.

I am deeply indebted to my coauthors, Alexandra Peter (Chapter 1) and Benjamin Born (Chapters 1 and 2), who contributed to this work. In this regard, I would particularly like to thank my long-time office mate Benjamin, who shared the joy and hardships of both graduate school and poor movie selection with me. You were the best coauthor, office mate, and friend I could hope for. I am eagerly looking forward to many joint projects still to come.

I thank Michael Evers for countless discussions that often provided an unexpected new angle to the economic problems at hand and Patrick Hürtgen for critically reviewing parts of this work.

Special thanks go to Urs Schweizer for developing the Bonn Graduate School of Economics (BGSE) into the stimulating and thriving research environment whose hospitality I have enjoyed the last 4 years and to Silke Kinzig and Pamela Mertens for working tirelessly in the background to make the BGSE run smoothly. Financial
support from the German Research Foundation (DFG) is gratefully acknowledged.

Thanks also to my fellow grad students Rafael Aigner, Inga van den Bongard, Deniz Dizdar, Tilman Drerup, Sebastian Ebert, Andreas Esser, Patrick Hürtgen, Stephan Kurka, Matthias Lang, Juliane Parys, Ronald Rühmkorf, Gregor Schwerhoff, Christian Seel, Mirko Seithe, Marco Sorge, Jörn Tenhofen, Christoph Wagner, Felix Wellschmied, and Florian Zimmermann for numerous scientific discussions and making graduate school in Bonn such a great experience. The parties, barbecues, wine tastings, and hiking tours were a welcome distraction from everyday research. Thank you, Andreas, for being our personal “tour planer” and “travel guide”.

I am grateful to two special and dear groups of people. First, many thanks to the members of our voluntary, semester-spanning “Applied Game Theory”-course. It was a lot of fun. Thank you, Rafael, for reminding us what rivalry in consumption and excludability of unique private goods means. I also thank Deckard Cain for providing help with intricate identification problems arising during this course. Second, I want to thank the participants of our regular cooking and baking parties for making me forget the Mensa food. Without you, I would have never learned how to totally mess up a kitchen while preparing a Thanksgiving goose, how to bake 961 Christmas cookies in one day, how to prepare Indian food until the fuse blows, and what a kilometer of pasta looks like. Sorry to Patrick if the food was too hot and spicy.

I want to thank Esther for her love, the wonderful time together, and for always being there. Moreover, I am indebted to her for her constant encouragement, proofreading my writings, and being a valuable, diligent, and industrious coauthor.

Contents

<table>
<thead>
<tr>
<th>Introduction</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fiscal News and Macroeconomic Volatility</td>
<td>5</td>
</tr>
<tr>
<td>1.1 Introduction</td>
<td>5</td>
</tr>
<tr>
<td>1.2 A DSGE-Model with Fiscal Foresight</td>
<td>8</td>
</tr>
<tr>
<td>1.2.1 Shock Structure</td>
<td>9</td>
</tr>
<tr>
<td>1.2.2 Conceptualizing Tax Shocks</td>
<td>10</td>
</tr>
<tr>
<td>1.2.3 The Model</td>
<td>12</td>
</tr>
<tr>
<td>1.3 Model Estimation</td>
<td>20</td>
</tr>
<tr>
<td>1.3.1 Data</td>
<td>20</td>
</tr>
<tr>
<td>1.3.2 Fixed Parameters</td>
<td>21</td>
</tr>
<tr>
<td>1.3.3 Prior Distribution</td>
<td>23</td>
</tr>
<tr>
<td>1.3.4 Posterior Distribution</td>
<td>24</td>
</tr>
<tr>
<td>1.4 Business Cycle Effects of Fiscal News</td>
<td>27</td>
</tr>
<tr>
<td>1.4.1 Variance Decomposition</td>
<td>27</td>
</tr>
<tr>
<td>1.4.2 Impulse Responses</td>
<td>31</td>
</tr>
<tr>
<td>1.5 Conclusion</td>
<td>37</td>
</tr>
<tr>
<td>Appendix to Chapter 1</td>
<td>38</td>
</tr>
<tr>
<td>1.A Tables</td>
<td>38</td>
</tr>
<tr>
<td>1.B Stationary Equilibrium</td>
<td>43</td>
</tr>
<tr>
<td>1.C Observation Equation</td>
<td>44</td>
</tr>
<tr>
<td>1.D Data Construction</td>
<td>45</td>
</tr>
</tbody>
</table>
## Contents

### 2 Policy Risk and the Business Cycle

2.1 Introduction ........................................... 49
2.2 Uncertainty: Potential Transmission Channels .............. 53
2.3 A DSGE-Model with Policy Risk .......................... 55
   2.3.1 Household Sector .................................. 55
   2.3.2 Labor Market ..................................... 58
   2.3.3 Firm Side ........................................ 59
   2.3.4 Government Sector ................................. 60
2.4 Policy Risk: Time Series Evidence .......................... 61
   2.4.1 Estimation Methodology ............................ 62
   2.4.2 Estimation Results ................................ 64
2.5 Fitting the Model to the Data ............................... 68
   2.5.1 Simulated Method of Moments Estimation .............. 68
   2.5.2 Parameter Estimates ................................. 69
   2.5.3 The Effects of Time-Varying Volatility .............. 71
2.6 The Aggregate Effects of Policy Risk ......................... 73
   2.6.1 Impulse Response Analysis ......................... 73
   2.6.2 What Drives the Response to Policy Risk? ........... 78
   2.6.3 Why Are the Effects of Uncertainty Small? .......... 80
2.7 Conclusion ............................................... 84

Appendix to Chapter 2 ........................................ 85
2.A Data construction ....................................... 85
2.B Econometric Methods .................................... 87
2.C Diagnostics .............................................. 97

### 3 The Business Cycle Effects of Terms of Trade Uncertainty

3.1 Introduction ........................................... 109
3.2 Terms of Trade Risk: Empirical Evidence .................... 114
   3.2.1 The Chilean Terms of Trade ....................... 114
   3.2.2 Terms of Trade Risk in the Data .................. 116
3.3 A Small Open Economy Model of the Chilean Economy ........ 120
   3.3.1 Household Sector .................................. 121
   3.3.2 Final Good Sector ................................ 123
## Contents

3.3.3 Non-tradable Intermediate Good Producers ........................................ 124  
3.3.4 Tradable Good Bundler ................................................................. 125  
3.3.5 Intermediate Tradable Good Producers .......................................... 125  
3.3.6 Market Clearing and Definitions ................................................... 126  
3.3.7 Monetary Policy and Exogenous Processes ...................................... 128  
3.3.8 Model Calibration ........................................................................ 129  
3.4 The Aggregate Effects of Terms of Trade Uncertainty ........................... 133  
  3.4.1 The Effects of Time-Varying Volatility .......................................... 134  
  3.4.2 Impulse Response Analysis ............................................................ 136  
3.5 Conclusion ......................................................................................... 143  

Appendix to Chapter 3 ............................................................................ 145  
3.A Data Appendix ................................................................................... 145  
  3.A.1 Terms of Trade Index ................................................................. 145  
  3.A.2 Moment Comparison ................................................................. 147  
3.B Convergence Diagnostics ................................................................. 149  
3.C Misspecification Tests ....................................................................... 151  

**Bibliography** ....................................................................................... 153
## List of Figures

1.1 Intrade Daily Closing Prices .................................................. 11
1.2 Evolution of the Tax Rates and the Spending to GDP Ratio ....... 21
1.3 Impulse Responses to Unanticipated and Anticipated Capital Tax Shocks .................................................. 32
1.4 Impulse Responses to Unanticipated and Anticipated Stationary TFP Shocks .................................................. 35

2.1 Time Series of Exogenous Driving Processes ......................... 63
2.2 Smoothed Standard Deviations ............................................ 67
2.3 Impulse Responses to a Two-standard Deviation Uncertainty Shock to Capital Taxes, Labor Taxes, and Government Spending ......... 74
2.4 Impulse Responses to a Two-standard Deviation Uncertainty Shock to Monetary Policy, TFP, and Investment-specific Technology .... 75
2.5 Impulse Responses to a Joint Two-standard Deviation Policy Risk Shock and to a Joint Technology Risk Shock .................. 76
2.6 Impulse Responses to a Two-standard Deviation Uncertainty Shock to Capital Taxes, Labor Taxes, and TFP ......................... 79
2.7 Impulse Responses to a Two-standard Deviation Policy Risk Shock – Volatile Counterfactual ................................................. 81
2.8 Evolution of MCMC Sampler over Time for $\tau^k$ ...................... 100
2.9 Evolution of MCMC Sampler over Time for $\tau^n$ ...................... 101
2.10 Evolution of MCMC Sampler over Time for $z$ ......................... 102
2.11 Evolution of MCMC Sampler over Time for $z^I$ ...................... 103
List of Figures

2.12 Evolution of MCMC Sampler over Time for $g$ .......................... 104
2.13 Evolution of MCMC Sampler over Time for $m$ .......................... 105
2.14 QQ-Plots for Model Misspecification ........................................ 107

3.1 World Copper and Timber Prices ............................................. 111
3.2 Chilean Terms of Trade 1965-2010 .......................................... 115
3.3 Historical Evolution of the Chilean Terms of Trade Volatility ...... 119
3.4 Structure of the Model Economy ............................................. 121
3.5 Impulse Responses to a Two-standard Deviation Terms of Trade Uncertainty Shock .................................................. 138
3.6 Impulse Responses to a Two-standard Deviation Terms of Trade Uncertainty Shock – No Central Bank Response ......................... 140
3.7 Impulse Responses to a One-standard Deviation Terms of Trade Level Shock .............................................................. 141
3.8 Impulse Responses to a One-standard Deviation Technology Shock – Non-tradable Sector ............................................. 142
3.9 Export Price Index – Price Components and Basket Shares ........ 148
3.10 Evolution of MCMC Sampler over Time. ................................. 150
3.11 QQ-plot for Model Misspecification ........................................... 151
List of Tables

1.1 Parameters Fixed Prior to Estimation ........................................... 22
1.2 Prior and Posterior Distributions of Preference and Technology Parameters ....................................................... 38
1.3 Prior and Posterior Distributions of the Shock Processes .............. 38
1.4 Model and Data Moments ................................................................. 41
1.5 Forecast Error Variance Decomposition ........................................... 42

2.2 Prior and Posterior Distributions of the Shock Processes .............. 65
2.3 Parameters Fixed Prior to Estimation ............................................. 69
2.4 Parameters Estimated by SMM ......................................................... 70
2.5 Simulated and Empirical Moments .................................................. 71
2.6 Simulated and Empirical Moments for the Model without Time-varying Volatility ................................................................. 72
2.7 Counterfactual Calibration Implying Large Uncertainty Effects ...... 80
2.8 Simulated and Empirical Moments: Counterfactual with Stronger Amplification ................................................................. 82
2.9 GMM Estimation of Taylor Rule ....................................................... 97
2.10 Tests for Heteroskedasticity ............................................................. 98
2.12 Tests for Model Misspecification ..................................................... 106

3.1 Prior and Posterior Distributions of the Shock Processes .............. 118
List of Tables

3.2 Parameter Values of the Theoretical Economy: Structural Parameters 130
3.3 Model and Empirical Moments: Benchmark Calibration 135
3.4 Model and Empirical Moments: No TOT Risk 136
3.5 Convergence Diagnostics 149
3.6 Tests for Model Misspecification 151
Introduction

The recent “Great Recession” has thrown macroeconomic research into a state of disarray\(^1\) and has clearly shown the need to go beyond traditional business cycle explanations. However, many of the recently proposed business cycle explanations like news (Beaudry and Portier, 2006), noise (Blanchard, L’Huillier, and Lorenzoni, 2009), confidence (Barsky and Sims, forthcoming), mood swings (Beaudry, Nam, and Wang, 2011), and uncertainty (Bloom, 2009) rely on factors that are not directly observed by the econometrician. One promising way to deal with this issue of unobserved state variables has been the use of structural estimation, where the co-movement of observed variables in conjunction with a structural model is used for inference about the underlying driving processes. The present work contributes to the literature on non-traditional business cycle explanations by using structural macroeconomic modeling and structural estimation to analyze the role of fiscal news (Chapter 1), policy risk (Chapter 2), and terms of trade uncertainty (Chapter 3) for explaining macroeconomic fluctuations.

Chapter 1.\(^2\) The first chapter investigates the role of news about fiscal policy, and in particular the anticipation of tax rate changes, for macroeconomic fluctuations in the United States. Recent macroeconomic research has started to analyze the effects of anticipated shocks on business cycle fluctuations, but has mostly focused on news about productivity (see e.g. Fujiwara, Hirose, and Shintani, 2011; Khan and Tsoukalas, 2011; Schmitt-Grohé and Uribe, 2010). However, fiscal news have

---

\(^1\)See e.g. the debate between Krugman (2009) and Cochrane (2011b).

\(^2\)This chapter is based on joint work with Benjamin Born and Alexandra Peter (Born, Peter, and Pfeifer, 2011).
Introduction

potentially an important role for explaining aggregate fluctuations for two reasons. First, legislated fiscal measures are usually publicly debated long before they are enacted or become effective. Second, surprise fiscal policy shocks have long been discussed as a potential prominent driver of the business cycle (see e.g. Baxter and King, 1993; Jones, 2002; McGrattan, 1994, 2011). The importance of surprise fiscal shocks combined with the regular preannouncement of fiscal measures suggests that anticipated fiscal policy shocks may be important too, because rational forward-looking agents should react to news of an event and not only to its eventual realization.

Chapter 1 adds upon the previous literature by explicitly analyzing the business cycle variance contribution of fiscal news about tax rates and government spending. To deal with the problem that news shocks are not observed by the econometrician, we resort to structural estimation of a New Keynesian DSGE model. We find that while fiscal policy accounts for 12 to 20 percent of output variance at business cycle frequencies, the anticipated components hardly matter for explaining fluctuations of real variables. In contrast, anticipated capital tax shocks do explain a sizable part of inflation and nominal interest rate fluctuations, accounting for 5 to 15 percent of their total variance. Consistent with earlier studies, we find that news shocks account for 20 percent of output variance, driven by news about stationary TFP and non-stationary investment-specific technology.

Chapter 2. In the second chapter we analyze the role of policy risk in explaining U.S. business cycle fluctuations. The aftermath of the financial and economic crisis is clearly characterized by extraordinary uncertainty regarding U.S. economic policy. Hence, the argument that policy risk, i.e. uncertainty about monetary and fiscal policy, has been holding back the economic recovery in the U.S. during the “Great Recession” has a large popular appeal. But the empirical literature is still inconclusive with respect to the aggregate effects of (mostly TFP) uncertainty. Studies using different proxies and identification schemes to uncover the effects of uncertainty produce a variety of results.

We analyze the role of policy risk in explaining business cycle fluctuations by using an estimated New Keynesian model featuring policy risk as well as uncertainty about technology. To deal with the unobserved state “uncertainty”, we directly measure

---

3The work in this chapter, “Policy Risk and the Business Cycle”, has been conducted jointly with Benjamin Born (Born and Pfeifer, 2011).
uncertainty from aggregate time series by structurally estimating a stochastic volatility model using *Sequential Monte Carlo Methods*. While we find considerable evidence of policy risk in the data, we show that the “pure uncertainty”-effect of policy risk is unlikely to play a major role in business cycle fluctuations. In the estimated model, output effects are relatively small due to i) dampening general equilibrium effects that imply a low amplification and ii) counteracting partial effects of uncertainty. Finally, we show that policy risk has effects that are an order of magnitude larger than the ones of uncertainty about aggregate TFP.

**Chapter 3.** The third chapter leaves the closed economy setting of the first two chapters and analyzes the business cycle contribution of terms of trade uncertainty. Over the last decades, world-wide commodity and manufacturing prices have been going through distinct periods of high and low volatility. In particular, the recent commodity price boom and the financial crisis have been accompanied by a large increase in price volatility. The result was a significant increase in the uncertainty associated with international commodity prices and correspondingly the terms of trade for many countries.

However, time variation in this variance has mostly been neglected in both public discussions and academic research. I contribute to closing this gap by studying the effects of terms of trade uncertainty on Chilean business cycles through the lens of a small open economy DSGE model. As in Chapter 2, *Sequential Monte Carlo Methods* are used to estimate a stochastic volatility model to deal with the latent state “uncertainty”. My findings are fourfold. First, there is considerable evidence for time-varying terms of trade uncertainty in the Chilean data, with the variance of terms of trade shocks more than doubling in a short period of time. Second, I show that the ex-ante and ex-post effects of increased terms of trade uncertainty can account for about one fifth of Chilean output fluctuations at business cycle frequencies. Third, I find that a two-standard deviation terms of trade risk shock, i.e. a 54 percent increase in uncertainty, leads to a 0.1 percent drop in output. The fact that terms of trade uncertainty more than doubled during the recent commodities boom suggests that the contribution of terms of trade risk during this more recent period may have been substantial. Finally, I show that the economic mechanisms that attenuated the negative output effects of uncertainty in Chapter 2 also dampen the negative
Introduction

impact of terms of trade uncertainty. Both the precautionary savings motive of the representative household and the expansionary response of the central bank mitigate the drop in GDP.
Chapter 1

Fiscal News and Macroeconomic Volatility

1.1 Introduction

This chapter analyzes the role of news about fiscal policy, and in particular the anticipation of tax rate changes, for business cycle fluctuations. Recent macroeconomic research has increasingly shifted from explaining business cycle fluctuations through contemporaneous shocks to explaining them by anticipated, or news, shocks. Rational agents, anticipating future changes will already react today to these news (see e.g. Beaudry and Portier, 2004, 2006; Jaimovich and Rebelo, 2009; Schmitt-Grohé and Uribe, 2010). However, most empirical studies on the effects of anticipated shocks on business cycles have focused on news about future productivity (see e.g. Forni, Gambetti, and Sala, 2011; Fujiwara, Hirose, and Shintani, 2011; Khan and Tsoukalas, 2011).¹

This is remarkable for two reasons. First, fiscal measures are usually publicly debated well in advance and often known before becoming effective, i.e. there are considerable decision and implementation lags. A tax bill typically takes about one year from the U.S. President’s initial proposal to the law’s enactment and another year until the tax change becomes effective (Mertens and Ravn, 2011; Yang, 2011; Yang, 2005).

¹There is a prominent literature branch dealing with the importance of fiscal foresight. However, its focus has mostly been on analyzing single tax events (House and Shapiro, 2006; Parker, 1999; Poterba, 1988) or tracing out the consequences for econometric analyses (Leeper, Walker, and Yang, 2011; Yang, 2005).
2005). As a recent example, consider the Patient Protection and Affordable Care Act (“Obamacare”), whose core contents were debated for almost one year and whose financing provisions will only phase in gradually over time. Second, surprise fiscal policy shocks have long been discussed as a potential prominent driver of the business cycle (see e.g. Baxter and King, 1993; Cardia, Kozhaya, and Ruge-Murcia, 2003; Jones, 2002; McGrattan, 1994). McGrattan (1994) for example attributes one third of the U.S. business cycle variance to distortionary taxation, while McGrattan (2011) argues that changes in business taxation can explain one third of the output drop during the Great Depression.\(^2\) This potential importance of fiscal policy shocks, combined with the fact that many fiscal policy measures are known well in advance, makes fiscal news a natural candidate for explaining aggregate fluctuations.

We add upon the previous literature by explicitly analyzing the business cycle variance contribution of fiscal news. For this purpose, we employ a New Keynesian DSGE model featuring several real and nominal rigidities as well as various shocks identified as important drivers of the business cycle and augment it with a government sector financed through distortionary labor and capital taxes. Our main focus lies on the effects of fiscal news, but we also control for anticipation in technology, investment-specific productivity, and the wage markup. The model is estimated by full information (Bayesian) methods using quarterly U.S. data from 1955 to 2006. Model-based estimation allows us to circumvent the issue of non-invertibility typically encountered when estimating structural VARs in the presence of anticipation effects (Fernández-Villaverde et al., 2007; Hansen and Sargent, 1991; Leeper, Walker, and Yang, 2011).\(^3\)

Computing forecast error variance decompositions, we find that while fiscal policy accounts for 12 to 20 percent of output variance at business cycle frequencies, fiscal

---

\(^2\)Although Forni, Monteforte, and Sessa (2009) find that unanticipated tax shocks contribute little to macroeconomic fluctuations of the Euro area, this could in principle be the result of ignoring fiscal foresight.

\(^3\)Non-invertibility means that the DSGE-model has a VARMA representation that cannot be inverted to yield a finite-order VAR in the observables. Hence, the true innovations do not perfectly map into the VAR residuals, meaning that the structural shocks cannot be recovered using a VAR. Non-invertibility arises, e.g., when the information set of an econometrician estimating the VAR is smaller than that of the forward-looking agents. For alternative ways to mitigate this problem, see e.g. Sims (2009), Giannone and Reichlin (2006), and Forni, Gambetti, and Sala (2011).
news generally only plays a very limited role. Its contribution to output variance ranges around 3 percent.

With a variance share of 10 percent at the 5 year forecast horizon, government spending is the fiscal variable with the largest effect on output variance. However, this contribution only comes from surprise shocks, with anticipated spending shocks explaining virtually nothing. Contemporaneous and anticipated capital tax shocks each contribute 2 – 3 percent to output fluctuations. However, they are considerably more important for explaining inflation and interest rate fluctuations. Depending on the forecast horizon, surprise capital tax shocks contribute roughly 30 percent to their variance. Anticipated capital tax shocks are responsible for 5 to 15 percent. The effect of contemporaneous and anticipated labor taxes, on the other hand, is negligible.

In line with previous studies that do not consider news shocks (e.g. Smets and Wouters, 2007), we find that the main drivers of the output variance are preference and wage markup shocks. News shocks explain on average 20 percent of the variance of output, with the main effect coming from news about TFP and investment-specific productivity. This result conforms well with i) VAR evidence (Barsky and Sims, 2011), ii) evidence coming from a factor model (Forni, Gambetti, and Sala, 2011), and iii) other DSGE-based estimates of the importance of news shocks, who all find a similar fraction of output fluctuations explained by anticipated shocks.

The two papers most closely related to ours are recent contributions by Mertens and Ravn (forthcoming) and Schmitt-Grohé and Uribe (2010). The former use a VAR to analyze the business cycle contribution of narratively identified anticipated and unanticipated tax shocks.4 They find that both types of tax shocks together explain 20 to 25 percent of output variance, with anticipation accounting for the majority. Schmitt-Grohé and Uribe (2010) evaluate the role of news about TFP, investment-specific technology, wage markup, and government spending shocks in an estimated RBC model with various real rigidities. In their setup, news shocks account for 41 percent of output fluctuations. But while they find government spending shocks to explain 10 percent, evenly distributed across surprise, one and two year

---

4Mertens and Ravn (forthcoming) classify the Romer and Romer (2010) tax shocks according to the time passed between the presidential signing of a bill and the tax changes becoming effective into anticipated and contemporaneous shocks.
anticipated shocks, they do not consider foresight about the financing side of the government budget constraint.

Our paper is also related to other DSGE-based papers focusing on the effects of anticipated technology shocks. Davis (2007), using a New Keynesian model, estimates news shocks to be responsible for 50 percent of output fluctuations. Fujiwara, Hirose, and Shintani (2011) extend the New Keynesian model of Smets and Wouters (2007) and Christiano, Eichenbaum, and Evans (2005) to include news about TFP. They estimate news shocks to explain 9 percent of output variance in the unconditional variance decomposition. The paper of Khan and Tsoukalas (2011) uses the same basic New-Keynesian model framework, but additionally allows for news about investment-specific technology growth. In their estimated model, both types of news shocks together account for less than 10 percent. Finally, Auray, Gomme, and Guo (2009) estimate a New Keynesian model with an additional durables sector, featuring news about TFP in both sectors. They find that technology news in the non-durables sector explain 52% of output variance.

The outline of the chapter is the following. Section 2.2 introduces the DSGE-model with fiscal foresight, while Section 2.3 presents the estimation approach and results. In Section 2.4, we compute variance decompositions and impulse responses. Section 2.5 concludes.

### 1.2 A DSGE-Model with Fiscal Foresight

We use a medium-scale DSGE-model featuring various real and nominal frictions as well as a variety of shocks that have been identified as important drivers of the business cycle (see e.g. Justiniano, Primiceri, and Tambalotti, 2010a; Smets and Wouters, 2007). We incorporate both contemporaneous and anticipated elements into the shock processes as in Schmitt-Grohé and Uribe (2010) and allow for non-stationary shocks. We first discuss the information structure of the shock processes in the next section before describing the model in detail.
1.2 A DSGE-Model with Fiscal Foresight

1.2.1 Shock Structure

Our model features 10 sources of stochastic fluctuations. On the government side, we include shocks to labor and capital tax rates $\tau_n$ and $\tau_k$, a shock to government spending $g$, and a monetary policy shock $\xi^R$. The technology shocks considered are shocks to stationary neutral productivity $z_t$, non-stationary productivity $X_t$, stationary investment-specific productivity $z^I_t$, and non-stationary investment-specific productivity $A_t$. In addition, the model includes a preference shock $\xi^{pref}_t$ and a wage markup shock $\mu^w_t$.

The monetary policy shock and the preference shock are assumed to only contain a contemporaneous, unanticipated component. For the other shocks, we follow the framework proposed by Schmitt-Grohé and Uribe (2010) and allow for both contemporaneous shocks and shocks that are anticipated 4 and 8 periods in advance. Anticipation horizons of 4 and 8 quarters fulfill the aim of capturing longer anticipation horizons while keeping the state space at a manageable level. This is crucial as each additional anticipation horizon is an additional state variable. While specifically choosing 4 and 8 quarters of anticipation might be seen as arbitrary, this assumption can be rationalized by the workings of the political system. Four quarters of anticipation are close to the average length of a tax bill from the President’s proposal announcement to enactment (Yang, 2005). Eight quarters serves as a plausible upper bound for the anticipation of shocks to tax rates as Congressional elections take place every two years. We think this makes it very unlikely that people are able to correctly predict both the reigning majority and the tax laws being implemented by the next Congress. The same, of course, applies to spending bills. For reasons of symmetry, we then assume this anticipation structure for all shock processes.

The general structure for shock $\epsilon^i$, $i \in \{\tau^n, \tau^k, g, z, x, z^I, a, w\}$ is given by

$$\epsilon^i = \epsilon^0_{i,t} + \epsilon^4_{i,t-4} + \epsilon^8_{i,t-8}, \quad (1.1)$$

where $\epsilon^j_{i,t-j}$, $j \in \{0, 4, 8\}$ denotes a shock to variable $i$ that becomes known in period $t - j$ and hits the economy $j$ periods later. For example, $\epsilon^4_{\tau^n,t-4}$ denotes a four period anticipated shock to the labor tax rate that becomes known at time $t - 4$ and becomes effective at time $t$. The shocks are assumed to have mean 0, standard deviation $\sigma^i_t$, ...
to be serially uncorrelated, and to be uncorrelated across anticipation horizons, i.e. 
\( E(\varepsilon^j_{i,t-j}) = 0 \) and \( E(\varepsilon^k_{i,t} \varepsilon^l_{i,t-j}) = (\sigma^k)^2 \) for \( j = 0, k = l, \) and 0 otherwise. Moreover, they are uncorrelated across shock types \( i_m, i_n \in i, E(\varepsilon^k_{i_m,t} \varepsilon^l_{i_n,t-j}) = 0 \) \( \forall j, k, l \) and \( i_m \neq i_n \).

The assumed information structure implies that agents foresee future shocks to the extent of already known but not yet realized shocks \( \varepsilon^m_{i,t-j}, m > j \). The forward-looking behavior of rational optimizing agents results in them reacting to anticipated shocks even before they are realized. By imposing a structural model on the data, this anticipatory behavior enables the econometrician to achieve identification. However, it is exactly this foresight that makes identifying the shocks with a VAR impossible. The econometrician attempting to do this only uses current and past values of the observables and thus has a smaller information set than the agents. In particular, he is missing the anticipated but not yet realized shocks as states in his VAR.\(^5\) To remedy this issue, structural estimation has been advocated (Blanchard, L’Huillier, and Lorenzoni, 2009). We will pursue this avenue in Section 1.3 by using Bayesian methods to estimate the proposed model.

1.2.2 Conceptualizing Tax Shocks

The tax shocks considered in the present work do not necessarily stem from actual changes in the labor and capital tax rates. Rather, they are interpreted as the probability weighted effect of tax actions under legislative debate or due to judicative decisions. They are the product of the likelihood of a tax change and the size of this effect, as perceived by rational agents forming expectations about the future path of taxes. Hence, our definition is wider than the one considered by Mertens and Ravn (forthcoming), who restrict their attention to the shocks directly deriving from the legislative process. Shocks deriving from e.g. the SEC suing against the legality of a tax shelter would be excluded from their definition but not from ours.\(^6\) Note that news shocks are distinct from pure uncertainty about future taxes. While the

---

\(^5\)Sims (2009) shows that in some cases it may be possible to recover the shocks using a structural VAR. By including enough lags and forward-looking variables, it may be possible to move the non-invertible root(s) close enough to unity so that the discrepancy between true structural errors and the estimated ones becomes small.

\(^6\)This notion of tax shocks is consistent with concept of “policy expectations” in McGrattan (2011).
former are associated with an anticipated change in the mean of the tax rate, tax uncertainty shocks can be conceptualized as mean-preserving spreads.\footnote{For an analysis of uncertainty about fiscal policy in the context of a New Keynesian model, see Born and Pfeifer (2011); Fernández-Villaverde et al. (2011b).}

Figure 1.1: Intrade Daily Closing Prices: “Will 'Obamacare' health care reform become law in the United States?”

Notes: This contract will settle (expire) at 100 ($10.00) if a health care reform bill is passed into law before midnight ET 30 Jun 2010. It will settle (expire) at 0 ($0.00) if a health care reform bill is not passed into law.

To fix ideas, consider the \textit{Patient Protection and Affordable Care Act} of 2010 as an example. On June 9, 2009, a first draft of the health care bill was released. At that time, people at the latest could anticipate that taxes were going to rise in order to finance the bill, if it ever passed. However, both the size and the likelihood of such a change was largely unknown. The first point of uncertainty changed on July 13, 2009, when the Congressional Budget Office published official cost estimates: If passed, marginal income tax rates were going to increase by 22 percentage points for households between 100\% and 400\% of the poverty level.\footnote{People at the poverty line would have gotten a 15,000$ subsidy to mandatory health care per year.} Taking these costs as...
given, households were experiencing tax shocks with changes in the likelihood of the passage of the bill. Intrade bets on the passage of the bill show that some people were constantly reevaluating this likelihood. Figure 1.1 presents the closing prices of an Intrade betting contract that paid 100, if a health care reform bill was passed into law before mid-2010 and 0 if a health care reform bill was not passed. Hence, the closing price is a direct measure of the likelihood of a bill becoming law. There is a large variance in the probability of passing the bill that varies with the ebb and flow of the political process. These changes potentially act like a huge sequence of tax shocks for households. If one considers only the change in the likelihood from the time directly after the Massachusetts Senate election in January to the final vote of the bill, this amounts in expectations to a tax shock of $0.7 \times 22\% = 15.4\%$ during one quarter.\footnote{Unfortunately, due to the non-availability of data for the relative price of investment, our sample does not cover this series of events.}

### 1.2.3 The Model

The model economy includes four sectors: the household sector with a large representative household, the labor market featuring a continuum of monopolistically competitive unions selling differentiated labor services to intermediate firms, the firm sector including a continuum of intermediate goods firms producing intermediate goods and a final good firm bundling the intermediate goods, and the government sector responsible for fiscal and monetary policy.

#### Household Sector

The economy is populated by a large representative household with a continuum of members. Household preferences are defined over per capita consumption $C_t$ and per capita labor effort $L_t$, where each member consumes the same amount and works the same number of hours.\footnote{Due to the symmetric equilibrium, the decisions of the household members are identical. Hence, we suppress the subscript denoting individual members.} We follow Schmitt-Grohé and Uribe (2006) and assume that

\footnotetext[9]{This subsidy would have been linearly decreasing until reaching 0$ at 400\% of the poverty line, implying that every additional dollar of income would have decreased the subsidy by 22 cents (CBO, 2009).}
1.2 A DSGE-Model with Fiscal Foresight

Household members supply their labor uniformly to a continuum of unions $j \in [0, 1]$. The unions are monopolistically competitive and supply differentiated labor services $l_t(j)$ to intermediate goods firms. Overall, total labor supply of the representative household is given by the integral over all labor markets $j$, i.e. $L_t = \int_0^1 l_t(j) \, dj$. We will discuss the labor market structure in detail below.

Following Jaimovich and Rebelo (2009), we assume a preference specification that allows to control the size of the wealth effect, but additionally assume habits in consumption:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \xi_{t}^{\text{pref}} \frac{C_t - \phi_c C_{t-1} - \gamma \frac{L_t^{1+\sigma_c}}{1 + \sigma_c} S_t^{1-\sigma_c} - 1}{1 - \sigma_c}.$$  \hspace{1cm} (1.2)

Here, the parameter $\phi_c \in [0, 1]$ measures the degree of internal habit persistence, $\sigma_c \geq 0$ governs the intertemporal elasticity of substitution, $\sigma_l \geq 0$ is related to the Frisch elasticity of labor supply, and $\gamma \geq 0$ measures the relative disutility of labor effort.\(^{11}\) The term

$$S_t = (C_t - \phi_c C_{t-1})^{\sigma_s} S_{t-1}^{1-\sigma_s}$$  \hspace{1cm} (1.3)

makes the preferences non-separable in both consumption and work effort. This preference specification introduces the parameter $\sigma_s \in (0, 1]$ that allows to govern the magnitude of the wealth effect on the labor supply. As special cases, the specification nests the preference class discussed by King, Plosser, and Rebelo (1988), i.e. $\sigma_s = 1$, and the preferences proposed by Greenwood, Hercowitz, and Huffman (1988), i.e. $\sigma_s = 0$, where the latter case implies a zero wealth elasticity of labor supply. We assume the preference shock $\xi_t^{\text{pref}}$ to follow an AR(1)-process in logs:

$$\log S_t = \rho_{t}^{\text{pref}} \log S_{t-1}^{\text{pref}} + \xi_t^{\text{pref}}.$$  \hspace{1cm} (1.4)

\(^{11}\)In a recent paper, Nutahara (2010) shows that it is important to distinguish between internal and external habits in a model with news shocks. He finds that internal habits are able to generate news-driven business cycles, whereas external habits are not.
Chapter 1

The household faces the budget constraint

\[ C_t + z_t^I A_t I_t + \frac{B_{t+1}}{P_t} = (1 - \tau^n_t) \int_0^1 W_t(j) l_t(j) dj + (1 - \tau^k_t) R^K_t u_t K_t + \Phi_t + T_t \]

\[ + (1 - \tau^k_t) \Xi_t + (1 - \tau^n_t) (R_{t-1} - 1) \frac{B_t}{P_t} + \frac{B_t}{P_t}. \]  (1.5)

Besides labor income from supplying differentiated labor services \( l_t(j) \) at the real wage \( W_t(j) \), the household has capital income from renting out capital services \( u_t K_t \) at the rental rate \( R^K_t \), from receiving firm profits \( \Xi_t \), and from investing in bonds \( B_{t+1} \), which are in zero net supply. Both forms of income are taxed at their respective tax rates \( \tau^n_t \) and \( \tau^k_t \). Only net returns of bonds are taxed, such that the term \((1 - \tau^k_t) (R_{t-1} - 1) \frac{B_t}{P_t} + \frac{B_t}{P_t}\) is the after-tax return. In addition, the government pays lump sum transfers.

The household spends its income on consumption \( C_t \) and investment \( z_t^I A_t I_t \), where \( I_t \) denotes gross investment at the price of capital goods. We assume that the relative price of investment in terms of the consumption good is subject to two shocks, a stationary investment-specific productivity shock \( z_t^I \) and non-stationary investment-specific technological progress \( A_t \) (see Greenwood, Hercowitz, and Krusell, 1997, 2000). The relative price of investment is equal to the technical rate of transformation between investment and consumption goods. Changes in this price do not affect the productivity of already installed capital, but do affect newly installed capital and become embodied in it. For the non-stationary investment-specific technology process, we assume a random walk with drift in its logarithm

\[ \log A_t = \log A_{t-1} + \log \mu_t^a. \]  (1.6)

The drift term \( \mu_t^a \) is subject to contemporaneous and anticipated shocks according to

\[ \log \left( \frac{\mu_t^a}{\mu^a} \right) = \rho^a \log \left( \frac{\mu_{t-1}^a}{\mu^a} \right) + \varepsilon^0_{a,t} + \varepsilon^4_{a,t-4} + \varepsilon^8_{a,t-8}. \]  (1.7)

The stationary investment-specific technology shock \( z_t^I \) follows an AR(1)-process

\[ \log z_t^I = \rho_z^I \log z_{t-1}^I + \varepsilon^9_{z,t} + \varepsilon^4_{z,t-4} + \varepsilon^8_{z,t-8}. \]  (1.8)
Depreciation allowances are an important feature of the U.S. tax code, therefore, we also include them in our model. They are captured by the term $\Phi_t$ in equation (1.5) and have the form

$$\Phi_t = \tau \sum_{s=1}^{\infty} \delta \tau (1 - \delta \tau)^{s-1} z_{t-s}^t A_{t-s} I_{t-s},$$

where $\delta \tau$ is the depreciation rate for tax purposes. Since depreciation allowances provide new investment with a tax shield at historical costs, they may be important in capturing the dynamics of investment following shocks (Christiano, Trabandt, and Walentin, 2011; Yang, 2005).

The household members own the capital stock $K_t$, whose law of motion is given by

$$K_{t+1} = \left[ 1 - \left( \delta_0 + \delta_1 (u_t - 1) + \delta_2/2 (u_t - 1)^2 \right) \right] K_t + \left[ 1 - \frac{\kappa}{2} \left( \frac{I_t}{I_{t-1} - \mu^l} \right)^2 \right] I_t. \quad (1.9)$$

Household members do not simply rent out capital, but capital services $u_t K_t$, where $u_t$ denotes capital utilization. Thus, they decide about the intensity with which the existing capital stock is used. However, using capital with an intensity that is higher than normal is not costless, but leads to higher depreciation of the capital stock. This is captured by the increasing and convex function $\delta (u_t) = \delta_0 + \delta_1 (u_t - 1) + \delta_2/2 (u_t - 1)^2$, with $\delta_0, \delta_1, \delta_2 > 0$. Without loss of generality, capital utilization in steady state is normalized to 1. Following Christiano, Eichenbaum, and Evans (2005), we assume the presence of investment adjustment costs $S (I_t/I_{t-1}) = \kappa/2 \left( I_t/I_{t-1} - \mu^l \right)^2$ to dampen the volatility of investment over the business cycle. $\kappa > 0$ is a parameter governing the curvature of the investment adjustment costs and $\mu^l$ is the steady state growth rate of investment, which is equal to the steady state growth rate of capital. This specification assures that the investment adjustment costs are minimized and equal to 0 along the balanced growth path, i.e. $S = S' = 0$ and $S'' > 0$, where the primes denote derivatives.

The household maximizes its utility, equation (1.2), by choosing $C_t, L_t, S_t, B_{t+1}, K_{t+1}, u_t$, and $I_t$, subject to the budget constraint (1.5), the law of motion for capital (1.9), and the resource constraint for aggregate labor given by (1.10) below.

---

12Following Auerbach (1989), we allow the depreciation rate for tax purposes to differ from the physical rate.
Chapter 1

Labor Market

The labor market is characterized by differentiated labor services and staggered wage setting. To model these features without letting idiosyncratic wage risk affect the household members, and thus making aggregation intractable, we assume a continuum of unions \( j, j \in [0, 1] \). The household members supply their labor \( l_t(j) \) equally to the unions, which are monopolistically competitive and supply differentiated labor \( l_t(j) \) to intermediate firms at wage \( W_t(j) \). Every period, a union \( j \) is able to re-optimize its wage with probability \( (1 - \theta_w) \), \( 0 < \theta_w < 1 \). A union \( j \) that is not able to re-optimize indexes its nominal wage to the price level according to

\[
W_t(j) = (\Pi_{t-1})^{\chi_w} \bar{\Pi} (1 - \chi_w) W_{t-1}(j) P_{t-1},
\]

where the parameter \( \chi_w \in [0, 1] \) measures the degree of indexing, \( \bar{\Pi} \) is steady state gross inflation, and \( \mu_y \) is the gross growth rate of output (see e.g. Smets and Wouters, 2003). Thus, in the absence of price adjustment the wage still partly adapts to changes in productivity and inflation (Christiano et al., 2008), thereby assuring that no current wage contract will deviate arbitrarily far from the current optimal wage.

Household members supply the amount of labor services that is demanded at the current wage. Unions that can reset their wages choose the real wage that maximizes the expected utility of its members, taking into account the demand for its labor services

\[
l_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\eta_w,t} L_t^{\text{comp}},
\]

where \( L_t^{\text{comp}} \) is the aggregate demand for composite labor services, the respective resource constraint

\[
L_t = L_t^{\text{comp}} \int_0^1 \left( \frac{W_t(j)}{W_t} \right)^{-\eta_w,t} dj,
\]

and the aggregate wage level

\[
W_t = \left( \int_0^1 W_t(j)^{1-\eta_w,t} dj \right)^{-\eta_w,t}.
\]

The time-varying substitution elasticity \( \eta_{w,t} \) allows us to include a wage markup shock \( \mu_t^w = (\eta_{w,t} - 1)^{-1} \) that follows

\[
\log \left( \frac{\mu_t^w}{\mu_t^w} \right) = \rho_w \log \left( \frac{\mu_{t-1}^w}{\mu_t^w} \right) + \varepsilon_{w,t}^0 + \varepsilon_{w,t-4}^4 + \varepsilon_{w,t-8}^8.
\]

Including a wage markup shock is motivated by the finding that this shock is important for explaining output fluctuations (see e.g. Schmitt-Grohé and Uribe, 2010; Smets and Wouters, 2003). Thus, in the absence of price adjustment the wage still partly adapts to changes in productivity and inflation (Christiano et al., 2008), thereby assuring that no current wage contract will deviate arbitrarily far from the current optimal wage.
1.2 A DSGE-Model with Fiscal Foresight

Firm Sector

A continuum of monopolistically competitive intermediate goods firms $i$, $i \in [0, 1]$, produces differentiated intermediate goods $Y_{it}$ via a Cobb-Douglas production function, using capital services $u_{it}K_{it}$ and a composite labor bundle $L_{it}^{\text{comp}}$

$$Y_{it} = z_t (u_{it}K_{it})^\alpha (X_{t}L_{it}^{\text{comp}})^{1-\alpha} - \psi X_{t}^Y,$$

where $\alpha$ is the capital share, $z_t$ is a stationary TFP shock, $X_t$ is a non-stationary labor augmenting productivity process, and $X_{t}^Y$ is the trend of output defined in Appendix 1.B. The fixed cost of production $\psi$ is set such that profits are 0 in steady state and there is no entry or exit (Christiano, Eichenbaum, and Evans, 2005). The composite labor bundle is aggregated from differentiated labor inputs $L_{it}(j)$ with a Dixit-Stiglitz aggregator $L_{it}^{\text{comp}} = \left[\begin{array}{c} 1 \\ 0 \end{array} \right] l_{it}(j) \omega_{w,t} - 1 \omega_{w,t} dj \right] \omega_{w,t} - 1$

For the non-stationary labor augmenting productivity process $X_t$, we assume a random walk with drift in its logarithm

$$\log X_t = \log X_{t-1} + \log \mu^x_t.$$

The drift term $\mu^x_t$ is subject to contemporaneous and anticipated shocks according to

$$\log \left( \frac{\mu^x_t}{\mu^x_{t-1}} \right) = \rho_x \log \left( \frac{\mu^x_{t-1}}{\mu^x_{t-1}} \right) + \epsilon^0_{x,t} + \epsilon^4_{x,t-4} + \epsilon^8_{x,t-8}. \quad (1.14)$$

Hence, in the deterministic steady state, the natural logarithm of the non-stationary component of the neutral technology shock grows with rate $\mu^x$. The stationary technology shock $z_t$ follows an AR(1)-process with persistence $\rho_z$

$$\log z_t = \rho_z \log z_{t-1} + \epsilon^0_{z,t} + \epsilon^4_{z,t-4} + \epsilon^8_{z,t-8}. \quad (1.15)$$

We assume staggered price setting a la Calvo (1983) and Yun (1996). Each period, an intermediate firm $i$ can re-optimize its price with probability $(1 - \theta_p)$, $0 < \theta_p < 1$. If a firm $i$ cannot re-optimize the price, it is indexed to inflation $\Pi_t = \frac{P_t}{P_{t-1}}$, according
to $P_{t+1} = (\Pi_t)^{\chi_p} (\Pi)^{1-\chi_p} P_t$, where $\chi_p \in [0, 1]$ governs the degree of indexation. The intermediate firms maximize their discounted stream of profits subject to the demand from the final good producer, equation (1.17) below, applying the discount factor of their owners, the household members.

The intermediate goods are bundled by a competitive final good firm to a final good $Y_t$ using a Dixit-Stiglitz aggregation technology with substitution elasticity $\eta_p$

$$Y_t = \left( \int_0^1 Y_{it}^{-\eta_p} di \right)^{\eta_p}.$$  \hspace{1cm} (1.16)

Expenditure minimization yields the optimal demand for intermediate good $i$ as

$$Y_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\eta_p} Y_t \ \forall i.$$  \hspace{1cm} (1.17)

**Government Sector**

Government expenditures are financed by taxing profits and the return to capital services at the rate $\tau_k$ and labor income at the rate $\tau_n$. Following McGrattan (1994) and Mertens and Ravn (forthcoming), we model average tax rates as AR(2)-processes

$$\tau_n = (1 - \rho_1^n - \rho_2^n) \tau^n + \rho_1^n \tau^n_{t-1} + \rho_2^n \tau^n_{t-2} + \varepsilon_0^{\tau_n,t} + \varepsilon_4^{\tau_n,t-4} + \varepsilon_8^{\tau_n,t-8},$$  \hspace{1cm} (1.18)

$$\tau_k = (1 - \rho_1^k - \rho_2^k) \tau^k + \rho_1^k \tau^k_{t-1} + \rho_2^k \tau^k_{t-2} + \varepsilon_0^{\tau^k,t} + \varepsilon_4^{\tau^k,t-4} + \varepsilon_8^{\tau^k,t-8},$$  \hspace{1cm} (1.19)

where $\tau^k, \tau^n \in [0, 1)$ are parameters determining the unconditional mean. We are aware that using average effective tax rates for capital and labor income may be problematic for several reasons. First, the U.S. tax code does not allow for a clean division between labor and capital taxation, which are theoretical constructs. Second, using average effective tax rates may be particularly problematic for progressive labor income taxes, where marginal tax rates rather than effective tax rates influence peoples’ behavior. Nevertheless, due to data availability issues and comparability with the existing literature, we follow the path set forward by Mendoza, Razin, and

---

13For example, the personal income tax applies to both sources of income.

14In principle, it would be desirable to e.g. use the Barro and Sahasakul (1983) average marginal tax rates as extended by Barro and Redlick (2011). However, they are only available at annual frequency.
1.2 A DSGE-Model with Fiscal Foresight

Tesar (1994), Jones (2002), and Leeper, Plante, and Traum (2010) and construct average effective tax rates for capital and labor income. While this is clearly a simplifying assumption, it can be justified on grounds that dynamics of marginal and average tax rates are very similar (Mendoza, Razin, and Tesar, 1994).

Government spending $G_t$, which may be thought of as entering the utility function additively separable, displays a stochastic trend $X^G_t$. Log deviations of government spending from its trend are assumed to follow an $AR(1)$-process

$$
\log \left( \frac{g_t}{\bar{g}_t} \right) = \rho_g \log \left( \frac{g_{t-1}}{\bar{g}_{t-1}} \right) + \epsilon^0_{g,t} + \epsilon^4_{g,t-4} + \epsilon^8_{g,t-8},
$$

where $g_t = \frac{G_t}{X^G_t}$ denotes detrended government spending and $\rho_g$ is the persistence parameter.

The stochastic trend in $G_t$ is assumed to be cointegrated with the trend in output. This assures that the output share of government spending $G_t/Y_t$ is stationary, while at the same time allowing the trend in $G_t$ to be smoother than the one in $Y_t$. In particular,

$$
X^G_t = \left( X^G_{t-1} \right)^{\rho_{Xg}} \left( X^Y_{t-1} \right)^{1-\rho_{Xg}}.
$$

Lump sum transfers $T_t$ are used to balance the budget. Thus, the government budget constraint is given by

$$
G_t + T_t = \tau^n_t W_t L^{comp}_t + \tau^k_t \left( R^K_t u_t K_t + \Xi_t \right) - \Phi_t.
$$

We close the model by assuming that the central bank follows a Taylor rule that reacts to inflation and output growth:

$$
\frac{R_t}{\bar{R}} = \left( \frac{R_{t-1}}{\bar{R}} \right)^{\rho_R} \left( \frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_R\Pi} \left( \frac{Y_t}{Y_{t-1}} \right)^{\phi_RY} \exp \left( \xi^R_t \right),
$$

where $\rho_R$ is a smoothing parameter introduced to capture the empirical evidence of gradual movements in interest rates (see e.g. Clarida, Gali, and Gertler, 2000). The parameters $\phi_{RY}$ and $\phi_R\Pi$ capture the responsiveness of the nominal interest rate to

---

Note that private bonds are in zero net supply.
deviations of inflation and output growth from their steady state values. We assume that the central bank responds to changes in output rather than its level as this conforms better with empirical evidence and avoids the need to define a measure of trend growth that the central bank can observe (see Lubik and Schorfheide, 2007). $\xi_t^R$ is the i.i.d. monetary policy shock.

1.3 Model Estimation

We use a Bayesian approach as described in An and Schorfheide (2007) and Fernández-Villaverde (2010). Specifically, we use the Kalman filter to obtain the likelihood from the state-space representation of the model solution and the Tailored Randomized Block Metropolis-Hastings (TaRB-MH) algorithm (Chib and Ramamurthy, 2010) to maximize the posterior likelihood.\footnote{We used a t-distribution with 10 degrees of freedom as proposal density. The posterior distribution was computed from a 10,000 draw Monte Carlo Markov Chain, where the first 2,500 draws were discarded as burn-in draws.}

1.3.1 Data

We use quarterly U.S. data from 1955:Q1 until 2006:Q4 and include twelve observable time series: the growth rates of per capita GDP, consumption, investment, wages and government expenditure, all in real terms, the logarithm of the level of per capita hours worked, the growth rates of the relative price of investment and of total factor productivity, the log difference of the GDP deflator, and the federal funds rate. Since our main objective are the effects of tax shocks, we also include capital and labor tax rates.\footnote{Detailed data sources and the observation equation that describes how the empirical time series are matched to the corresponding model variables can be found in Appendices 1.D and 1.C.} Figure 1.2 displays the evolution of the tax rates and the government spending to GDP ratio over our sample. All three series show a large persistence. Tests against the null hypothesis of a unit root in both tax rates are borderline significant, while they cannot reject the null of a unit root in the government spending to GDP ratio. As there are theoretical reasons to believe that both the tax rates and the government spending to GDP ratio do not contain unit roots, we treat them as stationary. However, to account for the relatively persistent
deviations from the unconditional mean, we allow the trend in $G_t$ to be smoother than the one in $Y_t$.$^{18}$

Figure 1.2: Evolution of the Tax Rates and the Spending to GDP Ratio.

Notes: The top panel shows the evolution of the labor tax rate series ($\tau_n$), the middle panel the evolution of the capital tax rate series ($\tau_k$), and the bottom panel the evolution of the spending to GDP ratio ($G/Y$).

### 1.3.2 Fixed Parameters

Prior to estimation, we fix a number of parameters to match sample means (see Table 1.1). The curvature of the utility function $\sigma_c$ is set to 2. This value is consistent

$^{18}$We think that the government spending to GDP ratio actually displays mean reversion. Since the end of our sample in 2006Q4, it has returned to about 20.5 in 2010 and is thus close to its unconditional mean.
Table 1.1: Parameters Fixed Prior to Estimation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target/Motivation (matched to quarterly data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_c$</td>
<td>2</td>
<td>Common in RBC models</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.0216</td>
<td>Set labor effort in steady state to 20%</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Common in RBC models</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>0.025</td>
<td>Annual physical depreciation of 10%</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.0486</td>
<td>Set capacity utilization $u = 1$ in steady state</td>
</tr>
<tr>
<td>$\delta_\tau$</td>
<td>0.05</td>
<td>Twice the rate of physical depreciation $\delta_0$ (Auerbach, 1989)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.2935</td>
<td>Match capital share in output</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.0432</td>
<td>Set profits to zero</td>
</tr>
<tr>
<td>$\eta_p$</td>
<td>10</td>
<td>Set price markup to 11% in steady state</td>
</tr>
<tr>
<td>$\eta_w$</td>
<td>10</td>
<td>Set wage markup to 11% in steady state</td>
</tr>
<tr>
<td>$\mu_y$</td>
<td>1.0045</td>
<td>Match average sample growth rate of per capita output</td>
</tr>
<tr>
<td>$\mu^a$</td>
<td>0.9957</td>
<td>Match average sample growth rate of relative price of investment</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>0.1984</td>
<td>Match average sample labor tax rate</td>
</tr>
<tr>
<td>$\tau^k$</td>
<td>0.3880</td>
<td>Match average sample capital tax rate</td>
</tr>
<tr>
<td>$G/Y$</td>
<td>0.2031</td>
<td>Match average sample mean</td>
</tr>
<tr>
<td>$\bar{\Pi}$</td>
<td>1.0089</td>
<td>Match average sample mean</td>
</tr>
</tbody>
</table>

with most DSGE models. The discount factor $\beta$ is fixed at 0.99. We set the parameter that governs the disutility of labor effort $\gamma$ such that labor effort in steady state is 20%. We assume an annual physical depreciation rate of 10%, which corresponds to a $\delta_0$ of 0.025 per quarter. Following Auerbach (1989) and Mertens and Ravn (2011), we set the depreciation rate for tax purposes $\delta_\tau$ to twice the rate of physical depreciation, i.e. 0.05. The depreciation parameter $\delta_1$ is fixed to set the steady state capacity utilization to 1 (Christiano, Eichenbaum, and Evans, 2005). The parameter $\alpha$ is 0.2935, which matches the capital share in output over our sample, and the fixed cost parameter $\psi$ is set to ensure zero profits in steady state. We assume a steady state price and wage markup of 11% and thus set $\eta_p$ and $\eta_w$ to 10.

The steady state gross growth rates of per capita output $\mu_y$ and of the relative price of investment $\mu^a$ are set to their sample means of $1 + 0.45\%$ and $1 - 0.43\%$. The parameters $\tau^k$ and $\tau^n$, which determine the unconditional mean of the tax rates, equal the post-war sample means of 0.388 and 0.1984. We set the steady state ratio of government spending to output $G/Y$ to 0.2031, which also corresponds to the sample
1.3 Model Estimation

The steady state inflation rate corresponds to the average sample mean of 1.0089, i.e. annual inflation of 3.6%

1.3.3 Prior Distribution

Tables 1.2 and 1.3 present the prior distributions. Where available, we use prior values that are standard in the literature (e.g. Smets and Wouters, 2007) and independent of the underlying data. The autoregressive parameters of the tax processes, $\rho_{1n}, \rho_{2n}, \rho_{1k}, \rho_{2k}$, are essentially left unrestricted, but we impose stability of the AR(2)-processes.\textsuperscript{19} The other autoregressive parameters, $\rho_i$, $i \in \{pref, g, z, x, z^l, a, w\}$, are assumed to follow a beta distribution with mean 0.5 and standard deviation 0.2. We assume the standard deviations of the shocks to follow inverse-gamma distributions with prior means 0.1 and standard deviations 2. For the parameters of the Taylor-rule, $\phi_{R\Pi}$ and $\phi_{Ry}$, we impose gamma distributions with a prior mean of 1.5 and 0.5, respectively, while the interest rate smoothing parameter $\rho_R$ has the same prior distribution as the persistence parameters of the shock processes. The habit parameter $\phi_c$ is assumed to be beta distributed with a prior mean of 0.7, which is standard in the literature. Following Justiniano, Primiceri, and Tambalotti (2010b), the parameter determining the Frisch elasticity of labor supply $\sigma_l$ is assumed to follow a gamma distribution with a prior mean of 2 and a standard deviation of 0.75. The prior distribution for the parameter governing the wealth elasticity of labor supply $\sigma_s$ is a beta distribution with mean 0.5 and standard deviation 0.2. We impose an inverse-gamma distribution with prior mean of 0.5 and standard deviation of 0.15 for $\delta_2/\delta_1$, the elasticity of marginal depreciation with respect to capacity utilization. The parameters governing the indexation of prices and wages, $\chi_p$ and $\chi_w$, each are beta distributed with mean 0.5 and standard deviation 0.2. For the Calvo parameters $\theta_w$ and $\theta_p$ we assume a beta distribution with a prior mean of 0.5, which corresponds to price and wage contracts having an average length of half a year (Smets and Wouters, 2007). Finally, we follow the literature (e.g. Justiniano, Primiceri, and Tambalotti, 2010a; Smets and Wouters, 2007) and impose a gamma prior with mean 4 for the

\textsuperscript{19}Specifically, we impose a uniform prior for each of the corresponding autoregressive roots over the stability region $(-1, +1)$. Let $\xi_1$ and $\xi_2$ be the roots of such an $AR(2)$-process. The autoregressive parameters corresponding to these roots can be recovered from: $\rho_1 = \xi_1 + \xi_2$ and $\rho_2 = -\xi_1 \xi_2$.
Chapter 1

parameter controlling investment adjustment costs $\kappa$.

1.3.4 Posterior Distribution

The last four columns of Tables 1.2 and 1.3 display the mean, the standard deviation and the 90%-posterior intervals for each of the estimated parameters. Most estimated parameters and shock processes are in line with previous studies on the determinants of business cycle fluctuations, both with those using only contemporaneous shocks (e.g. Justiniano, Primiceri, and Tambalotti, 2010a; Smets and Wouters, 2007) as well as those including contemporaneous and anticipated shocks (Fujiwara, Hirose, and Shintani, 2011; Khan and Tsoukalas, 2011; Schmitt-Grohé and Uribe, 2010).

However, some estimates deserve further comment. We find a considerable degree of internal habits with $\phi_c = 0.86$, which is right between the estimates obtained by Smets and Wouters (2007) and Schmitt-Grohé and Uribe (2010). The posterior mean of the parameter governing the wealth elasticity ($\sigma_s = 0.1$) implies a relatively low wealth elasticity of labor supply and, thus, preferences that are close to the ones proposed by Greenwood, Hercowitz, and Huffman (1988).

Schmitt-Grohé and Uribe (2010) find an even lower wealth elasticity of almost zero. Khan and Tsoukalas (2011), on the other hand, estimate the wealth elasticity of labor to be quite high at 0.85. A possible explanation for these differing estimates is the inclusion of government spending as an observable. Increases in government spending may entail positive consumption responses (Blanchard and Perotti, 2002; Galí, López-Salido, and Vallés, 2007), a behavior which can be explained by a New-Keynesian model with a low wealth elasticity (Monacelli and Perotti, 2008). Even in studies finding a negative consumption response (see e.g. Ramey, 2011), this negative response tends to be relatively small or hardly distinguishable from 0, also suggesting the presence of a low wealth effect. Including government spending as an observable restricts the parameter governing the wealth elasticity to a low value. In our model, this happens although the consumption response to a government spending shock is estimated to be negative. On the other hand, without the observable government spending as in Khan and Tsoukalas (2011), this parameter remains mostly unrestricted with regard

Note, however, that in the presence of habits, even a value of $\sigma_s = 0$ still implies the presence of a wealth effect, see Monacelli and Perotti (2008).
1.3 Model Estimation

to the effects of government spending on consumption.\footnote{A small wealth effect also helps in explaining the empirical behavior of labor market variables (Gali, Smets, and Wouters, 2011).}

Turning to the nominal rigidities in our model, we find that prices are on average adjusted about every three quarters, while the Calvo parameter for wages implies a high degree of wage stickiness. The degree of price indexation is low \( \chi_p = 0.06 \) and in a similar range as in Justiniano, Primiceri, and Tambalotti (2011). Wages, on the other hand, are indexed to inflation with a higher proportion than prices \( \chi_w = 0.6 \), which corresponds well with the estimates in Smets and Wouters (2007).

The parameters of the Taylor rule are in line with previous estimates (e.g. Clarida, Gali, and Gertler, 2000). They imply a high degree of interest rate smoothing \( \rho_R = 0.86 \), a strong response to inflation \( \phi_R = 2.96 \), and a moderate value for the standard deviation of the monetary policy shock \( \sigma_R = 0.251\% \).

With the exception of the non-stationary technology shock, all shocks are estimated to be highly persistent, with \( AR(1) \)-coefficients ranging from 0.94 for the government spending shock to 0.99 for the preference, the stationary technology, and the non-stationary investment-specific technology shock. The non-stationary productivity component has a relatively low serial correlation of 0.34, a value commonly found in the literature (e.g. Justiniano, Primiceri, and Tambalotti, 2011).

The contemporaneous shock as well as the 4 quarter anticipated non-stationary technology shock have relatively low standard deviations of 0.04\% and 0.03\%, respectively, whereas the two year anticipated shock is the most important one with a standard deviation of 0.6\%. A similar pattern emerges for the stationary technology shock. In this case, however, the standard deviation of the unanticipated component has a similar size as the 8 quarter anticipated component, 0.74\% and 0.73\%, whereas the 4 quarter anticipated shock is less important with a standard deviation of 0.18\%.

Examining investment-specific technology shows that investment-specific growth displays the same pattern as neutral technology growth. The shock with the longest anticipation horizon is the most important one, having the highest standard deviation \( \sigma_a^8 = 0.14\% \), albeit in this case it is only slightly higher than the one for the contemporaneous shock \( \sigma_a^0 = 0.11\% \). The 4 quarter anticipated shock, on the other hand, is negligible \( \sigma_a^4 = 0.04\% \). In contrast, for stationary investment-specific technology anticipation does not play a role, the standard deviations are less than
0.05%, while the unanticipated stationary shock component has a higher standard deviation than the unanticipated non-stationary investment-specific technology shock ($\sigma_0 = 0.31\%$).

Another shock, where the anticipated shock components are negligible, is the wage markup shock. While the standard deviation of the unanticipated shock is relatively high, the anticipated shocks have very low standard deviations that are below 0.04%. In contrast, the surprise wage markup shock has a high standard deviation of almost 46%, which is consistent with evidence from Smets and Wouters (2007) and Galí, Smets, and Wouters (2011), who showed this shock to be the most important driver of business cycles.\(^{22}\)

Next, we direct our focus to the fiscal policy shock processes. Both tax processes show a very high persistence, with the roots of the autoregressive processes implying autoregressive parameters of $\rho_{n1} = 0.770$, $\rho_{n2} = 0.228$, $\rho_{k1} = 1.604$, and $\rho_{k2} = -0.605$, respectively.\(^{23}\) The posterior estimates suggest that for government spending and labor taxes fiscal foresight is rather limited. The unanticipated government spending shock has a volatility of 3%, a value also found by Leeper, Plante, and Traum (2010). The volatilities of the anticipated shock components, on the other hand, are rather small, $\sigma_{4g} = 0.03\%$ and $\sigma_{8g} = 0.04\%$. A similar pattern emerges for the labor tax process $\tau_{nt}$. The shock with the largest volatility is the unanticipated component $\epsilon_{0}\tau_{nt}$ with 0.48%, while the anticipated components have a similar size as the anticipated government spending shocks. Only for the capital tax rate, news shocks display a higher standard deviation. Particularly, compared to the shocks to the labor tax process, the shocks $\epsilon_{i}\tau_{kt}$ to the capital tax process $\tau_{kt}$ display a much higher volatility. The unanticipated component $\epsilon_{0}\tau_{kt}$ has the highest standard deviation of 0.92%, while the anticipated components have smaller, but still sizeable standard deviation, $\sigma_{4k} = 0.46\%$ and $\sigma_{8k} = 0.65\%$.

Table 1.4 compares empirical moments of the data to the corresponding moments

---

\(^{22}\)Note that the shock applies to the net markup so a 46% shock increases the markup from 11% to about 16%. Chari, Kehoe, and McGrattan (2009) point out that wage markup shocks cannot be distinguished from labor supply shocks. For policy makers this distinction matters, since both shocks entail different policy implications (Galí, Smets, and Wouters, 2011). However, as we are not interested in optimal policy, it is not important to identify the two shocks separately.

\(^{23}\)The high persistence of the labor tax rate has, for example, been documented in Cardia, Kozhaya, and Ruge-Murcia (2003).
1.4 Business Cycle Effects of Fiscal News

We are now in a position to analyze the dynamic effects of fiscal news. Given the estimated deep parameters of the model, we compute forecast error variance decompositions to trace out the shocks’ contributions to business cycle volatility. To better understand the dynamic effects of news shocks, we then analyze their transmission into the economy in Section 1.4.2.

1.4.1 Variance Decomposition

Results

We use our estimated model to analyze the quantitative importance of the different anticipated and surprise shocks for explaining business cycles. To this end, we compute conditional and unconditional forecast error variance decompositions for the growth rates of output, consumption, investment, hours, wages, the Federal funds rate, and inflation (see Table 1.5).24

Overall, we find that news shocks on average explain between 10 and 30 percent of the variance of the variables considered. However, fiscal foresight only plays a very limited role. Of the three types of fiscal foresight we consider, only the anticipated capital tax shock has a sizeable variance contribution. While news about future capital taxes contribute only 2 percent to output growth variance, they matter for inflation and interest rate variability, explaining more than 10 percent of the variability of inflation and interest rates at forecast horizons longer than three years. This makes them the third largest source of inflation and interest rate volatility, only behind preference and unanticipated capital tax shocks. Together, surprise and anticipated capital tax shocks explain around 40 to 50 percent of inflation and interest rate

24For ease of exposition we have combined the two anticipated shock components into one and left out three anticipated shocks (stationary investment-specific, wage markup, and government spending) that each contributed less than 0.01 percent to the variance of the variables.
fluctuations. In contrast, news about labor tax and government spending shocks explain at most 0.01 percent of the variance of any of the seven variables considered.

More important than fiscal foresight are the surprise components of the fiscal variables. As already noted, besides the preference shock, the surprise capital tax shock is the most important factor for the variance of the Federal funds rate and inflation. Moreover, it accounts for 2 to 3 percent of output fluctuations. While the surprise government spending shock $\varepsilon_0^g$ accounts for almost 10 percent of the output growth variance at the five year horizon and even more at shorter horizons, it hardly contributes anything to the other variables' fluctuations.

Whereas fiscal foresight seems to be of only minor importance for the fluctuations of output, consumption, and investment, other news shocks contribute significantly to their variance. The news shocks that matter most are news about stationary technology, which account for 8 to 12 percent of the variance of output and consumption. News about non-stationary technology mostly affects the volatility of wages, predominantly at long horizons. At the five year horizon, it is the single most important factor affecting wage volatility. News about non-stationary investment-specific technology explain around 8 percent of the variance of investment at all horizons and about the same amount of the variance of hours (at the five year horizon). In contrast, the news components of stationary investment-specific technology and the wage markup shock account for at most 0.01 percent of the variance of any variable we consider.

In general, the importance of news shocks increases at longer forecast horizons. E.g., anticipated shocks account for a larger share of output volatility at the five year horizon (21%) than at the one year horizon (11%).

Turning to the surprise shocks, we find the most important drivers of business cycles to be wage markup, preference, and unanticipated technology shocks. At business cycle frequencies, these shocks combined explain about 60 to 70 percent of the fluctuations of real variables. E.g., at the 20 period forecast horizon, these three shocks account for 31, 21, and 16 percent of output volatility, respectively. Inflation and interest rate variability are mostly explained by preference and capital tax shocks, whereas wage fluctuations are mainly driven by technology shocks, especially anticipated non-stationary technology shocks. Lastly, the monetary policy shock
plays a minor role in accounting for macroeconomic fluctuations, a result similar to Smets and Wouters (2007). It explains around 15 percent of the Federal funds rate volatility, but only at the short term, i.e. horizons of about one year, and has much smaller contributions for the other variables.

Discussion

Using a DSGE-based estimation approach to determine the importance of news about fiscal policy, we find that fiscal foresight only plays a minor role in explaining business cycle fluctuations. Specifically, using full information Bayesian estimation and accounting for different kinds of shocks, we find tax shocks and, in particular, news about taxes to explain less than 3 percent of output growth fluctuations. This compares to about 25 percent in the VAR study of Mertens and Ravn (forthcoming), indicating that the rigid anticipation structure and the strict exogeneity assumption in the latter paper may be problematic (see also Leeper, Walker, and Yang, 2011).

Our estimates also attribute less than one third of output fluctuations to surprise tax shocks, which was found by McGrattan (1994). However, her paper only featured TFP, government spending, and tax rate shocks. In contrast, our analysis features a richer set of shocks commonly thought to be essential for explaining business cycles (Chari, Kehoe, and McGrattan, 2007; Smets and Wouters, 2007).

Regarding the evidence on the effects of news shocks on the business cycles, our result of 10 to 30 percent of the variance of output growth being attributable to anticipated shocks squares well with the evidence found by Forni, Gambetti, and Sala (2011) and Barsky and Sims (2011). Using a factor model, Forni, Gambetti, and Sala (2011) find that around 20 percent of output volatility is explained by technology and 10 percent by news about technology, while Barsky and Sims (2011), in a VAR, attribute 10 to 40 percent to news shocks.

Fujiwara, Hirose, and Shintani (2011) and Khan and Tsoukalas (2011), using an estimated DSGE model with nominal rigidities, find a technology news contribution to output variance of 8.5 and 1.6 percent, respectively, which is lower than our own estimates. On the other hand, Schmitt-Grohé and Uribe (2010) find that news about technology account for as much as 41 percent of output variance. Part of this higher number can be attributed to the absence of nominal rigidities in their model.
Overall and consistent with these studies, news shocks contribute a higher share to the unconditional variance of nominal variables (wages, inflation, interest rate) than to the variance of real variables (output, consumption, investment, hours). However, allowing anticipation not only for TFP but also for other shocks, leads to a higher relative contribution of news shocks. Whereas the contribution of anticipated shocks in the study by Fujiwara, Hirose, and Shintani (2011) ranges from 4 percent (to the variance of investment) to 15 percent (to inflation volatility), we find contributions of anticipated shocks (combining all shocks) between 19 percent (investment and consumption volatility) and 52 percent (variance of wages).

Turning to the role of unanticipated shocks, we see that while the investment-specific technology shock has been identified as an important driver of business cycles by previous studies (Davis, 2007; Fisher, 2006; Justiniano, Primiceri, and Tambalotti, 2010a), it is of lesser importance in our case and contributes a smaller fraction to fluctuations than TFP shocks. The contributions of non-stationary investment-specific productivity vary between 5 and 15 percent, whereas stationary investment-specific technology explains hardly 1 percent. The difference to the previous studies finding the high contribution of investment-specific technology stems from our decision to include the relative price of investment as an observable. Recent studies including the relative price of investment as an observable find similarly small contributions of investment-specific technology (Justiniano, Primiceri, and Tambalotti, 2011; Schmitt-Grohé and Uribe, 2010). However, we have to stress that both the stationary as well as the non-stationary investment-specific productivity shock pertain to the relative price of investment and are accordingly mapped to this observable. Thus, our stationary investment-specific technology shock is not directly comparable to the stationary investment-specific technology shock in Schmitt-Grohé and Uribe (2010). This could explain the starkly differing results regarding the effects of this particular shock for output and investment fluctuations, 30 to 60 percent in their case vs. less than 1 percent in our case.

---

25 Models that do not use the relative price of investment as an observable variable usually imply wrong moments for this series (Justiniano, Primiceri, and Tambalotti, 2011). When this problem is eliminated, the variance contribution of investment-specific technology shocks tends to disappear.  
26 The observation equation in Appendix 1.C shows the exact mapping.
1.4 Business Cycle Effects of Fiscal News

1.4.2 Impulse Responses

In order to better understand what drives the results of the previous section, we analyze the impulse responses to stationary TFP shocks and to capital tax rate shocks. We choose to focus on these shocks as they are the technology and fiscal policy shock, respectively, where the anticipated component contributes most to business cycle variance.\(^{27}\)

Figure 1.3 shows the impulse responses to an unanticipated (solid line) and an eight period anticipated (dashed line) one percentage point cut of the capital tax rate.\(^{28}\) The top left panel shows the impulse response for the capital tax rate that is shocked. The actual response of the exogenous capital tax rate is the same after the surprise and anticipated tax shock, because the only difference between the two cases is the time at which the tax change that happens at \(t = 0\) is known. But the other variables react differently, because with anticipation the future realization of the tax rate is already known at \(t = -8\) and agents immediately start to optimally respond to this information.

First, consider the solid line representing the impulse responses to a surprise 1 percentage point decrease in the capital tax rate. This tax cut acts expansionary and leads to an increase in output, investment, and consumption on impact. The effect is quite large due to the strong estimated persistence of the shock process. Consistent with the evidence of high multipliers for tax rates (Mountford and Uhlig, 2009; Romer and Romer, 2010), an initial 1 percentage point decrease in the capital tax rate leads to a peak output response of 1.25 percent. Labor and capital services increase in a hump shaped manner after the realization. For capital services, this is driven by the higher after-tax rental rate that can be earned after the tax cut. Note that the gross value of the rental rate decreases, reflecting the decreased tax wedge. The increase

\(^{27}\)Although we find the preference and wage markup shocks to be the most important drivers of business cycles, we omit analyzing their impulse responses as their importance and behavior is already well understood (see e.g. Gali, Smets, and Wouters, 2011; Smets and Wouters, 2007). The impulse responses to a government spending news shock are very similar to the ones in Ramey (2011), albeit the negative response of private consumption is more persistent in our setup.

\(^{28}\)For the surprise shock, this roughly corresponds to a one standard deviation shock as \(\sigma_{\tau_k}^0 = 0.923\%\). For the eight period anticipated shock, \(\sigma_{\tau_k}^8 = 0.645\%\), so that we have re-scaled the size of this shock to make both shocks comparable. Note that the impulse responses are semi-elasticities, i.e. they are measured in percent of the steady state values of the corresponding variables.
Figure 1.3: Impulse Responses to Unanticipated and Anticipated Capital Tax Shocks

Notes: solid line: impulse responses to an unanticipated 1 percentage point cut of the capital tax rate $\tau^k$; dashed line: impulse responses to an eight period anticipated 1 percentage point cut of the capital tax rate $\tau^k$ that becomes known at $t = -8$ and effective at $t = 0$. All impulse responses are semi-elasticities and measured in percent. Inflation and the policy rate are measured as gross rates so that the responses can be interpreted as percentage point changes.
in capital services also raises the marginal product of labor, leading to an initial jump in the real wage as a fraction of unions is able to reset wages in the current period and to a further rise over time when additional unions are able to reset their nominal wages. The initial increase of the real wage is amplified by an overshooting of the nominal wage, which is indexed to past inflation, due to a drop in inflation. Current inflation falls due to the positive supply side effect of the tax decrease. This positive effect on inflation is also the reason why the policy rate falls considerably, accommodating the expansion and further fueling investment and consumption.

Although the impulse responses for the eight period anticipated tax shock look very similar, there are two major differences. First, agents have more time to adjust and already react during the anticipation phase. Hence, the impulse responses are now more drawn out. Reacting immediately to an anticipated tax shock is optimal for the agents, because the estimated degrees of consumption habits, capital adjustment costs, capital utilization, and nominal rigidities imply that large abrupt changes in important choice variables are welfare reducing and must be avoided. As a result of these more gradual and hence more resource-saving responses, the peak responses of all variables are now higher than for the case of a comparable surprise tax cut and generally occur earlier relative to the shock realization at t=0. Note that relative to the announcement of the shocks, i.e. the point in time where the horizon for the forecast error variance decomposition starts, the peak responses generally occur later for the news shocks. This peak response at later horizons for news shocks explains why their importance in the forecast error variance decomposition tends to be larger at later horizons.

Second, in contrast to the unanticipated shock, agents now substitute labor services for capital services, leading to an immediate increase in the former and a decrease in the latter. Only when the tax shock realizes, there is a jump in capital services. The higher production resulting from the increase in labor services and the resources saved through the initially lower depreciation resulting from the weaker capital use allows to increase consumption during the anticipation phase. The net result of this substitution of labor for capital services with the simultaneous increase in consumption and investment expenditures is a slight inflationary pressure in the first period. As a

\[29\text{I.e. } t=-8 \text{ for the anticipated shock and } t=0 \text{ for the surprise shock.}\]
response, the central bank somewhat tightens its policy. However, the negative supply side effect of the input substitution subsides with the subsequent further increase in labor supply. This increase is driven by the household’s desire to increase the physical capital stock through investment while also keeping up consumption. As a result, inflationary pressures abate and give room to an accommodating policy stance.

Note that physical investment in the capital stock slightly decreases initially. This behavior is due to the depreciation allowances, whose present value for new investment decreases with the future tax bill from which it is deducted. But, in contrast to the results of Mertens and Ravn (2011), this incentive to disinvest is rather mild. Hence, in our estimated model, the announcement of a tax cut is insufficient to generate the investment-driven slump during the anticipation phase of a tax cut found in their model. This difference can be explained by the different estimation procedures used. Mertens and Ravn (2011) rely on an impulse response matching technique, where the empirical impulse responses were derived from a VAR using a narrative identification scheme. The impulse responses to be matched by the model were only the ones to anticipated and unanticipated labor and capital tax shocks. In contrast, our estimation uses full information techniques and thus tries to match all moments given the full set of exogenous driving forces of the model. While the crucial investment adjustment cost and capital utilization cost parameters are actually estimated to generate a drop in investment as in Mertens and Ravn (2011), it is the monetary policy response that dampens this drop. When setting the output coefficient in the Taylor rule to 0 and the inflation response to 2, the investment response becomes stronger and leads to an initial drop in output with a subsequent boom. This indicates the importance of controlling for the stance of monetary policy when tracing out the effects of fiscal shocks.\footnote{On this issue, see also Leeper (2010).}

Figure 1.4 displays the impulse responses to one standard deviation surprise (solid line) and anticipated (dashed line) stationary TFP shocks.\footnote{We scaled the news shock by 1.03 to have exactly the same standard deviation as the surprise shock.} The result of a surprise increase in total factor productivity is a prolonged boom driven by both consumption and investment. Consistent with a typical supply side shock, inflation decreases considerably with the central bank lowering the policy rate by 20 basis points in
Figure 1.4: Impulse Responses to Unanticipated and Anticipated Stationary TFP Shocks

Notes: solid line: impulse responses to an unanticipated one standard deviation increase in stationary TFP $z$; dashed line: impulse responses to an eight period anticipated one standard deviation increase in stationary TFP $z$ that becomes known at $t = -8$ and effective at $t = 0$. All impulse responses are semi-elasticities and measured in percent. Inflation and the policy rate are measured as gross rates so that the responses can be interpreted as percentage point changes.
response. This in turn leads to an increase in the real wage and a subsequent increase in the labor services used.

For the eight period anticipated increase in technology, we observe an immediate increase in output, investment, and consumption during the anticipation phase due to the entailed wealth effect. This boom occurs already before the technology has actually increased and is fueled by a rise in both capital and labor services.\textsuperscript{32} In this regard, the response differs from the response to an anticipated capital tax shock, where a substitution of capital services for labor services is observed. The reason for the difference is that, for the anticipated TFP shock, agents have a stronger incentive to increase investment during the anticipation phase. In contrast, for the anticipated capital tax shock, investment falls slightly on announcement due to the decrease in the present value of the depreciation allowances.

Lastly, to better understand the contribution of capital tax and stationary TFP shocks to business cycle variance, it is worth comparing the relative size and persistence of the impulse responses of output, inflation, and the nominal interest rate to these shocks. As can be seen from the the upper right panels of Figures 1.3 and 1.4, the peak response of output to an average TFP shock is about 80\% higher than to an average capital tax shock, although the latter is somewhat more persistent.\textsuperscript{33} This difference in the size of the output responses explains why stationary TFP shocks are more important for the volatility of output than capital tax shocks. In contrast, both the inflation and the policy rate responses to capital tax shocks have higher peaks and show more persistence. In particular, the average surprise TFP shock leads to a peak reduction in the nominal interest rate of -0.2\%, while the average surprise tax shock leads to a drop of -0.4\%. As this larger response is also more persistent, the difference in response sizes explains why capital taxes are rather important for the variance of inflation and the nominal interest rate, while they are less important for explaining output variance.

\textsuperscript{32}This observation is consistent with Jaimovich and Rebelo (2009), who show theoretically that a low estimated wealth elasticity of labor supply facilitates positive comovement of output, consumption, and hours in response to TFP news.

\textsuperscript{33}Note also that the average anticipated capital tax shock is roughly 40\% smaller than the one depicted due to re-scaling.
1.5 Conclusion

In this chapter, we analyzed the contribution of fiscal foresight about labor and capital tax rates and government spending to business cycle volatility in an estimated New Keynesian DSGE model. Computing forecast error variance decompositions, we found that fiscal foresight only plays a limited role for business cycle fluctuations. Its variance contribution was mostly confined to inflation and interest rate fluctuations, where anticipated capital tax shocks were responsible for between 5 and 15 percent of the total variance.

Our results show that accounting for fiscal foresight does not qualitatively alter the importance of traditional business cycle factors like technology, wage markup, and preference shocks (see e.g. Smets and Wouters, 2007).

Structural estimation always runs the risk of misspecifying the underlying model structure. Hence, future work should test whether the results obtained here are robust against the specification of different fiscal rules where taxes respond to debt and possibly output as in Leeper, Plante, and Traum (2010) and Forni, Monteforte, and Sessa (2009). Similarly, it might be worthwhile to explore the effects of a more detailed modeling of the U.S. tax code as suggested by McGrattan (2011). However, given the need for non-linear modeling and filtering required in this case and the typically large state space of models with anticipation effects, estimating the effects of fiscal news in such a model will be an extremely challenging computational task. Finally, the role of the information structure assumed in the present work should be further scrutinized as the particular choice of information structures may matter (Leeper and Walker, 2011).
Appendix to Chapter 1

1.A Tables

Table 1.2: Prior and Posterior Distributions of Preference and Technology Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distribution</td>
<td>Mean</td>
<td>Std. Dev.</td>
<td>Mean</td>
</tr>
<tr>
<td>$\phi_c$</td>
<td>Beta</td>
<td>0.7</td>
<td>0.1</td>
<td>0.858</td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>Gamma</td>
<td>2</td>
<td>0.75</td>
<td>3.410</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.101</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Gamma</td>
<td>4</td>
<td>1.5</td>
<td>4.860</td>
</tr>
<tr>
<td>$\delta_2/\delta_1$</td>
<td>Inv.-Gamma</td>
<td>0.5</td>
<td>0.15</td>
<td>0.280</td>
</tr>
<tr>
<td>$\chi_w$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.590</td>
</tr>
<tr>
<td>$\chi_p$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.059</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.938</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
<td>0.662</td>
</tr>
</tbody>
</table>

Table 1.3: Prior and Posterior Distributions of the Shock Processes

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distribution</td>
<td>Mean</td>
</tr>
<tr>
<td>Preference Shock</td>
<td>$\rho_{pref}$</td>
<td>Beta</td>
</tr>
<tr>
<td></td>
<td>$\sigma_{pref}$</td>
<td>Inv.-Gamma</td>
</tr>
<tr>
<td>Wage Markup Shock</td>
<td>$\rho_w$</td>
<td>Beta</td>
</tr>
<tr>
<td></td>
<td>$\sigma^0_w$</td>
<td>Inv.-Gamma</td>
</tr>
<tr>
<td></td>
<td>$\sigma^4_w$</td>
<td>Inv.-Gamma</td>
</tr>
<tr>
<td></td>
<td>$\sigma^8_w$</td>
<td>Inv.-Gamma</td>
</tr>
<tr>
<td>Parameter</td>
<td>Prior distribution</td>
<td>Prior</td>
</tr>
<tr>
<td>-------------------</td>
<td>--------------------</td>
<td>-------</td>
</tr>
<tr>
<td></td>
<td>Distribution</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Stationary Technology Shock</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma^0_z$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td>$\sigma^4_z$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td>$\sigma^8_z$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td><strong>Non-stationary Technology Shock</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma^0_x$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td>$\sigma^4_x$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td>$\sigma^8_x$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td><strong>Stationary Investment-Specific Productivity Shock</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_{zI}$</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma^0_{zI}$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td>$\sigma^4_{zI}$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td>$\sigma^8_{zI}$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td><strong>Non-stationary Investment-Specific Productivity Shock</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma^0_a$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td>$\sigma^4_a$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td>$\sigma^8_a$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
</tr>
<tr>
<td><strong>Taylor Rule and Monetary Policy Shock</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_R$</td>
<td>Beta</td>
<td>0.5</td>
</tr>
<tr>
<td>$\phi_{Rm}$</td>
<td>Gamma</td>
<td>1.5</td>
</tr>
<tr>
<td>$\phi_{Rf}$</td>
<td>Gamma</td>
<td>0.5</td>
</tr>
<tr>
<td>$\sigma_R$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
</tr>
</tbody>
</table>
## Chapter 1

Prior and Posterior Distributions of the Shock Processes - Continued

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Distribution</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>Std. Dev.</td>
</tr>
<tr>
<td><strong>Government Spending Shock</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>$\rho_{2g}$</td>
<td>Beta</td>
<td>0.5</td>
<td>0.2</td>
</tr>
<tr>
<td>$\sigma^0_g$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma^4_g$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma^8_g$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td><strong>Labor Tax Shock</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\xi_{n1}$</td>
<td>Uniform</td>
<td>0</td>
<td>0.577</td>
</tr>
<tr>
<td>$\xi_{n2}$</td>
<td>Uniform</td>
<td>0</td>
<td>0.577</td>
</tr>
<tr>
<td>$\sigma^0_{\tau n}$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma^4_{\tau n}$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma^8_{\tau n}$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td><strong>Capital Tax Shock</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\xi_{k1}$</td>
<td>Uniform</td>
<td>0</td>
<td>0.577</td>
</tr>
<tr>
<td>$\xi_{k2}$</td>
<td>Uniform</td>
<td>0</td>
<td>0.577</td>
</tr>
<tr>
<td>$\sigma^0_{\tau k}$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma^4_{\tau k}$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma^8_{\tau k}$</td>
<td>Inv.-Gamma</td>
<td>0.1</td>
<td>2</td>
</tr>
</tbody>
</table>
### Table 1.4: Model and Data Moments

<table>
<thead>
<tr>
<th></th>
<th>Model Data</th>
<th>Model Data</th>
<th>Model Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\rho(x_t, y_t)$</td>
<td>$\sigma(x_t)$</td>
<td>$\rho(x_t, x_{t-1})$</td>
</tr>
<tr>
<td>$\Delta \log (Y_t)$</td>
<td>1</td>
<td>1.00</td>
<td>0.02</td>
</tr>
<tr>
<td>$\Delta \log (C_t)$</td>
<td>0.631</td>
<td>0.50</td>
<td>0.01</td>
</tr>
<tr>
<td>$\Delta \log (A_t)$</td>
<td>0.89</td>
<td>0.69</td>
<td>0.09</td>
</tr>
<tr>
<td>$\Delta \log (A_t)$</td>
<td>-0.1792</td>
<td>-0.13</td>
<td>0.01</td>
</tr>
<tr>
<td>$\Delta \log (TFP_t)$</td>
<td>0.1669</td>
<td>0.09</td>
<td>0.01</td>
</tr>
<tr>
<td>$\log (R_t)$</td>
<td>0.1453</td>
<td>-0.19</td>
<td>0.11</td>
</tr>
<tr>
<td>$\log (\Pi_t)$</td>
<td>0.0575</td>
<td>-0.29</td>
<td>0.04</td>
</tr>
</tbody>
</table>

**Notes:** Time Series $x_t$ are the growth rates of output ($\Delta \log (Y_t)$), consumption ($\Delta \log (C_t)$), investment ($\Delta \log (A_tI_t)$), investment-specific technology ($\Delta \log (A_t)$), TFP ($\Delta \log (TFP_t)$), the level of the net nominal interest rate ($\log (R_t)$), and the level of net inflation ($\log (\Pi_t)$).
<table>
<thead>
<tr>
<th>Pref./Wage Markup</th>
<th>Technology</th>
<th>Policy</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>( \epsilon_{i}^{0} )</td>
</tr>
<tr>
<td>4 Periods</td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>5.00</td>
<td>35.26</td>
</tr>
<tr>
<td>Cons.</td>
<td>18.93</td>
<td>44.73</td>
</tr>
<tr>
<td>Invest.</td>
<td>37.59</td>
<td>17.99</td>
</tr>
<tr>
<td>Hours</td>
<td>6.39</td>
<td>25.73</td>
</tr>
<tr>
<td>Wages</td>
<td>7.39</td>
<td>26.35</td>
</tr>
<tr>
<td>FFR</td>
<td>16.46</td>
<td>2.39</td>
</tr>
<tr>
<td>Infl.</td>
<td>19.15</td>
<td>6.82</td>
</tr>
<tr>
<td>8 Periods</td>
<td>11.18</td>
<td>35.13</td>
</tr>
<tr>
<td>GDP</td>
<td>15.94</td>
<td>46.39</td>
</tr>
<tr>
<td>Cons.</td>
<td>41.33</td>
<td>18.32</td>
</tr>
<tr>
<td>Invest.</td>
<td>8.78</td>
<td>52.59</td>
</tr>
<tr>
<td>Hours</td>
<td>6.53</td>
<td>24.11</td>
</tr>
<tr>
<td>Wages</td>
<td>21.19</td>
<td>3.23</td>
</tr>
<tr>
<td>FFR</td>
<td>19.15</td>
<td>5.09</td>
</tr>
<tr>
<td>20 Periods</td>
<td>21.15</td>
<td>30.89</td>
</tr>
<tr>
<td>GDP</td>
<td>19.72</td>
<td>44.09</td>
</tr>
<tr>
<td>Cons.</td>
<td>45.16</td>
<td>16.75</td>
</tr>
<tr>
<td>Invest.</td>
<td>22.63</td>
<td>50.14</td>
</tr>
<tr>
<td>Hours</td>
<td>6.16</td>
<td>2.26</td>
</tr>
<tr>
<td>Wages</td>
<td>31.49</td>
<td>4.15</td>
</tr>
<tr>
<td>FFR</td>
<td>31.97</td>
<td>6.27</td>
</tr>
<tr>
<td>Uncond. Variance</td>
<td>23.57</td>
<td>26.88</td>
</tr>
<tr>
<td>GDP</td>
<td>24.27</td>
<td>37.52</td>
</tr>
<tr>
<td>Cons.</td>
<td>44.46</td>
<td>15.95</td>
</tr>
<tr>
<td>Invest.</td>
<td>46.58</td>
<td>16.62</td>
</tr>
<tr>
<td>Hours</td>
<td>19.01</td>
<td>4.34</td>
</tr>
<tr>
<td>Wages</td>
<td>31.89</td>
<td>1.64</td>
</tr>
<tr>
<td>FFR</td>
<td>31.43</td>
<td>1.69</td>
</tr>
</tbody>
</table>

Notes: Variance decompositions are performed at the posterior mean. \( \epsilon_{i}^{0} \) represents contemporaneous shock components; \( \epsilon_{i}^{4,8} \) represents the sum of the 4 and 8 quarter anticipated shock components. For ease of exposition, we leave out anticipated stationary investment-specific, wage-markup, and government spending shocks, since these shocks contribute less than 0.01% to the variances of the variables.
1.B Stationary Equilibrium

In order to derive a state-space representation of the model, the model presented in the main text is solved by using a first-order perturbation method. However, due to the two integrated processes \( A_t \) and \( X_t \), which grow with rates

\[
\mu^a_t = \frac{A_t}{A_{t-1}}, \quad \mu^x_t = \frac{X_t}{X_{t-1}},
\]

the model has to be detrended first in order to induce stationarity and to have a well-defined steady state. \( Y_t, C_t \) and \( W_t \) inherit the trend \( X_t^Y = A^{\alpha - 1} X_t \), which corresponds to a growth rate of

\[
\mu^y_t = (\mu^a_t)^{\alpha - 1} \mu^x_t.
\]

\( K_t \) and \( I_t \) inherit the trend \( X_t^K = A^{\frac{1}{\alpha - 1}} X_t \) and thus grow with

\[
\mu^k_t = \mu^I_t = (\mu^a_t)^{\frac{1}{\alpha - 1}} \mu^x_t.
\]

\( G_t \) inherits \( X_t^G = \left( X_t^{G_{t-1}} \right)^{\rho_{xg}} \left( X_t^{Y_{t-1}} \right)^{1 - \rho_{xg}} \) due to the assumed cointegrated trend with output. It hence grows with rate

\[
x_t^g = \left( \frac{x_t^{a_{t-1}}}{\mu_t^x} \right)^{\rho_{xg}}.
\]

The detrending is performed by dividing the trending model variables by their respective trend. For the estimation of our structural model, these stationary model variables are matched to the data presented in Appendix 1.D.
Chapter 1

1.C Observation Equation

The observation equation describes how the empirical times series are matched to the corresponding model variables:\(^34\)

\[
OBS_t = \begin{bmatrix}
\Delta \log (Y_t) \\
\Delta \log (C_t) \\
\Delta \log (z^l_t A_t I_t) \\
\log (L_t) \\
\Delta \log (G_t) \\
\Delta \log (z^k_t A_t) \\
\Delta \log (\tau^k_t) \\
\Delta \log (\tau^n_t) \\
\Delta \log (TFP_t) \\
\Delta \log (W_t) \\
\Delta \log (R_t) \\
\Delta \log (\Pi_t)
\end{bmatrix} \times 100 = - \begin{bmatrix}
\log (\mu^y) \\
\log (\mu^y) \\
\log (\mu^y) \\
\log (\tilde{L}) \\
\log (\mu^y) \\
\log (\mu^a) \\
\log (\tau^k) \\
\log (\tau^n) \\
(1 - \alpha) \log (\mu^x) \\
\log (\mu^y) \\
\log (\tilde{R}) \\
\log (\tilde{\Pi})
\end{bmatrix} + \begin{bmatrix}
\hat{y}_t - \hat{y}_{t-1} + \hat{\mu}^y_t \\
\hat{c}_t - \hat{c}_{t-1} + \hat{\mu}^y_t \\
\hat{z}^l_t - \hat{z}^l_{t-1} + \hat{\mu}^y_t \\
\hat{\tilde{L}}_t \\
\hat{g}_t - \hat{g}_{t-1} + \hat{\mu}^y_t + \hat{\mu}^a_t + \hat{\tau}^k_t - \hat{\tau}^k_{t-1} \\
\hat{\tau}^n_t \\
\hat{z}_t - \hat{z}_{t-1} + (1 - \alpha) \hat{\mu}^x_t \\
\hat{\tilde{R}}_t \\
\hat{\tilde{\Pi}}_t
\end{bmatrix},
\]

where \(\Delta\) denotes the temporal difference operator, \(\tilde{L}\) denotes the steady state of hours worked, \(\mu^y\) is the steady state growth rate of output\(^35\), \(\mu^a\) is the steady state growth rate of the relative price of investment, \(\tau^k\) and \(\tau^n\) are the steady state tax rates, \(TFP_t = z_t X^{1-\alpha}_t\) is total factor productivity, and \(R\) is the steady state interest rate. The hats above the variables denote log deviations from steady state.

\(^34\)The equation for \(L_t\) follows from

\[
\log L_t = \log \left( \frac{L_t}{\tilde{L}} \right) \approx \tilde{L}_t + \log \tilde{L}.
\]

The equation for government spending follows from

\[
\log \frac{G_t}{G_{t-1}} = \log \frac{g_t X^y_t}{g_{t-1} X^y_{t-1}} = \log \frac{g_t x^a_t X^Y_t}{g_{t-1} x^a_{t-1} X^Y_{t-1}} = \log \left( \frac{g_t x^a_t}{g_{t-1} x^a_{t-1}} \right) \mu^y_t.
\]

Note that the presence of \(x^y\) also implies that there is no perfect linear restriction between the GDP components following from the resource constraint. Hence, we do not need to add additional measurement error.

\(^35\)This is also the growth rate of the individual components of GDP along the balanced growth path.
1.D Data Construction

Unless otherwise noted, all data are from the Bureau of Economic Analysis (BEA)’s NIPA Tables and available in quarterly frequency from 1955Q1 until 2006Q4.

- **Capital and labor tax rates**: Our approach to calculate average tax rates closely follows Mendoza, Razin, and Tesar (1994), Jones (2002), and Leeper, Plante, and Traum (2010). We first compute the average personal income tax rate
  \[ \tau^p = \frac{IT}{W + PRI/2 + CI}, \]
  where \( IT \) is personal current tax revenues (Table 3.1 line 3), \( W \) is wage and salary accruals (Table 1.12 line 3), \( PRI \) is proprietor’s income (Table 1.12 line 9), and \( CI \equiv PRI/2 + RI + CP + NI \) is capital income. Here, \( RI \) is rental income (Table 1.12 line 12), \( CP \) is corporate profits (Table 1.12 line 13), and \( NI \) denotes the net interest income (Table 1.12 line 18).

  The average labor and capital income tax rates can then be computed as
  \[ \tau^n = \frac{\tau^p(W + PRI/2) + CSI}{EC + PRI/2}, \]
  where \( CSI \) denotes contributions for government social insurance (Table 3.1 line 7), and \( EC \) is compensation of employees (Table 1.12 line 2), and
  \[ \tau^k = \frac{\tau^pCI + CT + PT}{CI + PT}, \]
  where \( CT \) is taxes on corporate income (Table 3.1 line 5), and \( PT \) is property taxes (Table 3.3 line 8).

- **Government spending**: Government spending is the sum of government consumption (Table 3.1 line 16) and government investment (Table 3.1 line 35) divided by the GDP deflator (Table 1.1.4 line 1) and the civilian noninstitutional population (BLS, Series LNU00000000Q).

- **Total factor productivity (TFP)**: The construction of TFP closely follows
Beaudry and Lucke (2010), i.e.

\[ TFP_t = \frac{Y_t}{K^\alpha H^{1-\alpha}}. \]

To construct \( K \), we use data on capital services for the private non-farm business sector (Bureau of Labor Statistics (BLS), Historical Multifactor Productivity Tables),\(^{36}\) multiply it by the total capacity utilization rate (Federal Reserve System, Statistical Release G.17 - Industrial Production and Capacity Utilization), and divide it by the civilian noninstitutional population above 16 years of age (BLS, Series LNU00000000Q). Real GDP per capita \( Y \) is nominal GDP (Table 1.1.5 line 1) divided by the GDP deflator (line 1 in Table 1.1.4) and the population, and per capita hours \( H \) are non-farm business hours worked (BLS, Series PRS85006033) divided by the population. The capital share \( \alpha \) is set at 0.2935, the mean over the sample compiled by the BLS (Bureau of Labor Statistics (BLS), Historical Multifactor Productivity Tables).


- **Output**: Nominal GDP (Table 1.1.5 line 1) divided by the GDP deflator (Table 1.1.4 line 1) and the civilian noninstitutional population (BLS, Series LNU00000000Q).

- **Investment**: Sum of Residential fixed investment (Table 1.1.5 line 12) and nonresidential fixed investment (Table 1.1.5 line 9) divided by the GDP deflator (Table 1.1.4 line 1) and the civilian noninstitutional population (BLS, Series LNU00000000Q).

- **Consumption**: Sum of personal consumption expenditures for nondurable goods (Table 1.1.5 line 5) and services (Table 1.1.5 line 6) divided by the GDP deflator (Table 1.1.4 line 1) and the civilian noninstitutional population (BLS, Series LNU00000000Q).

\(^{36}\)Quarterly data is interpolated from the annual series using cubic spline interpolation.
1.D Data Construction

- **Real wage**: Hourly compensation in the nonfarm business sector (BLS, Series PRS85006103) divided by the GDP deflator (Table 1.1.4 line 1).

- **Inflation**: Computed as the log-difference of the GDP deflator (Table 1.1.4 line 1).

- **Nominal interest rate**: Geometric mean of the effective Federal Funds Rate (St.Louis FED - FRED Database, Series FEDFUNDS).

- **Hours worked**: Nonfarm business hours worked (BLS, Series PRS85006033) divided by the civilian noninstitutional population (BLS, Series LNU00000000Q).
Chapter 2

Policy Risk and the Business Cycle

2.1 Introduction

The supposedly negative influence of “policy risk”, i.e. uncertainty about fiscal and monetary policy, has become a recurring theme in the political discourse. The popular argument espoused in speeches and newspaper articles by politicians and economists alike is that the uncertainty surrounding future policy stuns economic activity by inducing a “wait-and-see approach”.\(^1\) In the following, we think of uncertainty as the dispersion of the economic shock distribution. Rational consumers and firms will react to the fact that future shocks will be drawn from a wider distribution. This reaction is distinct from the ex-post effect of higher uncertainty resulting from on average more extreme shock realizations.\(^2\) The goal of the present study is to isolate the first effect and answer the question: Are uncertainty shocks to policy variables quantitatively important?

---

\(^1\)See e.g. The Wall Street Journal, October 29th, 2009: “For these small businesses, and many others [...], there’s an additional dark cloud: uncertainty created by Washington’s bid to reorganize a wide swath of the U.S. economy.” (Fields, 2009). For other proponents of this view, see Boehner (2010); Cantor (2010); Imrohoroglu (2010); Lowrie (2010); McKinnon (2010); see Klein (2010); Reeve (2010); Wingfield (2010) for dissenting opinions.

\(^2\)Uncertainty shocks are mean preserving spreads to the shock distribution. They are not associated with the expectation of shocks going into a specific direction, like expecting an expansionary stimulus package. Hence, they are also distinct from news shocks (Beaudry and Portier, 2006; Schmitt-Grohé and Uribe, 2010), which are future level shocks of which both the sign and the magnitude are already perfectly known today.
Clearly, during the so-called Great Recession U.S. citizens were facing a period of extraordinary uncertainty regarding economic policy. On the one hand, both the output decline due to the financial crisis and the fiscal stimuli designed to counteract this decline had led to a considerable deterioration of the U.S. fiscal situation. Given this unsustainable fiscal path, many commentators and politicians were arguing for a quick consolidation of government finances, possibly by raising taxes. On the other hand, the U.S. unemployment rate stood at 9.6% at the end of 2010, its highest value since 1983. Hence, there were considerable calls for more fiscal stimulus, preferably in the form of reduced taxes due to supposedly higher multipliers (see e.g. Romer and Romer, 2010). At the same time, Republicans and Democrats were fighting over the continuation of the Bush tax cuts. On the monetary side, the amount of policy risk was equally high. Hawks and doves at the Federal Reserve System fought over the extent of quantitative easing and the correct monetary stance given conflicting signals from core and headline inflation measures.

Scientific evidence on the aggregate effects of uncertainty is still inconclusive and mostly confined to TFP uncertainty. Empirical studies using different proxies and identification schemes to uncover the effects of uncertainty have produced a variety of results. One group of studies reports an important impact of uncertainty about productivity on real aggregate variables like GDP and employment (Alexopoulos and Cohen, 2009; Bloom, 2009; Bloom, Floetotto, and Jaimovich, 2010). A one-standard deviation shock to uncertainty in these studies typically leads to a 1%-2% drop in GDP, followed by a recovery with a considerable overshooting. In contrast, a second group of studies reports little to no impact at all (Bachmann and Bayer, 2011; Bachmann, Elstner, and Sims, 2010; Bekaert, Hoerova, and Duca, 2010; Chugh, 2011; Popescu and Smets, 2010). In the theoretical literature, while most studies have emphasized the contractionary effects of uncertainty on economic activity, it is generally acknowledged that there are different effects working in opposite directions, thereby making the overall effect ambiguous. For example, while an increase in uncertainty may depress investment due to a “wait-and-see approach”, economic agents may want to self-insure by working more to build up a buffer capital stock, which ceteris paribus leads to an increase in investment.

We answer the question of whether policy risk shocks are quantitatively important
in an estimated DSGE-model. We focus on aggregate uncertainty as it has been shown to have potentially important output effects (Fernández-Villaverde et al., 2011b). We add to the previous literature in the following ways. First, we are to our knowledge the first to study the effect of policy risk on business cycles. Second, we directly measure aggregate uncertainty from the respective time series without the need to resort to proxies. Third, we jointly consider level shocks and uncertainty shocks. Regarding uncertainty shocks, we focus on policy risk, i.e. uncertainty about future tax liabilities, government spending, and monetary policy, to test the hypothesis that policy risk may be an important factor in explaining the prolonged Great Recession. We also include uncertainty with respect to total factor productivity (TFP) and investment-specific technology in order to have a benchmark against which we can judge our findings. Fourth, we integrate these processes into a medium-scale New Keynesian DSGE-model of the type typically used for policy analysis (see e.g. Christiano, Eichenbaum, and Evans, 2005; Smets and Wouters, 2007) and solve this model using third-order perturbation methods. We then estimate the model using the Simulated Method of Moments. This approach allows us to control for the effects of level shocks to TFP, investment-specific technology, government spending, monetary policy, and taxes when estimating the importance of policy risk.

We find that the role of policy risk in explaining the prolonged slump is largely overstated. Although the output effects of policy risk are an order of magnitude larger than the effects of TFP uncertainty, even a large (two-standard deviation) shock to policy risk decreases output by a mere 0.025%. The reason for this result is the existence of strong general equilibrium effects that dampen the effects of aggregate uncertainty and imply a low shock amplification. Most notably, monetary policy reacts fast and decisively to current economic conditions, implying an interest rate response that dampens aggregate fluctuations arising from uncertainty shocks. If we

---

3We have recently become aware of independently conducted work by Fernández-Villaverde et al. (2011a), studying a similar issue in a calibrated model. The studies differ in the set of shocks considered and in the details of the model specification. However, the results are quite similar, with even large uncertainty shocks generating only a contained output decline. In their baseline calibration, a two-standard deviation policy risk shock decreases output by 0.06% compared to 0.025% in our estimated baseline specification. The advantage of our approach is that we estimate the parameters of our model. Moreover, we allow for time-varying volatility in technology, allowing us to relate our findings to the literature on TFP uncertainty and to “good luck” explanations of the Great Moderation.
allow for a stronger amplification, uncertainty shocks generate considerably larger output effects, but at the same time imply counterfactually volatile business cycles.

From a methodological viewpoint, the paper most closely related to our work is Fernández-Villaverde et al. (2011b). Their study also employs Sequential Monte Carlo Methods combined with third-order perturbation to estimate the effect of interest risk on the Argentinean economy. In terms of results, this chapter is most closely related to Bachmann and Bayer (2011), who show for the case of idiosyncratic uncertainty about technology that general equilibrium effects may considerably reduce the effect of uncertainty shocks typically found in partial equilibrium models (e.g. Bloom, 2009). This chapter is also related to the work of Primiceri (2005) and Justiniano and Primiceri (2008). Using a time-varying Bayesian VAR and an estimated DSGE-model, respectively, the authors document the importance of time-varying volatility for explaining the time series behavior of output and inflation and the Great Moderation in particular. We differ from their work in two major points: first, we allow for a non-linear transmission of volatility shocks into the economy. Second, by using a third-order approximation instead of a first-order approximation, we are able to distinguish uncertainty-effects from the ex-post effect of uncertainty in the form of more extreme level shocks. We show that their result is mainly due to the differing size of the realized level shocks when the dispersion of the distribution from which they are drawn changes. In contrast, the pure uncertainty-effect is only of secondary importance.

The outline of the chapter is as follows. Section 2.2 presents a short literature review on the transmission channels of uncertainty. In Section 2.3, we build a quantitative business cycle model featuring several channels identified in the theoretical literature through which aggregate uncertainty may impact economic activity. We measure policy risk and technological uncertainty directly from aggregate time series using Sequential Monte Carlo methods in Section 2.4. In Section 2.5, we feed the uncertainty processes estimated in Section 2.4 as driving processes into the model and fit it to U.S. data using a Simulated Method of Moments approach. With the estimated model at hand, we then study the effects of policy risk in Section 2.6. Section 2.7 concludes.
2.2 Uncertainty: Potential Transmission Channels

Three different mechanisms through which aggregate uncertainty may affect economic activity have been identified in the microeconomic literature: Hartman-Abel effects, real option effects, and precautionary savings. While these categories are helpful in shaping our thinking about the effects of uncertainty, they are partial equilibrium effects. In general equilibrium, each of these effects necessarily induces equilibrating price and quantity changes that may significantly dampen the aggregate effects. While in a partial equilibrium model uncertainty may have ceteris paribus largely contractionary effects on investment and output (e.g. Bloom, 2009), in general equilibrium wages and interest rates may adjust, thereby significantly reducing the resulting net effect (Bachmann and Bayer, 2011).

The first category are the so called Hartman-Abel-effects (Abel, 1983; Hartman, 1972). Under certain conditions, it follows from the firms’ FOC that the expected marginal revenue product of capital is convex in output prices and TFP. Hence, due to Jensen’s Inequality larger uncertainty about these variables increases the demand for capital and thus investment. In our model, while capital is predetermined, both the utilization of capital and labor input can be adjusted, opening up the possibility of expansionary Hartman-Abel effects.

Second, there may be real option effects at work (Bernanke, 1983), e.g. through investment being (partially) irreversible and/or partially expandable. For example, if the resale (purchase) price of capital in the future differs from the current acquisition price, a firm installing capital that it may sell later, effectively acquires a put option. Moreover, investment today destroys a call option, namely the opportunity to buy capital later at a possibly lower price. Hence, in the investment decision these option values have to be taken into account (Abel et al., 1996). Higher uncertainty decreases investment as the call option to purchase the capital later, which is “killed” by investing today, becomes more valuable. However, in the presence of partial

---

4Constant-returns-to-scale production function with i) a predetermined capital stock, ii) perfect competition, iii) risk neutrality, and iv) symmetric convex adjustment costs.

5The reason is that labor can flexibly react to shocks and hence the marginal revenue product reacts stronger than one for one to the movement in the respective variable. To see this, assume a fixed capital stock of capital and that the output price rises. There is a direct positive effect of this price increase on profits via quantity times price change. Additionally, there is a positive indirect effect through the increase in optimal output that is achieved by increasing labor.
Table 2.1: Overview: Potential Transmission Mechanism

<table>
<thead>
<tr>
<th></th>
<th>Hartman-Abel eff.</th>
<th>Real option effects</th>
<th>Precaution. sav.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment</td>
<td>+</td>
<td>−</td>
<td>+</td>
</tr>
<tr>
<td>Consumption</td>
<td>?</td>
<td>?</td>
<td>+/-</td>
</tr>
</tbody>
</table>

**Notes:** + indicates a positive effect of uncertainty, − a negative effect, and +/- an ambiguous effect on the respective variable. ? denotes that the respective effect makes no prediction for this variable due to its partial equilibrium nature.

Due to reversibility, the value of the put option that is obtained by investing today increases with higher uncertainty. Hence, the total effect of uncertainty on investment in such a framework is generally ambiguous.

In our model, several features give rise to option effects. First, capital is predetermined for one period. Second, the relative price of investment and consumption is stochastic, thereby giving rise to potentially costly irreversibility and expandability. Third, through the presence of depreciation allowances investment generates a tax shield at historical costs of investment so that investment effectively “kills” the option to purchase this tax shield later. Fourth, the interest rate in our model is stochastic, giving rise to additional countervailing option effects as discussed in Ingersoll and Ross (1992).

The third effect is the precautionary saving motive (Leland, 1968), defined as the “additional saving that results from the knowledge that the future is uncertain” (Carroll and Kimball, 2008). Faced with higher uncertainty, agents may both consume less and work more in order to self-insure against future shocks, i.e. they build a buffer stock.\(^6\) As the preferences of the agents in our model feature prudence (Garcia, Restrepo, and Tanner, 2007; Kimball, 1990) uncertainty should increase precautionary savings in our model.

In the end, due to these three effects acting on different variables and potentially working in opposite directions as well as the presence of general equilibrium effects, only a rigorous quantitative evaluation can answer the question what the net effect

---

\(^6\) Real option effects and the precautionary saving motive are not disjunct effects. Consumption is completely irreversible as the consumed good is not available for consumption in later periods when the marginal utility of consumption may be high.
of uncertainty on aggregate activity is. We pursue this question by estimating a structural model featuring time-varying volatility, which we present in the next section.

2.3 A DSGE-Model with Policy Risk

We use a standard quantitative New Keynesian business cycle model (Smets and Wouters, 2007). The model economy is populated by a large representative family, a continuum of unions $j \in [0, 1]$ selling differentiated labor services to intermediate firms, a continuum of intermediate firms producing differentiated intermediate goods using bundled labor services and capital, and a final good firm bundling intermediate goods to a final good. In addition, the model features a government sector that finances government spending with distortionary taxation and transfers, and a monetary authority which sets the nominal interest rate according to an interest rate rule.

2.3.1 Household Sector

The economy is populated by a large representative family with a continuum of members, each consuming the same amount and working the same number of hours. Preferences are defined over per capita consumption $C_t$ and per capita labor effort $L_t$. Following the framework in Schmitt-Grohé and Uribe (2006), labor is supplied to a continuum of unions $j \in [0, 1]$, which are monopolistically competitive and supply differentiated labor services $l_t(j)$. Household members supply their labor uniformly to all unions. Hence, total labor supply of the representative family is given by the integral over all labor markets $j$, i.e. $L_t = \int_0^1 l_t(j) \, dj$. The labor market structure will be discussed in more detail below. We assume the preference specification of Jaimovich and Rebelo (2009), but allow for habits in consumption:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \left( C_t - \phi_c C_{t-1} - \gamma \frac{L_t^{1+\sigma_l}}{1+\sigma_l} S_t \right)^{1-\sigma_c} - 1 \right\},$$

(2.1)
where $\phi_c \in [0, 1]$ measures the degree of internal habit persistence, $\sigma_c \geq 0$ governs the intertemporal elasticity of substitution, $\sigma_l \geq 0$ is related to the Frisch elasticity of labor supply, and $\gamma \geq 0$ measures the relative disutility of labor effort. The term

$$S_t = (C_t - \phi_c C_{t-1})^{\sigma_c} S_{t-1}^{1-\sigma_c}$$

(2.2)

makes the preferences non-separable in both consumption and work effort, where $\sigma_G \in [0, 1]$ parameterizes the strength of the wealth effect on the labor supply. When $\sigma_G = 1$, the preference specification is equal to the one discussed in King, Plosser, and Rebelo (1988), while with $\sigma_G = 0$ the preference specification of Greenwood, Hercowitz, and Huffman (1988) with no wealth effect on the labor supply is obtained.\footnote{As mentioned in Chapter 1, the presence of habits generates a wealth effect even with $\sigma_s = 0$.}

The household faces the budget constraint

$$C_t + z_t^l I_t + \frac{B_{t+1}}{P_t} = (1 - \tau_n^t) \int_0^1 W_t(j) l_t(j) dj + (1 - \tau_k^t) r_t^k u_t K_t + (1 - \tau_k^t) (R_{t-1} - 1) \frac{B_t}{P_t} + \frac{B_t}{P_t} + \Phi_t + T_t + (1 - \tau_k^t) \Xi_t ,$$

(2.3)

where the household earns income from supplying differentiated labor services $l_t(j)$ at the real wage $W_t(j)$ to union $j$, and from renting out capital services $u_t K_t$ at the rental rate $r_t^k$. In addition, it receives lump sum transfers $T_t$ from the government and profits $\Xi_t$ from owning the firms in the economy. All forms of income are taxed at their respective tax rates $\tau_n^t$ and $\tau_k^t$. The term $(1 - \tau_k^t) (R_{t-1} - 1) \frac{B_t}{P_t} + \frac{B_t}{P_t}$ is the after-tax return on savings in bonds, where the net returns are taxed at the capital tax rate. Bonds are in zero net supply. The household spends its income on consumption $C_t$ and investment $z_t^I I_t$, where $I_t$ is gross investment and $z_t^I$ denotes a shock to the relative price of investment in terms of the consumption good. This price is equal to the technical rate of transformation between investment and consumption goods. Due to the presence of a temporary shock, it is exogenous and stochastic. Changes in $z_t^I$ do not affect the productivity of already installed capital, but do affect newly installed capital and become embodied in it. We assume the shock to follow an
2.3 A DSGE-Model with Policy Risk

AR(2)-process\(^8\)

\[ \log z_t^I = \rho_1^I \log z_{t-1}^I + \rho_2^I \log z_{t-2}^I + \epsilon_t^I, \]  

(2.4)

where \(\sigma_t^I\) allows for time-varying volatility and is discussed in detail in Section 2.4. Apart from the fact that this form of investment-specific technology may be an important source of economic fluctuations (Greenwood, Hercowitz, and Krusell, 1997, 2000), a stochastic relative price of investment introduces costly reversibility and expandability of investment into the model as the future purchase/resale price is stochastic.

The term \(\Phi_t\) captures depreciation allowances, which are an important feature of the U.S. tax code. We assume depreciation allowances of the form

\[ \Phi_t = \tau_t \sum_{s=1}^{\infty} \delta_s (1 - \delta_s)^{s-1} z_{t-s}^I I_{t-s}, \]  

(2.5)

where \(\delta_s\) is the depreciation rate for tax purposes.\(^9\) By providing new investment with a tax shield, depreciation allowances may be important in capturing the dynamics of investment following shocks (Christiano, Trabandt, and Walentin, 2011; Yang, 2005). Through this tax shield at historical investment prices, combined with a stochastic relative price of investment \(z^I\), depreciation allowances contribute to costly reversibility and expandability of investment.

The household owns the capital stock \(K_t\), whose law of motion is given by

\[ K_{t+1} = \left[ 1 - \left( \delta_0 + \delta_1 (u_t - 1) + \frac{\delta_2}{2} (u_t - 1)^2 \right) \right] K_t + I_t - \frac{\kappa}{2} \left( \frac{I_t}{K_t} - \delta_0 \right)^2 K_t, \]  

(2.6)

where \(I_t\) is gross investment. Household members do not simply rent out capital, but capital services \(u_t K_t\), where \(u_t\) denotes the capital utilization, i.e. the intensity with which the existing capital stock is used. Without loss of generality, capital utilization in steady state is normalized to 1. Using capital with an intensity higher

---

\(^8\)The lag lengths for the individual exogenous driving processes is chosen to provide a good empirical fit. Details are provided in Section 2.4.

\(^9\)Following Auerbach (1989), we allow the depreciation rate for tax purposes to differ from the physical rate.
than normal incurs costs to the household in the form of a higher depreciation 
\[ \delta (u_t) = \delta_0 + \delta_1 (u_t - 1) + \frac{\delta_2}{2} (u_t - 1)^2, \] 
which, assuming \( \delta_0, \delta_1, \delta_2 > 0 \), is an increasing and convex function of the capital utilization. The last term in equation (2.6) captures capital adjustment costs at the household level of the form introduced by Hayashi (1982), where \( \kappa \geq 0 \) is a parameter governing the curvature of the cost function. This functional form implies that the capital adjustment costs are minimized and equal to 0 in steady state. We choose this type of adjustment costs for two reasons. First, while this functional form clearly is unable to explain some micro-level phenomena like lumpy investment, it has nevertheless been shown to provide a good fit of firm level investment data and performs better than the Christiano, Eichenbaum, and Evans (2005)-formulation with quadratic adjustment costs in investment changes (Eberly, Rebelo, and Vincent, 2008). Second, with the flow specification of Christiano, Eichenbaum, and Evans (2005), Tobin’s marginal \( q \) would be independent of the capital stock, which would essentially shut off intertemporal linkages and thereby the option effects (Wu, 2009).

Thus, the household maximizes its utility (2.1) by choosing \( C_t, B_{t+1}, u_t, K_{t+1}, I_t, S_t, L_t \), subject to the constraints (2.2) - (2.6) and the resource constraint for aggregate labor.

### 2.3.2 Labor Market

The household supplies labor \( l_t(j) \) equally to a continuum of unions \( j, j \in [0, 1] \). This labor market structure allows to introduce differentiated labor services and staggered wage setting without letting idiosyncratic wage risk affect the household members, which would make aggregation intractable. Monopolistically competitive unions supply differentiated labor \( l_t(j) \) to intermediate firms at wage \( W_t(j) \). Every period, each union may re-optimize its wage with probability \( (1 - \theta_w), 0 < \theta_w < 1 \). If a union \( j \) cannot re-optimize, its nominal wage is indexed to the price level according to 
\[ W_t(j) P_t = \Pi_{s=1}^{\kappa_w} W_{t-s} P_{t-s}, \]
where \( \kappa_w \in [0, 1] \) measures the degree of indexing. Hence, when the union has not been able to re-optimize for \( \tau \) periods, its real wage \( \tau \) periods ahead is given by:

\[
W_{t+\tau}(j) = \begin{cases} 
W_{t+\tau}^{\text{opt}}(j), & \text{if able to re-optimize in } t + \tau, \\
\prod_{s=1}^\tau \frac{\Pi_{s=1}^{\kappa_w} W_{t-s}}{W_t(j)}, & \text{otherwise.} 
\end{cases}
\] (2.7)
Household members supply the amount of labor services that is demanded at the current wage. The objective of each union able to reset its wage is to choose the real wage that maximizes the expected utility of its members, given the demand for composite labor services and $\eta_w$ is the substitution elasticity, the respective resource constraint \( L_t = \int_0^1 (W_t(j)/W_t)^{-\eta_w} \, dj \), and the aggregate wage level \( W_t = (\int_0^1 W_t(j)^{1-\eta_w} \, dj)^{-\frac{1}{1-\eta_w}}. \)

### 2.3.3 Firm Side

There is a continuum of monopolistically competitive intermediate goods firms \( i, i \in [0, 1], \) which produce differentiated intermediate goods \( Y_{it} \) using capital services \( K_{it}^{serv} = u_{it}K_{it-1} \) and a composite labor bundle \( L_{it}^{comp} \) according to a Cobb-Douglas production function with capital share \( \alpha \)

\[
Y_{it} = z_t (K_{it}^{serv})^\alpha (L_{it}^{comp})^{1-\alpha} - \phi, \quad \text{if } z_t (K_{it}^{serv})^\alpha (L_{it}^{comp})^{1-\alpha} - \phi > 0
\]

and \( Y_{it} = 0 \) otherwise. The fixed cost of production \( \phi \) is set to reduce economic profits to 0 in steady state, thereby ruling out entry or exit (Christiano, Eichenbaum, and Evans, 2005). The stationary TFP shock \( z_t \) follows an AR(2)-process

\[
\log z_t = \rho_1^t \log z_{t-1} + \rho_2^t \log z_{t-2} + e^{\sigma^t} \nu_t^2.
\]

The composite labor bundle is built from differentiated labor inputs \( L_{it}(j) \) according to a Dixit-Stiglitz aggregator \( L_{it}^{comp} = \left( \int_0^1 L_{it}(j)^{\eta_w} \, dj \right)^{\frac{1}{\eta_w}}. \)

We assume staggered price setting a la Calvo (1983) and Yun (1996). Each period, intermediate firms can re-optimize their prices with probability \( (1 - \theta_p), 0 < \theta_p < 1. \) In between two periods of re-optimization, the prices are indexed to the aggregate price index \( P_t \) according to \( P_{it+1} = \left( \frac{P_t}{P_{t-1}} \right)^{\chi_p} P_t = (\Pi_t)^{\chi_p} P_t, \) where \( \chi_p \in [0, 1] \) governs the degree of indexation. Intermediate goods producers maximize their discounted stream of profits subject to the demand from composite goods producers, equation (2.11).

There is a competitive final goods firm which bundles a final good \( Y_t \) from a
continuum of intermediate goods using a Dixit-Stiglitz aggregation technology with substitution elasticity $\eta_p$

$$Y_t = \left( \int_0^1 Y_{it}^{\eta_p} \frac{dY_t}{\eta_p} \right)^{\frac{\eta_p}{\eta_p - 1}}. \quad (2.10)$$

Expenditure minimization yields the optimal demand for intermediate good $i$ as

$$Y_{it} = \left( \frac{P_{it}}{P_t} \right)^{-\eta_p} Y_t \quad \forall i. \quad (2.11)$$

### 2.3.4 Government Sector

Government spending, which may be thought of as entering the utility function additively separable, follows the process

$$\log \left( \frac{G_t}{\bar{G}} \right) = \rho_1^g \log \left( \frac{G_{t-1}}{\bar{G}} \right) + \rho_2^g \log \left( \frac{G_{t-2}}{\bar{G}} \right) + e^{\sigma^g} \nu_t^g, \quad (2.12)$$

where $\bar{G}$ is government spending in steady state. The government finances its expenditures by distortionary taxation of labor at the rate $\tau_{tn}$ and capital and interest income at rate $\tau_{tk}$. We assume $AR(2)$-processes for the tax rates as this has been found to be a good empirical description for the U.S. (McGrattan, 1994; Mertens and Ravn, 2011)

$$\tau_{tk}^k = (1 - \rho_1^{\tau k} - \rho_2^{\tau k}) \bar{\tau}^k + \rho_1^{\tau k} \tau_{t-1}^k + \rho_2^{\tau k} \tau_{t-2}^k + e^{\sigma^k} \nu_t^k \quad (2.13)$$

$$\tau_{tn}^n = (1 - \rho_1^{\tau n} - \rho_2^{\tau n}) \bar{\tau}^n + \rho_1^{\tau n} \tau_{t-1}^n + \rho_2^{\tau n} \tau_{t-2}^n + e^{\sigma^n} \nu_t^n \quad (2.14)$$

where $\bar{\tau}^n$ and $\bar{\tau}^k$ are the unconditional means of the labor and capital tax rates, respectively. The government also sets lump-sum transfers $T_t$ to balance the budget. This assumed structure yields the government budget constraint

$$T_t + G_t + \Phi_t = \tau_{tn}^n W_t L_t^{\text{comp}} + \tau_{tk}^k \left( r_{tk}^k u_t K_t + \Xi_t \right). \quad (2.15)$$

Transfers plus government spending plus depreciation allowances equal tax revenues from taxing labor, capital income, and profits.
2.4 Policy Risk: Time Series Evidence

We close the model by assuming that the central bank follows a Taylor rule that reacts to inflation and output growth.

\[
\frac{R_t}{\bar{R}} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left( \left( \frac{\Pi_t}{\bar{\Pi}} \right)^{\phi_{\Pi}} \left( \frac{Y_t}{Y_{t-1}} \right)^{\phi_y} \right)^{1-\rho_R} \exp(m_t). \tag{2.16}
\]

Here, \( \rho_R \) is a smoothing parameter introduced to capture the empirical evidence of gradual movements in interest rates (Clarida, Galí, and Gertler, 2000; Rudebusch, 1995), \( \bar{\Pi} \) is the target interest rate set by the central bank, and the parameters \( \phi_y \) and \( \phi_{\Pi} \) capture the responsiveness of the nominal interest rate to deviations of inflation and output growth from their steady state values. We assume that the central bank responds to changes in output rather than its level as this specification conforms better with empirical evidence and avoids the need to define a measure of trend growth that the central bank can observe (see Lubik and Schorfheide, 2007). Finally, \( m_t \) is a shock to the nominal interest rate that follows an AR(1)-process

\[
\log m_t = \rho^m \log m_{t-1} + e^{\sigma^m} \nu_t^m. \tag{2.17}
\]

The definition of equilibrium and the market aggregation are standard and omitted for brevity.

2.4 Policy Risk: Time Series Evidence

In this section, we present empirical evidence on the importance of time-varying volatility in modeling macroeconomic time series. We demonstrate that the data tend to reject the homoscedasticity of macroeconomic driving processes and show that a stochastic volatility (SV) model is able to capture the salient features of the data. Using a particle smoother, we are able to recover the historical series of uncertainty shocks and show that both “good luck” and “good policy” contributed to the Great Moderation.
Chapter 2

2.4.1 Estimation Methodology

We perform a two-step estimation procedure. Due to the non-linear solution of the model required to capture uncertainty effects and the high-dimensional state space, it is computationally infeasible to jointly estimate all model parameters. Hence, we first estimate the exogenous stochastic driving processes of the model using Sequential Monte Carlo (SMC) methods. In the next section we feed these processes into the model presented in Section 2.3 and estimate the parameters of the remaining model equations with a Simulated Method of Moments (SMM) approach.

The model includes 6 exogenous stochastic driving processes with time-varying volatility, i.e. capital and labor tax rates, government spending, a monetary policy shock, total factor productivity, and investment-specific technology. We estimate these processes on quarterly U.S. time series, starting in 1960Q1 and using the longest available sample for each series. Details about the data sources can be found in Appendix 2.A. Because we use a stationary model, we need to extract the deviations of the non-stationary time series from their respective trend. Hence, we apply a one-sided HP-filter to the logarithms of government spending and the two technology processes. Using a one-sided, i.e. “causal” filter (Stock and Watson, 1999) assures that the time ordering of the data remains undisturbed and the autoregressive structure is preserved. We allow for AR(2)-processes in all variables, except for the monetary policy shocks,\textsuperscript{10} as the partial autocorrelations generally indicate the presence of a second root different from zero. Figure 2.1 shows the time series of the exogenous driving processes on which we estimate our laws of motion. In particular for monetary policy, the presence of time-varying volatility is immediately evident. In Appendix 2.C, we provide further evidence for the presence of time-varying volatility.

There are two major competing approaches to model time-varying standard deviations: GARCH models and stochastic volatility (SV) models (Fernández-Villaverde and Rubio-Ramírez, 2010). In the standard GARCH model, $\sigma_t^2$ is a function of the squared scaled lagged innovation in the level equation $\nu_{t-1}^2$ and its own lagged value: $\sigma_t^2 = \omega + \alpha (\sigma_{t-1}^2 + \beta \sigma_{t-1}^2)$. The GARCH model has one important drawback: there are no distinct volatility shocks. The only innovations to the volatility equation

\textsuperscript{10}Although theory suggests that monetary policy shocks in the Taylor rule should be unpredictable and thus i.i.d., we find a moderate degree of first-order autocorrelation.
2.4 Policy Risk: Time Series Evidence

Figure 2.1: Time Series of Exogenous Driving Processes

Notes: From left to right and top to bottom: capital taxes, labor taxes, TFP, investment-specific technology, monetary policy shocks, and government spending. Tax rates are demeaned; government spending and technology processes are detrended using one-sided HP-filter.

are past level shocks, meaning that they cannot be separated from volatility shocks. As we are especially interested in the effects of shocks to the volatility, we cannot use a GARCH model but instead employ a stochastic volatility model. Specifically, we model the standard deviations $\sigma_i^t$ as an AR(1) stochastic volatility process (see e.g. Fernández-Villaverde et al., 2011b; Shephard, 2008)

$$
\sigma_i^t = (1 - \rho^i) \bar{\sigma}^i + \rho^i \sigma_{i-1}^t + \eta_i \varepsilon_t^i, \quad \varepsilon_t^i \sim \mathcal{N}(0, 1),
$$

(2.18)

where $\bar{\sigma}^i$ is is the unconditional mean of $\sigma_i^t, i \in \{\tau_k, \tau_n, g, m, z, zI\}$. The shock to the volatility $\varepsilon_t^i$ is assumed to be independent from the level shock $\nu_t^i$. 

63
Due to the nonlinearity embedded in the stochastic volatility setup of the shocks, we cannot simply employ the Kalman filter as in the case of linearity and normally distributed shocks. For this case, Fernández-Villaverde and Rubio-Ramírez (2007) propose to use the Sequential Importance Resampling (SIR) particle filter, a special application of the more general class of SMC methods, to evaluate the likelihood.\footnote{Technical details of the algorithms used in this subsection can be found in Appendix 2.B.}

After obtaining the likelihood of the observables given the parameters, we use a Tailored Randomized Block Metropolis-Hastings (TaRB-MH) algorithm (Chib and Ramamurthy, 2010) to maximize the posterior likelihood. The prior distributions of the parameters, which are relatively weak, are given in Table 2.2.\footnote{For the autoregressive parameters of the level equation $\rho_1$ and $\rho_2$, we impose a uniform prior for each of the corresponding autoregressive roots over the stability region ($-1, +1$). Let $\xi_1$ and $\xi_2$ be the roots of such an $AR(2)$-process. The autoregressive parameters corresponding to these roots can be recovered from: $\rho_1 = \xi_1 + \xi_2$ and $\rho_2 = -\xi_1 \xi_2$. The posterior distribution was computed from a 20,500 draw Monte Carlo Markov Chain using 3,000 particles, where the first 2,500 draws were discarded as burn-in draws. Acceptance rates were generally between 20\% and 45\%. We also checked identifiability of the SV-process by simulating data from the process and trying to recover the true parameters from this artificial data.}

We are also interested in backing out the historical values of the latent state $\sigma_t$, given the whole set of observations. After filtering, it is straightforward to employ the \textit{backward-smoothing routine} (Godsill, Doucet, and West, 2004) to obtain a historical distribution of the volatilities. The smoothed values were computed at the mean of the posterior distribution using 10,000 particles.

\section*{2.4.2 Estimation Results}

The estimation results are presented in Table 2.2. Detailed convergence diagnostics are shown in Appendix 2.C. In general, all parameters are quite precisely estimated as evidenced by the percentiles. All shocks, except for the monetary policy shock, exhibit a high degree of persistence in their levels, with less persistence in their volatilities. Moreover, the estimated processes show considerable evidence of uncertainty, with $\eta_i^i$ ranging between 0.3 and 0.6. As a one-standard deviation uncertainty shock increases the volatility of the respective process by $(\exp(\eta_i) - 1) \times 100$ percent, such a shock increases the variance of capital taxes, labor taxes, TFP, investment specific technology, monetary policy, and government spending by 46\%, 92\%, 38\%, 39\%, 34\%, ...
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Distribution</td>
<td>Mean</td>
</tr>
<tr>
<td>Capital Tax Rates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>Uniform*</td>
<td>0.00</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>Uniform*</td>
<td>0.00</td>
</tr>
<tr>
<td>$\rho_\sigma$</td>
<td>Beta*</td>
<td>0.90</td>
</tr>
<tr>
<td>$\eta_\sigma$</td>
<td>Gamma</td>
<td>0.50</td>
</tr>
<tr>
<td>$\bar{\sigma}$</td>
<td>Uniform</td>
<td>-7.00</td>
</tr>
<tr>
<td>Labor Tax Rates</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>Uniform*</td>
<td>0.00</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>Uniform*</td>
<td>0.00</td>
</tr>
<tr>
<td>$\rho_\sigma$</td>
<td>Beta*</td>
<td>0.90</td>
</tr>
<tr>
<td>$\eta_\sigma$</td>
<td>Gamma</td>
<td>0.50</td>
</tr>
<tr>
<td>$\bar{\sigma}$</td>
<td>Uniform</td>
<td>-7.00</td>
</tr>
<tr>
<td>Total Factor Productivity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>Uniform*</td>
<td>0.00</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>Uniform*</td>
<td>0.00</td>
</tr>
<tr>
<td>$\rho_\sigma$</td>
<td>Beta*</td>
<td>0.90</td>
</tr>
<tr>
<td>$\eta_\sigma$</td>
<td>Gamma</td>
<td>0.50</td>
</tr>
<tr>
<td>$\bar{\sigma}$</td>
<td>Uniform</td>
<td>-7.00</td>
</tr>
<tr>
<td>Investment-Specific Technology</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>Uniform*</td>
<td>0.00</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>Uniform*</td>
<td>0.00</td>
</tr>
<tr>
<td>$\rho_\sigma$</td>
<td>Beta*</td>
<td>0.90</td>
</tr>
<tr>
<td>$\eta_\sigma$</td>
<td>Gamma</td>
<td>0.50</td>
</tr>
<tr>
<td>$\bar{\sigma}$</td>
<td>Uniform</td>
<td>-7.00</td>
</tr>
<tr>
<td>Government Spending</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>Uniform*</td>
<td>0.00</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>Uniform*</td>
<td>0.00</td>
</tr>
<tr>
<td>$\rho_\sigma$</td>
<td>Beta*</td>
<td>0.90</td>
</tr>
<tr>
<td>$\eta_\sigma$</td>
<td>Gamma</td>
<td>0.50</td>
</tr>
<tr>
<td>$\bar{\sigma}$</td>
<td>Uniform</td>
<td>-7.00</td>
</tr>
<tr>
<td>Monetary Policy Shock</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>Uniform*</td>
<td>0.00</td>
</tr>
<tr>
<td>$\rho_\sigma$</td>
<td>Uniform*</td>
<td>0.90</td>
</tr>
<tr>
<td>$\eta_\sigma$</td>
<td>Beta*</td>
<td>0.50</td>
</tr>
<tr>
<td>$\bar{\sigma}$</td>
<td>Gamma</td>
<td>-7.00</td>
</tr>
</tbody>
</table>

Notes: Beta* indicates that the parameter divided by 0.999 follows a beta distribution. Uniform* indicates that the roots of the autoregressive process are estimated instead of the autoregressive coefficients and follow the specified prior distribution.
and 45%, respectively.\textsuperscript{13} Appendix 2.C shows the results of model misspecification tests applied to the SV model. In general, the model fits the data well and cannot be rejected.

The relevance of stochastic volatility in modeling the behavior of the exogenous driving processes can be seen in the smoothed estimates of the historical variances of the shocks in Figure 2.2. The end of the 1960s and particularly the 1970s were plagued by high shock volatilities, both in the technology and the policy shocks. Particularly during the 1970s, the volatilities increased and reached their sample maxima for both tax rates and technology shocks. In contrast, the period from 1985 to 2000 was characterized by shock volatilities to the technology variables well below their unconditional mean, indicating the role of “good luck” in explaining the Great Moderation. However, from about 1990 on “good policy” also contributed to this phenomenon as is evidenced by the low volatilities of the tax and government spending shocks, although the change in volatility is not as pronounced for the latter.

For monetary policy shocks, there is clear evidence of a lower shock volatility following the Volcker disinflation from 1979-1983, a trend that also continued under Greenspan. In contrast, the early tenure of Volcker experienced a volatility of monetary shocks considerably larger than during the first oil price shock. With the height of the dot-com bubble the volatility of TFP shocks somewhat increased again, while the investment-specific technology growth remained tranquil over the whole 2000s. The largest changes in volatility in the 2000s came under George W. Bush who considerably changed the tax law, resulting in a pronounced increase in the volatility of tax rates. At the end of our sample, the Great Recession again results in an increase in policy risk with a rise in the volatility of government spending, tax rates, and monetary policy to comparable levels as after 9/11. For government spending and taxes, this mostly reflects the provisions in the American Recovery and Reinvestment Act that contained $288 billion in tax relief to companies and individuals, e.g. in the form of $116 billion in payroll tax relief.

Note that the SV-framework used in the present study does not imply a mechanical link between the level shocks and the volatility shocks as a GARCH-model would do. Of course, as a comparison of Figures 2.1 and 2.2 shows, a large level shock tends to

\textsuperscript{13}Thus, e.g. a one-standard deviation monetary policy risk shock increases the volatility of the monetary policy shocks from $exp(-5.19) = 0.56\%$ to $exp(-5.19 + 0.364) = 0.8\%$.\textsuperscript{13}
coincide with an increase in the conditional variance. However, the reason for this increase in the estimated conditional variance is not a mechanical effect of this level shock subsequently entering the volatility equation. Rather, the Bayesian estimation of the SV-model weighs the likelihood of observing such a large shock being drawn from a narrow distribution, i.e. without observing a simultaneous/previous volatility shock, against the likelihood of observing a shock of this size that is drawn from a wider distribution due to the occurrence of a variance shock.

Figure 2.2: Smoothed Standard Deviations

Notes: From left to right and top to bottom: capital taxes, labor taxes, TFP, investment-specific technology, monetary policy shocks, and government spending. Red dotted line: unconditional mean; shaded area: two standard deviation bands.
2.5 Fitting the Model to the Data

Using the parameter estimates of the stochastic driving processes obtained in the previous section, we are now in a position to estimate the deep parameters of the model presented in Section 2.3.

2.5.1 Simulated Method of Moments Estimation

We use the Simulated Method of Moments (SMM) approach as proposed in Ruge-Murcia (2010). Intuitively, this method minimizes the weighted distance between the empirical moments and the moments resulting from artificial data simulated from the model (details can be found in Appendix 2.B).

In order to simulate data, we first need to solve the model non-linearly. Due to the high-dimensional state space of our model, we employ perturbation methods to obtain an approximation of the policy function around the deterministic steady state (see e.g. Judd, 1998). Specifically, we need to obtain a third-order approximation, because we are interested in the pure effects of volatility shocks, i.e. when holding the level shocks constant. Loosely speaking, a first-order approximation yields no effects of uncertainty; a second-order approximation yields both a constant effect and an effect mediated through the corresponding level shock. Only in the third-order approximation does time-varying uncertainty play a separate role (for a more detailed explanation, see Appendix 2.B).

Table 2.3 presents the values of parameters we fix prior to the estimation. We set gross steady state inflation $\bar{\Pi}$ to 1 and the discount factor $\beta$ to 0.99. Regarding the depreciation parameters, $\delta_0 = 0.05$ is chosen to imply a 10% annual depreciation rate, $\delta_1 = 0.0351$ sets the steady state capital utilization to 1, and the depreciation rate for tax purposes $\delta_\tau$ is set to twice the rate of physical depreciation (Auerbach, 1989). The fixed-cost parameter $\phi = 0.038$ implies that firms make zero profit in steady state and the labor disutility parameter $\gamma = 19.1$ sets the steady state share of hours worked to total time to 20%. Regarding the preference parameters, we set the parameter governing the intertemporal elasticity of substitution $\sigma_c$ to 2 and set $\sigma_G = 0.001$, the value chosen in Jaimovich and Rebelo (2009).14 Hence, preferences

\footnote{14When attempting to estimate this parameter, it hit the lower bound of 0 as in Schmitt-Grohé and...}
2.5 Fitting the Model to the Data

Table 2.3: Parameters Fixed Prior to Estimation

<table>
<thead>
<tr>
<th>Param.</th>
<th>Value</th>
<th>Target/Motivation</th>
<th>Param.</th>
<th>Value</th>
<th>Target/Motiv.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\breve{\Pi}$</td>
<td>1</td>
<td>Zero infl. steady state</td>
<td>$\sigma_c$</td>
<td>2</td>
<td>Standard value</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Standard value</td>
<td>$\eta_p$</td>
<td>10</td>
<td>11% Markup</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>0.025</td>
<td>10% annual deprec.</td>
<td>$\eta_w$</td>
<td>10</td>
<td>11% Markup</td>
</tr>
<tr>
<td>$\delta_1$</td>
<td>0.0351</td>
<td>$\bar{u} = 1$</td>
<td>$\alpha$</td>
<td>0.295</td>
<td>Sample mean</td>
</tr>
<tr>
<td>$\delta_c$</td>
<td>0.05</td>
<td>Auerbach (1989)</td>
<td>$\tau^n$</td>
<td>0.1984</td>
<td>Sample mean</td>
</tr>
<tr>
<td>$\phi$</td>
<td>0.038</td>
<td>0 profits in SS</td>
<td>$\tau^k$</td>
<td>0.388</td>
<td>Sample mean</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>19.1</td>
<td>SS labor of 0.2</td>
<td>$G/Y$</td>
<td>0.2031</td>
<td>Sample mean</td>
</tr>
<tr>
<td>$\sigma_G$</td>
<td>0.001</td>
<td>Jaimovich-Rebelo (2009)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

are close to the GHH-specification and imply a small wealth effect on the labor supply, which is consistent with evidence from studies focusing on the effects of news (Schmitt-Grohé and Uribe, 2010) and government spending (Monacelli and Perotti, 2008). The elasticity of substitution parameters for differentiated labor services and intermediate goods are set to 10, resulting in a steady state markup of 11%. The capital share $\alpha$, the steady state tax rates $\tau^k$ and $\tau^n$, and the steady state share of government spending to output are set to their respective sample means.

The empirical moments to be matched are the standard deviations and first- and second-order autocovariances of output, consumption, investment, inflation, the real wage, and the nominal interest rate. Moreover, we target the covariance of output with the other variables. All variables are logged and detrended using a one-sided HP-filter with smoothing parameter $\lambda = 1600$. The second and fourth columns of Table 2.5 display the respective sample moments.\(^{15}\)

2.5.2 Parameter Estimates

The parameter estimates are shown in Table 2.4. All parameters except for the capital adjustment cost parameter $\kappa$ are precisely estimated as seen in columns 4

\(^{15}\)Some of the target moments are transformed to correlations for better interpretation. The relative standard deviations with respect to the standard deviation of output are only implicitly targeted through the standard deviations of the respective series.
Table 2.4: Parameters Estimated by SMM

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Mean</th>
<th>-1 std.-dev.</th>
<th>+1 std.-dev.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_c$</td>
<td>Consumption habits</td>
<td>0.9665</td>
<td>0.9660</td>
<td>0.9671</td>
</tr>
<tr>
<td>$\delta_2/\delta_1$</td>
<td>Capital utilization costs</td>
<td>0.0414</td>
<td>0.0314</td>
<td>0.0546</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Capital adjustment costs</td>
<td>10.0857</td>
<td>0.8007</td>
<td>127.0438</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>Calvo parameter prices</td>
<td>0.9644</td>
<td>0.9641</td>
<td>0.9646</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>Calvo parameter wages</td>
<td>0.7785</td>
<td>0.7615</td>
<td>0.7947</td>
</tr>
<tr>
<td>$\chi_p$</td>
<td>Price indexation</td>
<td>0.4170</td>
<td>0.3809</td>
<td>0.4539</td>
</tr>
<tr>
<td>$\chi_w$</td>
<td>Wage indexation</td>
<td>0.9751</td>
<td>0.9725</td>
<td>0.9774</td>
</tr>
<tr>
<td>$\sigma_t$</td>
<td>Frisch elasticity parameter</td>
<td>0.0683</td>
<td>0.0652</td>
<td>0.0716</td>
</tr>
<tr>
<td>$\rho_R$</td>
<td>Interest smoothing</td>
<td>0.4889</td>
<td>0.4541</td>
<td>0.5238</td>
</tr>
<tr>
<td>$\phi_x$</td>
<td>Taylor rule inflation</td>
<td>1.9691</td>
<td>1.9058</td>
<td>2.0422</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>Taylor rule output growth</td>
<td>1.2195</td>
<td>0.8416</td>
<td>1.7671</td>
</tr>
</tbody>
</table>

and 5.\textsuperscript{16} Consumers have strong habits in consumption with $\phi_c = 0.97$, which is at the upper end of values generally considered plausible. Capital utilization costs show little convexity with $\delta_2/\delta_1 = 0.04$, while capital adjustment is costly as indicated by $\kappa = 10.09$, ensuring that investment is not excessively volatile. Prices are estimated to be quite sticky with $\theta_p = 0.96$, while the degree of wage stickiness is moderate with an average duration of 4.3 quarters. The high degree of price stickiness compared to e.g. Smets and Wouters (2007) reflects the absence of real rigidities like a non-constant elasticity of substitution in our setup. The degree of indexation to past inflation is considerably higher for wages than for prices, with the former being almost perfectly indexed to past inflation. An estimated value of $\sigma_t = 0.07$ indicates almost linear disutility of labor. In the Taylor rule, there is a moderate degree of interest smoothing. The reaction coefficients of monetary policy are in line with values found in the literature.

The first and third column of Table 2.5 show the fit of the model. Output is 92\% as volatile in the simulated model as in the data, while investment is 108\% as volatile. The volatility of consumption is well-matched, while its correlation with output is

\textsuperscript{16}The confidence bands rely on the asymptotic normality of the estimator as shown in equation (2.36). However, this is only a rough approximation as most parameters, e.g. the Calvo parameters, have bounded support. Unfortunately, SMM is computationally too intensive to rely on bootstrapping the standard errors.
2.5 Fitting the Model to the Data

Table 2.5: Simulated and Empirical Moments

<table>
<thead>
<tr>
<th></th>
<th>Model Data</th>
<th>Model Data</th>
<th>Model Data</th>
<th>Model Data</th>
<th>Model Data</th>
<th>Model Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma(x_t)$</td>
<td>$\rho(x_t, y_t)$</td>
<td>$\sigma_{x_t}/\sigma_{y_t}$</td>
<td>$\rho(x_t, x_{t-1})$</td>
<td>$\rho(x_t, x_{t-2})$</td>
<td></td>
</tr>
<tr>
<td>$Y$</td>
<td>1.44%</td>
<td>1.57%</td>
<td>1.00</td>
<td>1.00</td>
<td>0.93</td>
<td>0.90</td>
</tr>
<tr>
<td>$C$</td>
<td>0.93%</td>
<td>0.95%</td>
<td>0.71</td>
<td>0.85</td>
<td>0.65</td>
<td>0.60</td>
</tr>
<tr>
<td>$I$</td>
<td>5.74%</td>
<td>5.30%</td>
<td>0.91</td>
<td>0.85</td>
<td>3.98</td>
<td>3.37</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>0.22%</td>
<td>0.27%</td>
<td>0.23</td>
<td>0.17</td>
<td>0.16</td>
<td>0.17</td>
</tr>
<tr>
<td>$W$</td>
<td>0.82%</td>
<td>0.90%</td>
<td>0.23</td>
<td>0.10</td>
<td>0.57</td>
<td>0.57</td>
</tr>
<tr>
<td>$R$</td>
<td>0.40%</td>
<td>0.39%</td>
<td>0.28</td>
<td>0.34</td>
<td>0.28</td>
<td>0.25</td>
</tr>
</tbody>
</table>

Notes: Time Series $X_t$ are output ($Y_t$), consumption ($C_t$), investment ($I_t$), inflation ($\Pi_t$), the real wage ($W_t$), and the nominal interest rate ($R_t$). Small letters denote variables that are logged and detrended using a one-sided HP-filter with smoothing parameter $\lambda = 1600$.

too low. The volatilities of the real wage, inflation, and the nominal interest rate are on target. Their correlation with output is also well matched. Only the real wage is somewhat too procyclical. The autocorrelations are also in general well-matched. Only consumption exhibits a slightly too high autocorrelation.

2.5.3 The Effects of Time-Varying Volatility

With the estimated model at hand, we can perform a simple counterfactual experiment to demonstrate the importance of time-varying volatility for explaining U.S. macroeconomic time series. However, the effects of time-varying volatility reflect both the ex-ante uncertainty effect of knowing that the shocks are drawn from a wider distribution and the ex-post effect of more extreme shock realizations. In the next section, we will therefore separate these two by using the model to keep the level shocks constant.

In Figure 2.2, we found clear evidence of a decrease in the variance of both the technological shocks and the policy shocks since the mid 1980s, which contributed to the lower volatility of output and inflation during the Great Moderation. Using our estimated DSGE-model, we can ask what a counterfactual economy without time-varying volatility would have looked like. For this purpose, we completely shut off time-varying volatility by setting uncertainty shocks to zero. We then simulate the
model again using the new set of driving forces where both the uncertainty effect and the effects of the corresponding more extreme level shocks are absent due to $\sigma^i_t = \bar{\sigma}^i$ for all $i \in \{\tau k, \tau m, g, m, z, zI\}$. This unconditional sample mean of the log-volatility of the level shocks $\bar{\sigma}^i$ lies between the high volatility pre-Great Moderation period’s value and the value in the subsequent low volatility Great Moderation phase. The corresponding simulated moments are presented in Table 2.6. The co-movement of the model variables still fits the data quite well. However, compared to the actual data, such an economy fails to generate sufficient volatility: output, consumption, and investment are only about 65%, 73%, and 75% as volatile as the data, respectively.\(^{17}\) In contrast, as seen in Table 2.5, the model with time-varying volatility captures the data moments well. These results clearly indicate the importance of time-varying volatility in explaining U.S. macroeconomic time series (see e.g. Justiniano and Primiceri, 2008; Primiceri, 2005).

<table>
<thead>
<tr>
<th>Model</th>
<th>Data</th>
<th>Model</th>
<th>Data</th>
<th>Model</th>
<th>Data</th>
<th>Model</th>
<th>Data</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(x_t)$</td>
<td>$\rho(x_t, y_t)$</td>
<td>$\sigma_{x_t}/\sigma_{y_t}$</td>
<td>$\rho(x_t, x_{t-1})$</td>
<td>$\rho(x_t, x_{t-2})$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$Y$</td>
<td>0.99%</td>
<td>1.57%</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.94</td>
<td>0.90</td>
<td>0.85</td>
<td>0.75</td>
</tr>
<tr>
<td>$C$</td>
<td>0.71%</td>
<td>0.95%</td>
<td>0.67</td>
<td>0.85</td>
<td>0.72</td>
<td>0.60</td>
<td>0.99</td>
<td>0.90</td>
<td>0.95</td>
</tr>
<tr>
<td>$I$</td>
<td>3.91%</td>
<td>5.30%</td>
<td>0.89</td>
<td>0.85</td>
<td>3.97</td>
<td>3.37</td>
<td>0.92</td>
<td>0.93</td>
<td>0.79</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>0.18%</td>
<td>0.27%</td>
<td>-0.19</td>
<td>0.17</td>
<td>0.18</td>
<td>0.17</td>
<td>0.91</td>
<td>0.50</td>
<td>0.76</td>
</tr>
<tr>
<td>$W$</td>
<td>0.53%</td>
<td>0.90%</td>
<td>0.56</td>
<td>0.10</td>
<td>0.54</td>
<td>0.57</td>
<td>0.97</td>
<td>0.84</td>
<td>0.91</td>
</tr>
<tr>
<td>$R$</td>
<td>0.30%</td>
<td>0.39%</td>
<td>-0.11</td>
<td>0.34</td>
<td>0.30</td>
<td>0.25</td>
<td>0.78</td>
<td>0.86</td>
<td>0.61</td>
</tr>
</tbody>
</table>

Notes: Time Series $X_t$ are output ($Y_t$), consumption ($C_t$), investment ($I_t$), inflation ($\Pi_t$), the real wage ($W_t$), and the nominal interest rate ($R_t$). Small letters denote variables that are logged and detrended using a one-sided HP-filter with smoothing parameter $\lambda = 1600$.

\(^{17}\)If we had used a linearized version of the model, this effect would not have been observed, as periods of high volatility would offset periods of low volatility. However, due to the non-linearity of our model, this is not the case here.
2.6 The Aggregate Effects of Policy Risk

We now turn to analyzing the effects of aggregate uncertainty on business cycle fluctuations. First, having estimated the deep parameters of the model, we conduct policy experiments to trace out the effects of uncertainty shocks. We then study their transmission into the economy and analyze the underlying amplification mechanisms. We find that the model is in principle able to generate large effects of uncertainty, but that the estimated parametrization implies that the aggregate effects of uncertainty are quantitatively small. The reason for the small aggregate response to uncertainty shocks is the presence of general equilibrium effects that imply only a weak amplification.

2.6.1 Impulse Response Analysis

We first analyze the pure uncertainty effect resulting from time-varying volatility by separating it from the ex-post effect of more extreme shock realizations. We do so by computing impulse response functions to uncertainty shocks while keeping constant the realizations of the level shocks.

Figures 2.3 and 2.4 show the impulse response functions to two-standard deviation policy risk and technology risk shocks with each column representing the impulse responses to a different shock. The ex-post level effect has been shut off, which is reflected in the flat impulse response for $\tau^k, \tau^n, g, m, z$, and $z_I$ depicted in the bottom row.\(^{18}\) The left column of Figure 2.3 shows that a capital tax risk shock acts like a positive demand shock. Output and inflation both increase on impact and slowly return to zero. Initially the output response is mostly driven by the positive response of investment, which has a peak response on impact of 0.014%. Consumption increases less strongly and follows a hump-shape, peaking after 12 quarters. Due to the estimated strong degree of habit persistence in consumption, the consumption response decays only slowly and drives the output response after about four years, when investment is already almost back to its initial level. The middle and right columns show the impulse responses to labor tax risk and government spending risk, respectively. Both emulate the characteristics of a negative supply shock, with output, consumption, and investment exhibiting a hump-shaped decline, while inflation rises.

\(^{18}\)In the subsequent graphs, we generally omit the flat level impulse responses.
Chapter 2

Figure 2.3: Impulse Responses to a Two-Standard Deviation Uncertainty Shock to Capital Taxes, Labor Taxes, and Government Spending (from Left to Right Column)

Notes: Level shocks are held constant. All responses are in percent, except for $\pi$, which is in percentage points.

Labor tax risk induces the strongest output response of all uncertainty shocks considered, with output showing a peak decline of 0.02% and investment dropping by four times as much. The reason for this relatively strong response, compared to e.g. the government spending risk shock, is that a two-standard deviation labor tax risk shock increases uncertainty about labor taxes by about 120%, compared to around 60% for the other uncertainty shocks. Due to the relatively low persistence of the underlying shock process for labor tax risk, the effect on inflation subsides after 10 quarters, while the effect on consumption is again considerable more drawn out.

The left column of Figure 2.4 displays the response to a two-standard deviation...
monetary policy risk shock. This shock has a contractionary effect on output, mostly driven by a decline in investment that peaks at -0.03% after 7 quarters. In contrast, consumption reacts sluggishly, peaking only after 30 quarters. Inflation initially drops, overshoots after 10 quarters and then slowly returns, driven by a large persistence in the underlying risk shock process.

The historical volatility estimates shown in Figure 2.2 indicated that uncertainty about the future path of economic policy increased for all policy instruments during the Great Recession. We simulate such a situation in the form of a simultaneous two-standard deviation increase in policy risk. Results are shown in Figure 2.5. A

---

19Due to the nonlinearity inherent in our model and the solution method that preserves this nonlinearity up to third order, the resulting impulse responses are not necessarily identical to the sum of the impulse responses to the individual uncertainty shocks.
Figure 2.5: Impulse Responses to a Joint Two-standard Deviation Policy Risk Shock (Solid Blue Line) and to a Joint Technology Risk Shock (Dashed Red Line)

Notes: Level shocks are held constant. All responses are in percent, except for $\pi$ and realinterest, which are in percentage points.

The simultaneous two-standard deviation policy risk shock (solid lines) acts like a negative supply shock. It leads to an immediate decrease in output of 0.025%, before output slowly returns to its initial level as the shock subsides. This decrease in output is driven by both consumption and investment, with investment dropping initially by 0.1%. While the capital stock reacts sluggishly due to the presence of relatively high capital adjustment costs, capital services decline immediately due to an accompanying decline in capital utilization. At the same time inflation rises. As a consequence, the real wage rises for a few periods, reflecting the indexation to the rising inflation, and then starts to decrease, reaching its minimum after 15 quarters. Due to monopolistic competition in the labor market and the non-separability of the utility function, the initial increase in the real wage does not induce an increase in labor supplied by the household. Rather, household members decrease their labor supply and consume
The Aggregate Effects of Policy Risk

more leisure. The real interest rate, computed as the difference between the policy rate and inflation, declines initially and then follows a hump-shaped pattern, reaching its peak after 7 quarters. The initial decline in the real interest rate reflects both the interest smoothing present in the estimated Taylor rule as well as the response of the central bank to the initial decline in output. Only when output starts to recover does the real interest rate rise to bring down inflation. The similarity in both the size and the shape of the impulse response functions of a policy risk shock and the labor tax risk shock indicates that the latter dominates the effects of the other policy risk shocks.\(^{20}\)

It is instructive to compare the policy risk results to the benchmark of uncertainty about technology. The middle and right columns of Figure 2.4 show the impulse responses to a two-standard deviation risk shock to total factor productivity and investment-specific technology, respectively. The response to TFP risk is qualitatively similar to what could have been expected from the previous literature: it triggers an investment driven decline in output while inflation increases. In contrast, investment-specific technology risk triggers exactly the opposite effect: output increases initially and peaks after 4 quarters, with the response again being mainly driven by the investment response. It is noteworthy that the response to TFP uncertainty is an order of magnitude smaller than the effects of uncertainty about the investment-specific technology shocks. This result suggests that the role of investment-specific technology risk might be underappreciated in the uncertainty literature.\(^{21}\)

Figure 2.5 also shows the impulse responses to a joint technology risk shock of the type occurring in the middle of the 1970s. The comparison of technology risk (dashed lines) with policy risk (solid lines) shows that policy risk generates responses that are

\(^{20}\)While strictly speaking the impulse responses to single shocks are not additive, the opposite signs of the output response for some sources of uncertainty have important consequences for periods of generally heightened uncertainty. The simultaneous increase in uncertainty from different sources does not necessarily translate into a large output response. In times like the Great Recession, where policy risk jointly increased, different sources of uncertainty may partially offset each other, resulting in a low overall effect. For example, Figure 2.3 documents that capital taxation risk acts expansionary and could more than offset the negative effect of government spending risk on output and investment.

\(^{21}\)While the effects of level shocks to investment-specific technology have received considerable attention in recent years (Fisher, 2006; Justiniano, Primiceri, and Tambalotti, 2010a; Schmitt-Grohé and Uribe, 2011), we are to our knowledge the first to study the effects of uncertainty about investment-specific technology.
one order of magnitude larger.

Summarizing, our results show that the finding of relatively minor effects of uncertainty on aggregate activity for the case of TFP (Bachmann and Bayer, 2011; Bachmann, Elstner, and Sims, 2010; Bekaert, Hoerova, and Duca, 2010; Chugh, 2011; Popescu and Smets, 2010) also holds true for policy risk and investment-specific technology risk.

### 2.6.2 What Drives the Response to Policy Risk?

Of the transmission channels discussed in Section 2.2, the precautionary savings motive does not play a dominant role. In all sets of impulse responses, consumption and investment move in the same direction, while in the case of a dominant precautionary savings motive we would expect agents to decrease their consumption in order to self-insure against aggregate uncertainty by investing in a buffer-stock. Of course, it is conceivable that the precautionary savings motive counteracts the observed effects, which then would have been larger in its absence.

While it is virtually impossible to disentangle the different real option, Hartman-Abel, and general equilibrium effects, we can gain some insight into the transmission of uncertainty by shutting off various features of the model. First, as can be seen by fixing the relative price of investment to consumption at 1, the real option effect embedded in the depreciation allowances via the stochastic resale price of capital hardly plays a role. However, while their role in providing current investment with a tax shield at historical investment prices does not seem to create strong real option effects in our model, this does not mean that depreciation allowances do not play an important role. With their effect on Tobin’s marginal \( q \) and the capital utilization decision, they have an important amplifying effect on the investment response and hence on output. When shutting them off completely, i.e. setting \( \delta_r = 0 \), capital drops less and the negative consumption response is cut in half (figures omitted for brevity).

Second, the low wealth effect on the labor supply implied by the preferences being close to the GHH-form (\( \sigma_G \approx 0 \)) has a considerable effect on the responses to uncertainty, amplifying the response to some shocks and dampening the one to others. As shown in Figure 2.6, when setting the preferences to the standard King-
2.6 The Aggregate Effects of Policy Risk

Figure 2.6: Impulse Responses to a Two-standard Deviation Uncertainty Shock to Capital Taxes, Labor Taxes, and TFP (from Left to Right Column)

Notes: solid blue line: KPR-preferences ($\sigma_G = 1$); red dashed line: preferences close to GHH ($\sigma_G \approx 0$). Level shocks are held constant. All responses are in percent, except for $\pi$, which is in percentage points.

Plosser-Rebelo specification ($\sigma_G = 1$), the negative response to labor tax risk declines by two orders of magnitude. At the same time, the effect of uncertainty shocks that mainly affect the capital margin, i.e. capital tax and TFP risk, substantially increases, with the former now being the dominant policy risk factor. The output response to government spending, monetary policy and investment-specific technology risk stays largely unaltered (figures omitted for brevity).\textsuperscript{22}

\textsuperscript{22}This finding of an important role of the preference specification for the transmission of uncertainty shocks suggests that adopting a certain form of utility function may already predetermine the sign of the output response to an uncertainty shock. Hence, future studies dealing with the effects of uncertainty should devote more attention to tracing out which preference specification may be the most suitable one. Our estimation results hint at a utility function featuring a low wealth effect on the labor supply. This is in line with an increasing number of studies from the fiscal policy (Monacelli and Perotti, 2008), open economy (Chang and Fernández, 2010; García-Cicco, Pancrazi, and Uribe, 2010), and news literature (Jaimovich and Rebelo, 2009; Schmitt-Grohé...
Table 2.7: Counterfactual Calibration Implying Large Uncertainty Effects

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Estimated mean</th>
<th>Counterfactual</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_c$</td>
<td>Consumption habits</td>
<td>0.96</td>
<td>0.9</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Capital adjustment costs</td>
<td>10.1</td>
<td>5</td>
</tr>
<tr>
<td>$\theta_p$</td>
<td>Calvo parameter prices</td>
<td>0.96</td>
<td>0.9</td>
</tr>
<tr>
<td>$\sigma_l$</td>
<td>Frisch elasticity parameter</td>
<td>0.07</td>
<td>4</td>
</tr>
<tr>
<td>$\rho_R$</td>
<td>Interest smoothing</td>
<td>0.49</td>
<td>0.9</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>Taylor rule output growth</td>
<td>1.22</td>
<td>0</td>
</tr>
</tbody>
</table>

As noted in Section 2.2, the theoretical literature predicts an ambiguous effect of uncertainty as real option, Hartman-Abel, and general equilibrium effects drive the dynamics and may work in opposite directions. That this is actually the case for the specific types of uncertainty considered can be seen from, e.g., the impulse response of consumption to a capital tax shock depicted in the middle left panel of Figure 2.6. The consumption response is mostly negative for the case of $\sigma_G \approx 0$ but unambiguously positive for $\sigma_G = 1$. This suggests that different partial effects are dominating the respective responses for the different parameterizations. While a contractionary effect dominates in the GHH-case, an expansive effect prevails in the KPR-case. The strong dependence of uncertainty effects on the specific parametrization underscores the need for model estimation as opposed to calibration in order to trace out the aggregate effects of uncertainty.

2.6.3 Why Are the Effects of Uncertainty Small?

We identify strong general equilibrium effects – constraining the amplification of uncertainty shocks – as the main reason for the small effect of uncertainty on economic activity. While the model is in principle capable of generating large real effects of uncertainty, strong stabilizing effects are required to match the data moments. Therefore, SMM estimates the model parameters to imply strong equilibrating effects.

Consider the simple counterfactual experiment displayed in Table 2.7. Here, we decrease habit persistence, capital adjustment costs, price rigidities, and the Frisch elasticity of labor supply. To dampen the general equilibrium response of the nominal
2.6 The Aggregate Effects of Policy Risk

interest rate, we shut off the reaction to output growth and considerably increase the interest smoothing. In this case, as shown Figure 2.7, policy risk leads to a drop in output of 1.5%, which is mostly driven by a large decline in investment. While this

Figure 2.7: Impulse Responses to a Two-standard Deviation Policy Risk Shock under Counterfactually Volatile Calibration

Notes: Level shocks are held constant. All responses are in percent, except for \( \pi \) and realinterest, which are in percentage points.

calibration allows for larger effects of uncertainty, it comes at a cost: the model with this calibration implies unrealistically large business cycles. As shown in Table 2.8, output would be almost three times as volatile as found in the data, investment five times, and wages almost four times as volatile.

Hence, given the estimated exogenous driving processes, SMM estimates the parameters to imply a shock amplification more in line with the actually observed data. First, consumption habits, capital adjustment costs, and price rigidities are estimated to be quite high, generating a high persistence and thereby limiting the reaction of consumption, investment, and inflation to shocks and thus the deviations from the ergodic mean that are realized over time. Second, the parameter governing
the Frisch elasticity of labor supply is estimated to be low so household’s labor supply reacts quite flexibly to shocks. Third and most importantly, monetary policy reacts fast and decisively to current economic conditions and in particular to output. The resulting transmission of both uncertainty and level shocks into the economy then implies less pronounced business cycles.

The decisive reaction to output growth is evident from the large coefficient estimate in the Taylor rule. The monetary authority’s aggressive reaction to changes in output has a considerable dampening effect on the business cycle as it prevents output from deviating too far from steady state. When keeping all parameters at their baseline values but setting $\phi_y = 0$, thus shutting off the response of interest rates to output growth, triples the negative output response following a policy risk shock (figures omitted for brevity). The main reason for this behavior is the response of the real interest rate. The uncertainty shock acts like a negative supply shock, agents reduce their labor and capital input, and inflation rises. The monetary authority responds to this increase in inflation by raising the nominal interest rate without considering the negative impact on output. As a result, the real interest rate now has a positive impact response, amplifying the original shock’s contractionary effect on output. In contrast, if the monetary authority also reacts to changes in output, the interest rate hike is more muted and the negative output response lower. The real interest initially
2.6 The Aggregate Effects of Policy Risk

declines to counteract the contractionary effect on output and only rises after several quarters.

The fast reaction of nominal interest rates to exogenous shocks can be seen from the relatively low degree of interest smoothing, meaning that current economic conditions affect nominal interests more than past interest rates. This low amount of interest smoothing exerts a considerable influence on the economy’s response to uncertainty shocks, allowing a stronger counteracting reaction of the nominal interest rate, which damps the uncertainty effects in a similar way as the output feedback of monetary policy. When giving more weight to past interest rates compared to the currently desired nominal interest, the nominal interest rate responds more sluggishly to shocks to the system, thereby temporarily allowing for larger deviations from steady state.

Hence, our result lend support to the findings of Bachmann and Bayer (2011). Their study showed for the case of idiosyncratic uncertainty about technology that general equilibrium effects, most importantly the endogenous feedback to wages and interest rates may considerably dampen the output effects of uncertainty shocks. Our results indicate that this also holds true for the case of aggregate uncertainty in an estimated DSGE-model.

These results suggest a potential issue for studies using a “proof-of-concept”-approach. Such studies typically show that uncertainty may matter by putting one source of uncertainty along one level shock into a model and then designing a transmission mechanism that enables this source to explain the whole business cycle. Our findings indicate that more attention needs to be devoted to what happens if other shocks, both uncertainty and level are present. As soon as other competing sources of aggregate fluctuations documented in the literature are added to these models, the effects of uncertainty are bound to decrease. Moreover, the approach of considering only one source of uncertainty and designing a particular amplification mechanism to generate an output drop in response may neglect that specially designed amplification mechanisms may interact with other types of shocks in undesired ways.²³

---

²³For example, expansionary output effects of uncertainty, which in our model e.g. arise with capital tax risk, might be amplified in the same way.
Chapter 2

2.7 Conclusion

The current chapter analyzes the effects of policy risk, i.e. aggregate uncertainty about labor and capital tax rates, monetary policy, and government spending on aggregate activity. We find that aggregate policy risk has only minor effects on the business cycle. Although its effects are an order of magnitude larger than the ones of technological uncertainty, a two standard-deviation policy risk shock still only generates a 0.025% drop in output. The reason for this small effect is that our parameter estimates imply strong general equilibrium effects that dampen the aggregate effects of uncertainty on economic activity. Most notably, the monetary authority’s estimated strong and rapid response to current conditions implies a nominal interest rate reaction that considerably reduces aggregate fluctuations. While our model is capable of generating strong uncertainty effects, such a calibration would imply unrealistically large business cycle fluctuations. Thus, SMM estimates the amplification of uncertainty shocks to be rather low.

The small effect of uncertainty on output does not imply that time-varying volatility is unimportant. In accordance with the previous literature (e.g. Justiniano and Primiceri, 2008; Primiceri, 2005), our findings suggest that the Great Moderation can be explained through a combination of “good luck” and “good policy”. The historical variance estimates indicate that the standard deviation of both technology and policy shocks significantly decreased since the mid-1980s. However, most of the effect of this time-varying volatility comes in the form of a different size of the realized level shocks instead of through the uncertainty-effect.

As our analysis focuses on aggregate uncertainty, it does not necessarily contradict studies finding large effects of idiosyncratic uncertainty. However, these studies clearly require different transmission mechanisms that do not give rise to large general equilibrium effects (see also Bachmann and Bayer, 2011).
Appendix to Chapter 2

2.A Data construction

Unless otherwise noted, all data are from the Bureau of Economic Analysis (BEA)’s NIPA Tables and available in quarterly frequency from 1960Q1 until 2010Q3.

Data for the exogenous processes

Capital and labor tax rates. Our approach to calculate average tax rates closely follows Mendoza, Razin, and Tesar (1994), Jones (2002), and Leeper, Plante, and Traum (2010). Details can be found in Appendix 1.D.

Government spending. Government spending is the sum of government consumption (Table 3.1 line 16) and government investment (Table 3.1 line 35) divided by the GDP deflator (Table 1.1.4 line 1) and the civilian noninstitutional population (BLS, Series LNU00000000Q).

Monetary policy shock. Computed as the residual from a Taylor rule as in Clarida, Galí, and Gertler (2000) (see Appendix 2.B). The sample only starts in 1961Q1 as we lose the first year of data due to the use of four time lags as instruments in the GMM estimation.

Total factor productivity (TFP). The construction of TFP closely follows Beaudry and Lucke (2010). Details can be found in Appendix 1.D.


The different sample lengths are not an issue as we estimate each exogenous process separately. Using the longest available sample assures that we make optimal use of the available information for each series.

Data for SMM

Output. Nominal GDP (Table 1.1.5 line 1) divided by the GDP deflator (Table 1.1.4 line 1) and the civilian noninstitutional population (BLS, Series LNU00000000Q).
Chapter 2

**Investment.** Sum of Residential fixed investment (Table 1.1.5 line 12) and nonresidential fixed investment (Table 1.1.5 line 9) divided by the GDP deflator (Table 1.1.4 line 1) and the civilian noninstitutional population (BLS, Series LNU00000000Q).

**Consumption.** Sum of personal consumption expenditures for nondurable goods (Table 1.1.5 line 5) and services (Table 1.1.5 line 6) divided by the GDP deflator (Table 1.1.4 line 1) and the civilian noninstitutional population (BLS, Series LNU00000000Q).

**Real wage.** Hourly compensation in the nonfarm business sector (BLS, Series PRS85006103) divided by the GDP deflator (Table 1.1.4 line 1).

**Inflation.** Computed as the log-difference of the GDP deflator (Table 1.1.4 line 1).

**Nominal interest rate.** Geometric mean of the effective Federal Funds Rate (St.Louis FED - FRED Database, Series FEDFUNDS).

Additional data for GMM

**Interest term spread.** We use the difference of the quarterly geometric mean of the 10-Year Treasury Constant Maturity Rate (FRED Database, Series GS10) and the quarterly geometric mean of the 3-Month Treasury Bill: Secondary Market Rate (FRED Database, Series TB3MS).

**Money growth rate.** Growth rate of the M2 Money Stock (FRED Database, Series M2SL).

**Commodity inflation.** Commodity inflation is computed as the growth rate of the X12-seasonally adjusted Producer Price Index: All Commodities (FRED Database, Series PPIACO).

**Output gap.** The output gap is constructed as the percentage difference between real GDP (FRED Database, Series GDPC96) and Real Potential Gross Domestic Product (FRED Database, Series GDPPOT).
2.B Econometric Methods

The Particle Filter

For ease of exposition, let $x_t$ be a generic observable AR(1) process

$$x_t = \rho x_{t-1} + \sigma_t \nu_t , \quad \nu_t \sim \mathcal{N}(0, 1) \tag{2.19}$$

where the unobserved/latent state $\sigma_t$ follows a stochastic volatility process

$$\sigma_t = (1 - \rho^2) \bar{\sigma} + \rho^2 \sigma_{t-1} + \eta \varepsilon_t , \quad \varepsilon_t \sim \mathcal{N}(0, 1), \tag{2.20}$$

where $\bar{\sigma}$ is the unconditional mean of $\sigma_t$. The shock to the volatility $\varepsilon_t$ is assumed to be independent from the level shock $\nu_t$.

Hence, a filter is required to obtain the so-called filtering density $p(\sigma_t | x^t; \Theta)$. Due to the nonlinearity embedded in the stochastic volatility setup of the shocks, we cannot simply employ the Kalman filter as in the case of linearity and normally distributed shocks. Instead, we employ the Sequential Importance Resampling (SIR) particle filter, a special application of the more general class of Sequential Monte Carlo methods, to evaluate the likelihood (Fernández-Villaverde and Rubio-Ramírez, 2007; Fernández-Villaverde et al., 2011b). Given the structure in (2.19) and (2.20) and some initial value $x_0$, the factorized likelihood of observing $x^T$ can be written as

$$p(x^T; \Theta) = \prod_{t=1}^{T} p(x_t | x^{t-1}; \Theta)$$

$$= \int p(x_1 | x_0, \sigma_0; \Theta) \, d\sigma_0 \prod_{t=2}^{T} \int p(x_t | x_{t-1}, \sigma_t; \Theta) \, p(\sigma_t | x^{t-1}; \Theta) \, d\sigma_t$$

$$= \int \frac{1}{\sigma_0 \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{x_1 - \rho x_0}{e^{\sigma_0}} \right)^2 \right] \, d\sigma_0$$

$$\times \prod_{t=2}^{T} \int \frac{1}{\sigma_t \sqrt{2\pi}} \exp \left[ -\frac{1}{2} \left( \frac{x_t - \rho x_{t-1}}{e^{\sigma_t}} \right)^2 \right] \, p(\sigma_t | x^{t-1}; \Theta) \, d\sigma_t , \tag{2.21}$$

where $x^t$ is a $(t \times 1)$ vector that stacks the observations on $x$ up to time $t$, $\Theta$ stacks the parameters, and the last equality follows from the assumption of normally distributed shocks. Although we do not have an analytical expression for $p(\sigma_t | x^{t-1}; \Theta), t =$
1, ..., T, and can therefore not compute it directly, we can employ the particle filter to estimate the likelihood by iteratively drawing from \( p(\sigma_t|x^{t-1}; \Theta) \).

The underlying idea of the particle filter is to use an approximation of the filtering density \( p(\sigma_t|x^t; \Theta) \) with a simulated distribution generated from empirical data. This distribution can be formed from mass points, or particles, \( p(\sigma_t|x^t; \Theta) \cong \sum_{i=0}^{N} \omega_i^t \delta_{\sigma_i^t}(\sigma_t) \)

where \( \delta \) is the Dirac delta function and \( \omega_i^t \) is the weight attached to the respective draw/particle \( \sigma_i^t \) (Godsill, Doucet, and West, 2004). We can then use a Sequential Importance Resampling (SIR)-approach to update particles from time \( t \) to \( t + 1 \) and obtain the new filtering distribution at \( t + 1 \) (see e.g. Fernández-Villaverde et al., 2011b). A convenient by-product of this filtering approach is that we also approximate \( p(\sigma_t|x^{t-1}; \Theta) \), the distribution we need to build the likelihood.

The SIR is a two-step procedure that, by using a prediction and a resampling/filtering step for each time period, ultimately allows to iteratively draw from \( p(\sigma_t|x^{t-1}; \Theta) \).

Starting with \( p(\sigma_0|x^0; \Theta) = p(\sigma_0; \Theta) \), the prediction step uses the law of motion for the states \( f(\sigma_{t+1}|\sigma_t) \), equation (2.20), to obtain the conditional density \( p(\sigma_1|x^0; \Theta) = p(\epsilon_1)p(\sigma_0|x^0; \Theta) \). That is, given \( N \) draws \( \{\sigma_i^t\}_{i=1}^{N} \) from \( p(\sigma_t|x^t; \Theta) \), (here \( p(\sigma_0|x^0; \Theta) \)) and a draw of exogenous shocks \( \epsilon_i^t \sim \mathcal{N}(0,1) \), we can use equation (2.20) to compute \( \{\sigma_i^{t+1}\}_{i=1}^{N} \).

Next, the resampling/filtering step uses importance resampling to update the conditional probability from \( p(\sigma_t|x^{t-1}; \Theta) \) to \( p(\sigma_t|x^t; \Theta) \). The crucial idea is that if \( \{\sigma_i^{t-1}\}_{i=1}^{N} \) is a draw from \( p(\sigma_t|x^{t-1}; \Theta) \) and \( \{\tilde{\sigma}_i^t\}_{i=1}^{N} \) is a draw with replacement from \( \{\sigma_i^{t-1}\}_{i=1}^{N} \) using the resampling probabilities \( \omega_i^t = \frac{p(x_t|x^{t-1}, \sigma_i^{t-1}; \Theta)}{\sum_{i=1}^{N} p(x_t|x^{t-1}, \sigma_i^{t-1}; \Theta)} \),

then \( \{\sigma_i^t\}_{i=1}^{N} = \{\tilde{\sigma}_i^t\}_{i=1}^{N} \) is a draw from \( p(\sigma_t|x^t; \Theta) \). The resampling with probabilities

\[ \text{The notation } t+1|t \text{ indicates a draw at time } t+1 \text{ conditioned on the information available at time } t. \]
given in (2.23) serves two purposes. First, the reweighting implements an importance sampling approach, i.e. draws are obtained from a proposal density that is easy to draw from and are then subsequently reweighted to reflect the density to be approximated (see Arulampalam et al., 2002, for a derivation). Second, without the resampling step, there would be an increase in the unconditional variance of $\omega_t$ over time, yielding only one particle with non-zero weight (known as degeneracy or sample impoverishment, see Arulampalam et al. (2002)). By resampling, we keep only those particles with high $\omega^t_i$ (i.e. those that are closer to the true state vector).

Having now obtained draws from $p(\sigma_t|x^t;\Theta)$, we can again start with the prediction step to obtain draws for time period $t + 1$.

After $T$ iterations, we get an estimate of our likelihood as

$$ p(x^T;\Theta) \approx \frac{1}{N} \sum_{i=1}^{N} \frac{1}{e^{\sigma_{00}}} \sqrt{2\pi} \exp \left[ -\frac{1}{2} \left( \frac{x_1 - \rho x_0}{e^{\sigma_{00}}} \right)^2 \right] \times \prod_{t=2}^{T} \frac{1}{N} \sum_{i=1}^{N} \frac{1}{e^{\sigma_{t|t-1}}} \sqrt{2\pi} \exp \left[ -\frac{1}{2} \left( \frac{x_t - \rho x_{t-1}}{e^{\sigma_{t|t-1}}} \right)^2 \right]. \tag{2.24} $$

Particle Smoother

We employ the backward-smoothing routine suggested by Godsill, Doucet, and West (2004) to draw from the smoothing density $p(\sigma^T|x^T;\Theta)$ to get a historical distribution of the volatilities. Specifically, we start with the factorization

$$ p(\sigma^T|x^T;\Theta) = p(\sigma_T|x^T;\Theta) \prod_{t=1}^{T-1} p(\sigma_t|\sigma_{t+1:T},x^T;\Theta). \tag{2.25} $$

The second factor can be further simplified

$$ p(\sigma_t|\sigma_{t+1:T},x^T;\Theta) = p(\sigma_t|\sigma_{t+1},x^t;\Theta) = \frac{p(\sigma_t|x^t;\Theta) f(\sigma_{t+1}|\sigma_t)}{p(\sigma_{t+1}|x^t)} \propto p(\sigma_t|x^t;\Theta) f(\sigma_{t+1}|\sigma_t), \tag{2.26} $$

25In our case, we use the prior density $p(\sigma_t|\sigma^{t-1};\Theta)$ as the importance density.
26See Fernández-Villaverde and Rubio-Ramírez (2007) and Doucet and Johansen (2009) and the references contained therein for the conditions required for a central limit theorem to apply, yielding a consistent estimator of $p(x^T;\Theta)$.  

89
where the first equality results from the Markovian properties of the model and $f$ denotes the state transition density following from equation (2.20). Equation (2.22) describes how to approximate $p(\sigma_t|x_t;\Theta)$ by forward filtering. Therefore, we can approximate $p(\sigma_t|\sigma_{t+1:T},x^T;\Theta) \propto p(\sigma_t|x^t;\Theta)f(\sigma_{t+1}|\sigma_t)$ by

$$p(\sigma_t|\sigma_{t+1},x^T;\Theta) \simeq \sum_{i=1}^{N} \omega_i f(\sigma_{t+1}^i|\sigma_t^i),$$

where the new weights $\omega_i^t$ are given by

$$\omega_i^t = \frac{\omega_i^t f(\sigma_{t+1}^i|\sigma_t^i)}{\sum_{j=1}^{N} \omega_j f(\sigma_{t+1}^j|\sigma_t^j)}.$$

and the $\omega_i^t$ are the weights obtained in the filtering step. Denote with $\tilde{\sigma}_i^t$ the $i^{th}$ draw from the smoothing density at time $t$. At time $T$, we can obtain draws $\tilde{\sigma}_T$ by drawing from $p(\sigma_T|x^T)$ with the weights $\omega_i^T$. Then, going backwards in time, we can use the above recursions to iteratively obtain draws $\tilde{\sigma}_i^t$ by resampling using the weights given in (2.28).

**Tailored Randomized Block Metropolis Hastings Algorithm**

Let $\Theta$, $p(x^T|\Theta)$, and $\pi(\Theta)$ denote the vector of parameters to be estimated, the likelihood function, and the prior distribution of the parameters, respectively. The posterior distribution $\pi(\Theta|x^T)$ can be computed as

$$\pi (\Theta | x^T) \propto p (x^T | \Theta) \pi (\Theta) .$$

Given this usually analytically intractable posterior, most macroeconomic applications employ a Random Walk Metropolis-Hastings (RW-MH) algorithm to generate draws from the posterior distribution. However, the standard RW-MH algorithm often has poor mixing properties, leading to highly autocorrelated draws, and is therefore often very inefficient. Hence, to increase the efficiency, we use the Tailored Randomized Block Metropolis Hastings (TaRB-MH) algorithm proposed by Chib and Ramamurthy.
(2010). Instead of in each iteration step simultaneously drawing an entire new parameter vector from a proposal density, the parameter vector is randomly split up into several blocks. Each block is then subsequently updated by a separate MH run, conditional on the previous step’s values of the parameters in the other blocks. Ideally, the blocks should be formed according to the correlation between parameters, with highly correlated parameters belonging to the same block. However, we have no a priori knowledge about the correlation between parameters and resort to a blocking scheme where both the number of blocks and its composition are randomized in each step. This algorithm provides a good compromise between the standard RW-MH and tailored multiple block MH algorithms that use multiple blocks, which are particularly designed for the problem at hand. The second feature that improves on the standard RW-MH is that in each step the proposal density is “tailored” to the location and the curvature of the posterior density in that block by using a non-derivative based global optimizer. We deviate from Chib and Ramamurthy (2010) by using the CMAES algorithm (Hansen, Müller, and Koumoutsakos, 2003) instead of a simulated annealing as the former has been shown to be more efficient (Andreasen, 2010). Moreover, it requires considerably less tuning than a simulated annealing. The TaRB-MH algorithm proceeds as follows.

1. At each iteration step $n$, $n = 1, \ldots, N$, the elements of the parameter vector $\theta$ are separated into random blocks $(\theta_{n,1}, \theta_{n,2}, \ldots, \theta_{n,p_n})$ by perturbing their initial ordering and assigning the first parameter in the perturbed vector to the first block and each following parameter with probability $p = 0.5$ to a new block, leaving us with 2.5 blocks on average as we estimate 5 parameters.

2. At each iteration step $n$, each block $\theta_{n,l}, l = 1, \ldots, p_n$ is sampled by a Metropolis-Hastings step using a proposal density adapted to the posterior in the following way. Denote with $\theta_{n,-l}$ the most current value of all blocks except for the $l$th one, i.e. their value at the end of step $n - 1$. To generate a new draw for $\theta_{n,l}$,

\[27\] Using the TaRB-MH decreased the inefficiency factors from values around 10 to below 2.

\[28\] For an intuitive introduction to the working of the CMAES algorithm, see Binsbergen et al. (2010).
the CMAES-algorithm is used to find

$$\hat{\theta}_{n,l} = \arg \max_{\theta_{n,l}} \log \left[ p \left( x^T | \theta_{n,l}, \theta_{n,-l} \right) \pi \left( \Theta \right) \right].$$

(2.30)

That is, we use a global optimizer to maximize the posterior over the current block \( l \), given the value of all other parameters at the end of step \( n - 1 \). Having found the “conditional mode” \( \hat{\theta}_{n,l} \), we compute the curvature of the target posterior distribution in the standard way as the negative inverse of the Hessian at the “conditional mode”

$$V_{n,l} = \left. \left( -\frac{\partial \log \left[ p \left( x^T | \theta_{n,l}, \theta_{n,-l} \right) \pi \left( \Theta \right) \right]}{\partial \theta_{n,l} \theta_{n,l}^T} \right) \right|_{\theta_{n,l} = \hat{\theta}_{n,l}}^{-1}.$$

(2.31)

Following Chib and Ramamurthy (2010), we use a multivariate \( t \)-distribution with \( \nu \) degrees of freedom as proposal density for \( \theta_{n,l} \), \( q_l \left( \theta_{n,l} | \theta_{n,-l}, x^T \right) \). Mean and variance are set to the “conditional mode” and the negative inverse of the Hessian at this point:

$$q_l \left( \theta_{n,l} | \theta_{n,-l}, x^T \right) = t \left( \theta_{n,l} | \hat{\theta}_{n,l}, V_{n,l}, \nu \right).$$

(2.32)

In the Metropolis-Hastings-step, a proposed value \( \theta_{n,l}^* \) is accepted as the new value of the block with probability

$$\alpha_l \left( \theta_{n,l}, \theta_{n,l}^* | \theta_{n,-l}, x^T \right) = \min \left[ \frac{p \left( x^T | \theta_{n,l}^*, \theta_{n,-l} \right) \pi \left( \theta_{n,l}^* \right) \left( \theta_{n,l}^* \right)}{p \left( x^T | \theta_{n,l}, \theta_{n,-l} \right) \pi \left( \theta_{n,l} \right) \hat{\theta}_{n,l} \left( \theta_{n,l} \right) \left( \theta_{n,l} \right)} \right].$$

(2.33)

If the proposed value \( \theta_{n,l}^* \) is rejected, we set \( \theta_{n+1,l} = \theta_{n,l} \). This step is repeated for all \( p_n \) blocks before the algorithm starts over with step 1.

Setting \( \nu = 5 \) and iterating over steps 1 and 2, we can - after a suitable burn-in-period - obtain samples from the desired posterior distribution, which is the invariant distribution of the resulting Markov Chain. In our case, a burn-in of 2500 proved sufficient.
2.B Econometric Methods

Model Solution

Let $s_t$ denote the $n_s \times 1$ vector of state variables in deviations from steady state, including the exogenous shocks and the perturbation parameter $\Lambda$, and let $s_i^t$ denote its $ith$ entry. The policy function/law of motion for an arbitrary model variable $\hat{X}_t$ then has the form

$$\hat{X}_t = \sum_{i=1}^{n_s} \xi_X s_i^t + \frac{1}{2} \sum_{i=1}^{n_s} \sum_{j=1}^{n_s} \xi_{X} s_i^t s_j^t + \sum_{i=1}^{n_s} \sum_{j=1}^{n_s} \sum_{l=1}^{n_s} \xi_{X} s_i^t s_j^t s_l^t,$$

(2.34)

where the $\xi$'s are scalars that depend on the deep parameters of the model and hats denote percentage deviations from steady state. Equation (2.34) shows why lower-order approximations would not be sufficient for our purpose.

As is well known, a first-order approximation exhibits certainty equivalence. This implies $\xi_X^v = 0$, where $v$ denotes the position of a volatility shock in the state vector $s$. That is, up to first order, uncertainty shocks do not enter the policy function at all.

For a second-order approximation, it is well known from Schmitt-Grohé and Uribe (2004) for the homoskedastic case that uncertainty only enters the policy function through a constant term via the second derivative with respect to the perturbation parameter, i.e. through $\xi_{X,\Lambda} \neq 0$. However, things are more complicated in the heteroscedastic case where shocks to the variance occur, leading to an additional effect. Fernández-Villaverde, Guerrón-Quintana, and Rubio-Ramírez (2010) prove that in this case, the volatility shocks additionally only enter the policy function with non-zero coefficients in their interaction term with the respective level shock. Algebraically, only the cross-product of $\hat{\sigma}_i^j \times \hat{\nu}_i^j$ is different from 0. In contrast, all other cross-terms with the uncertainty shocks are zero, i.e. $\xi_{X,v,j \neq u} = 0$, where $v$ and $u$ denote the positions of a volatility and its corresponding level shock in the state vector $s$, respectively. Hence, the effect of uncertainty is always mediated through level shocks. It is not possible to shock the variance of the level shocks independently from the level shock as its effect would be 0 by construction.

Only in the third-order approximation do the volatility shocks enter the policy function separately from the level shocks in a non-constant form. Most importantly, the term $\xi_{X,\Lambda,\Lambda}$ is in general different from 0 for all volatility shocks.
Chapter 2

Simulated Method of Moments

The idea of the Simulated Method of Moments (SMM) is the following. Let \( x_t \) be a time \( t \) vector of observables from a stationary and ergodic distribution and let \( \{x_t\}_{t=1}^T \) be the corresponding sequence. Furthermore, let \( m(x_t) \) denote a \( k \times 1 \) vector of empirical moments computed from this data. Denote with \( \{x_{t}^{\text{sim}}(\theta)\}_{t=1}^{aT} \) the corresponding time series of length \( aT \) generated from simulating the model using the \( p \times 1 \) parameter vector \( \theta \in \Theta \), with \( \Theta \subset \mathbb{R}^p \). Let \( m(x_{t}^{\text{sim}}(\theta)) \) be the vector of simulated moments computed from the artificial data. The SMM estimator is the value of \( \theta \) that satisfies

\[
\hat{\theta} = \arg \min_{\theta \in \Theta} \left[ m(x_t) - m(x_{t}^{\text{sim}}(\theta)) \right]^T W \left[ m(x_t) - m(x_{t}^{\text{sim}}(\theta)) \right],
\]

where \( W \) is a \( p \times p \) positive definite weighting matrix. Under the assumption that the model with \( \theta = \theta_0 \) is a correct representation of the true process that generated \( m(x_t) \) and the regularity conditions spelled out in Duffie and Singleton (1993), \( \hat{\theta} \) is a consistent estimator of \( \theta_0 \) with asymptotic distribution

\[
\sqrt{T} \left( \hat{\theta} - \theta_0 \right) \xrightarrow{d} \mathcal{N} \left( 0, (1 + 1/\tau) (J'WJ)^{-1} J'WSWJ (J'WJ)^{-1} \right),
\]

where

\[
S = \lim_{T \to \infty} \text{Var} \left( \frac{1}{\sqrt{T}} \sum_{t=1}^{T} m(x_t) \right),
\]

and \( J = \mathbb{E}(\partial m(x_{t}^{\text{sim}})/\partial \theta) \) (see Ruge-Murcia, 2010).

This estimator is asymptotically efficient when using the weighting matrix

\[
W = \left( V_{\text{longrun}} \right)^{-1} = \left[ \lim_{T \to \infty} \text{Var} \left( \frac{1}{\sqrt{T}} \sum_{t=1}^{T} m(x_t) \right) \right]^{-1}.
\]

The ideal weighting matrix places the most weight on the linear combination of moments that are the most precisely measured in the data. However, for two reasons, we use only the diagonal of the optimal weighting matrix:

\[
W^{\text{diag}} = \text{diag} \left( V_{\text{longrun}} \right)^{-1}.
\]
First, we would like to put more weight on moments that are actually observed in the data and that are economically meaningful, rather than on a linear combination of moments (see also Cochrane, 2005). Second, in practice, fully specified weighting matrices often lead to diverging parameter estimates. As shown in Ruge-Murcia (2010), using only the main diagonal of the optimal weighting matrix leads to a loss in efficiency but nevertheless delivers good results in most cases.

The simulation proceeds as follows. Starting at the deterministic steady state, we simulate the model for 3015 quarters using shocks drawn from the estimated shock distributions. Shocks larger than two standard deviations are trimmed. To assure non-explosive behavior of the simulations, we use the pruning algorithm of Kim et al. (2008). We discard the first 2000 quarters as a burn-in in order to reach the ergodic distribution. We then use the remaining 1015 quarters to compute the respective moments. The results are robust to using a longer burn-in period. The choice of using five times the length of the original data sample (i.e. $a = 5$) to compute the moments is motivated by the simulations in Ruge-Murcia (2010), who finds this choice to deliver a good balance between the precision of the estimates and computation time.

**Impulse Responses**

The nonlinearity of our model complicates the computation of impulse responses compared to linear models. We follow Fernández-Villaverde et al. (2011b) and generate impulse responses as the response to a two standard deviation shock to uncertainty at the ergodic mean. First, we simulate the model for 2,000 quarters by drawing shocks from the respective estimated distributions. Shocks larger than two standard deviations are trimmed to assure convergence, which technically depends on the shocks being bounded. To assure non-explosive behavior of the simulations, we use the pruning algorithm of Kim et al. (2008). We discard the first 2,000 quarters as a burn-in in order to reach the ergodic distribution and use the next 675 quarters to compute the ergodic mean. Starting at the ergodic mean, we compute the IRFs as the percentage difference of the respective variables between the system shocked with the respective shock and the baseline model response, i.e. the model response without shocks. To account for sampling uncertainty, we generate 50 different IRFs with
different starting values of the random number generator and take the cross-sectional average as our impulse response.

**GMM**

We construct the monetary policy shocks by specifying the Federal Reserve’s policy reaction function and estimating it by the generalized method of moments (GMM). Our approach is similar to the one used in Clarida, Galí, and Gertler (2000), with the difference that Clarida, Galí, and Gertler (2000) use a forward-looking policy reaction function, while we use a rule that reacts to contemporaneous variables to stay consistent with our DSGE-model. Specifically, the policy reaction function to be estimated is given by

\[ r_t = \rho r_{t-1} + (1-\rho) \left[ \bar{r} + \phi_\pi (\pi_t - \bar{\pi}) + \phi_y y_{gap} \right] + \varepsilon_t, \]  

(2.40)

where \( \pi_t \) is inflation with target rate \( \bar{\pi} \), \( y_{gap} \) is the output gap, \( r_{t-1} \) allows for interest smoothing, \( \bar{r} \) is the target nominal interest rate, and \( \varepsilon_t \) is an error term. Using the vector of instruments \( z_t \), the set of moment conditions for our GMM estimation procedure can be written as

\[ E \left\{ \{r_t - \rho r_{t-1} - \alpha - \beta \pi_t - \gamma y_{gap} \} z_t \right\} = 0 \]  

(2.41)

where \( \alpha = (1-\rho)(\bar{r} + \phi_\pi \bar{\pi}) \) collects all constant terms, \( \beta = (1-\rho)\phi_\pi \), and \( \gamma = (1-\rho)\phi_y \).

Hence, we regress the average effective Federal Funds Rate in the first month of the quarter on the lagged FFR, the inflation rate, and the output gap, where all rates are annualized. The set of instruments includes four lags of the FFR, the inflation rate, the output gap, commodity price inflation, money growth, and the interest term spread. Because we are only interested in the residuals of the policy reaction function \( \hat{\varepsilon}_t \), we do not need to separately identify the target nominal rate \( \bar{r} \) and target inflation \( \bar{\pi} \).

Table 2.9 presents the estimation results, which are all in the range typically reported in the literature. There is strong evidence of interest smoothing with
2.C Diagnostics

Table 2.9: GMM Estimation of Taylor Rule

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Mean</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.898</td>
<td>0.018</td>
<td>48.926</td>
<td>0.000</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.001</td>
<td>0.001</td>
<td>0.874</td>
<td>0.383</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.1741</td>
<td>0.027</td>
<td>6.361</td>
<td>0.000</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.102</td>
<td>0.017</td>
<td>5.950</td>
<td>0.000</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.890</td>
<td>Mean dependent var</td>
<td>0.058</td>
<td></td>
</tr>
<tr>
<td>Adjusted R-squared</td>
<td>0.888</td>
<td>Sum squared resid</td>
<td>0.027</td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.012</td>
<td>J-statistic</td>
<td>18.545</td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>2.314</td>
<td>pval(J-statistic)</td>
<td>0.552</td>
<td></td>
</tr>
</tbody>
</table>

Note: Kernel: Bartlett, Bandwidth: Fixed (4), No prewhitening; Simultaneous weighting matrix & coefficient iteration; Convergence achieved after: 28 weight matrices, 29 total coef iterations.

$\rho = 0.898$. The point estimates of the feedback parameters are $\phi_\pi = 1.718$ and $\phi_y = 1.003$. The test of overidentifying restrictions shows that the model cannot be rejected at conventional significance levels.

2.C Diagnostics

Testing for Heteroskedasticity

Table 2.10 presents evidence of the need to model time-varying volatility. Despite our relatively short sample size and the low power of tests for heteroskedasticity, the null hypothesis of homoskedastic shocks can be rejected at the 10% level for all series except labor taxes. This result is consistent with evidence that the standard deviation of structural shocks has changed over time (see e.g. Justiniano and Primiceri, 2008; Primiceri, 2005).

Convergence Diagnostics

Table (2.11) shows the results from the Geweke (1992)-convergence diagnostics that compares the means of the first 20% of draws with that of the last 50% of the
Table 2.10: Tests for Heteroskedasticity

<table>
<thead>
<tr>
<th></th>
<th>$\tau^k$</th>
<th>$\tau^n$</th>
<th>$z$</th>
<th>$z_I$</th>
<th>$g$</th>
<th>$m$</th>
</tr>
</thead>
<tbody>
<tr>
<td>White</td>
<td>0.000*</td>
<td>0.932</td>
<td>0.001*</td>
<td>0.042*</td>
<td>0.360</td>
<td>0.068*</td>
</tr>
<tr>
<td>WW</td>
<td>0.169</td>
<td>0.523</td>
<td>0.265</td>
<td>0.005*</td>
<td>0.076*</td>
<td>0.068*</td>
</tr>
<tr>
<td>BPK</td>
<td>0.004*</td>
<td>0.890</td>
<td>0.126</td>
<td>0.770</td>
<td>0.511</td>
<td>0.298</td>
</tr>
</tbody>
</table>

Notes: Asterisks indicate significance at the 10% level. White refers to the standard White (1980)-test, WW refers to the Wooldridge (1990)-version of this test, and BPK refers to the Breusch and Pagan (1979)/Koenker (1981)-test.

draws. In general, all MCMC chains have converged to their stationary distribution as indicated by the p-values of the $\chi^2$-test for equal means. Figures 2.8 to 2.13 show the corresponding mean plots.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Capital Tax Rates</th>
<th>Labor Tax Rates</th>
<th>Total Factor Productivity</th>
<th>Investment Specific Technology</th>
<th>Government Spending</th>
<th>Monetary Policy Shock</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4% taper</td>
<td>8% taper</td>
<td>15% taper</td>
<td>4% taper</td>
<td>8% taper</td>
<td>15% taper</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>0.160</td>
<td>0.165</td>
<td>0.145</td>
<td>0.909</td>
<td>0.890</td>
<td>0.887</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>0.947</td>
<td>0.941</td>
<td>0.937</td>
<td>0.926</td>
<td>0.913</td>
<td>0.904</td>
</tr>
<tr>
<td>$\rho_\sigma$</td>
<td>0.623</td>
<td>0.596</td>
<td>0.566</td>
<td>0.648</td>
<td>0.652</td>
<td>0.653</td>
</tr>
<tr>
<td>$\eta_\sigma$</td>
<td>0.929</td>
<td>0.927</td>
<td>0.919</td>
<td>0.327</td>
<td>0.319</td>
<td>0.271</td>
</tr>
<tr>
<td>$\tilde{\sigma}$</td>
<td>0.760</td>
<td>0.744</td>
<td>0.738</td>
<td>0.922</td>
<td>0.921</td>
<td>0.917</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>0.891</td>
<td>0.887</td>
<td>0.879</td>
<td>0.199</td>
<td>0.174</td>
<td>0.124</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>0.679</td>
<td>0.681</td>
<td>0.665</td>
<td>0.353</td>
<td>0.340</td>
<td>0.297</td>
</tr>
<tr>
<td>$\rho_\sigma$</td>
<td>0.643</td>
<td>0.615</td>
<td>0.583</td>
<td>0.546</td>
<td>0.534</td>
<td>0.520</td>
</tr>
<tr>
<td>$\eta_\sigma$</td>
<td>0.456</td>
<td>0.453</td>
<td>0.391</td>
<td>0.638</td>
<td>0.649</td>
<td>0.638</td>
</tr>
<tr>
<td>$\tilde{\sigma}$</td>
<td>0.772</td>
<td>0.765</td>
<td>0.706</td>
<td>0.304</td>
<td>0.260</td>
<td>0.187</td>
</tr>
</tbody>
</table>

Notes: Numbers are p-values of the $\chi^2$-test for equal means of the first 20% of draws and the last 50% of the draws (after the first 2500 draws are discarded as burn-in).
Figure 2.8: Evolution of MCMC Sampler over Time for $\tau^k$

(a) MCMC draws

(b) Mean of the parameters over time
Figure 2.9: Evolution of MCMC Sampler over Time for $\tau_n$

(a) MCMC draws

(b) Mean of the parameters over time
Figure 2.10: Evolution of MCMC Sampler over Time for $z$

(a) MCMC draws

(b) Mean of the parameters over time
Figure 2.11: Evolution of MCMC Sampler over Time for $z'$

(a) MCMC draws

(b) Mean of the parameters over time
Figure 2.12: Evolution of MCMC Sampler over Time for $g$

(a) MCMC draws

(b) Mean of the parameters over time

104
Figure 2.13: Evolution of MCMC Sampler over Time for $m$

(a) MCMC draws

(b) Mean of the parameters over time
Chapter 2

Model Misspecification Diagnostics

Following Kim, Shephard, and Chib (1998), we can test the specification of our SV-model. Using $N$ draws from the prediction density $p(x_t|x_{t-1}; \Theta)$, we can compute the probability that $x_{t+1}^{obs}$ will be less or equal than the actually observed value of $(x_{t+1}^{obs})^2$:

$$\Pr \left( x_{t+1}^2 \leq (x_{t+1}^{obs})^2 | x^t ; \Theta \right) \simeq \frac{1}{N} \Pr \left( x_{t+1}^2 \leq (x_{t+1}^{obs})^2 | x^t, \sigma_{t+1}^2 | \Theta \right), \tag{2.42}$$

$\forall t = 1, \ldots T - 1$. If the SV-model is correctly specified, the sequence of $u_t$ converges in distribution to i.i.d. uniform variables as the number of particles $N$ goes to infinity (Rosenblatt, 1952). Under the null hypothesis of a correctly specified model, the $u_t$ can be transformed to i.i.d. standard normal variables using the inverse normal CDF. Hence, we can perform a simple test for misspecification by testing the resulting series for their normality. Figure 2.14 shows the corresponding QQ-plots.

Table 2.12 presents the results from three commonly used normality tests. In general, a correct specification of the model tends to not be rejected. Only for $z$, the Jarque-Bera and the Kolmogorov-Smirnov tests reject normality. However, this effect is driven by the outliers visible in the bottom left corner of Figure 2.14. In contrast, when shutting off the time-varying volatility and setting the volatility to its unconditional mean, the specification is generally rejected (results are not shown here).

<table>
<thead>
<tr>
<th></th>
<th>JB</th>
<th>KS</th>
<th>SW</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau^k$</td>
<td>0.066</td>
<td>0.039**</td>
<td>0.125</td>
</tr>
<tr>
<td>$\tau^n$</td>
<td>0.141</td>
<td>0.960</td>
<td>0.135</td>
</tr>
<tr>
<td>$z$</td>
<td>0.037**</td>
<td>0.035**</td>
<td>0.085</td>
</tr>
<tr>
<td>$z^I$</td>
<td>0.377</td>
<td>0.076</td>
<td>0.586</td>
</tr>
<tr>
<td>$g$</td>
<td>0.500</td>
<td>0.747</td>
<td>0.528</td>
</tr>
<tr>
<td>$m$</td>
<td>0.052</td>
<td>0.377</td>
<td>0.012**</td>
</tr>
</tbody>
</table>

Note: Asterisks indicate significance at the 5% level. JB refers to the Jarque and Bera (1987)-test, KS refers to the Kolmogorov (1933)/Smirnov (1948)-test, and SW refers to the Shapiro and Wilk (1965)-test.
Figure 2.14: QQ-Plots for Model Misspecification

Notes: From left to right and top to bottom: capital taxes, labor taxes, TFP, investment-specific technology, monetary policy shocks, and government spending.
3 The Business Cycle Effects of Terms of Trade Uncertainty

3.1 Introduction

The 2006-2008 commodities boom led to an increase in international commodities prices that was unprecedented not only in magnitude but also in duration and breadth of commodity groups affected (Baffes and Haniotis, 2010; World Bank, 2011). In 2006, real copper and timber prices more than doubled (see Figure 3.1). At the height of the boom in 2008, oil prices were 94% higher than a year earlier. Ever since, analysts have been concerned with how these price changes translate into changes in international relative prices faced by different countries, i.e. the terms of trade, and their consequences for the business cycle (see e.g. International Monetary Fund, 2011; World Bank, 2011). In contrast, less attention has been devoted to the fact that this commodity price boom has been accompanied by a large increase in price volatility and that, over the last decades, world-wide commodity and manufacturing prices have been going through distinct periods of high and low volatility (Arezki, Lederman, and Zhao, 2011). The right row of Figure 3.1 displays the monthly growth rates of copper and timber prices, whose average magnitude has significantly risen since 2003. This suggests that the uncertainty associated with international commodities prices has increased significantly in the last few years.

That commodities prices are highly volatile is a well known fact: “What com-
modiﬁcy prices lack in trend, they make up for in variance” (Deaton, 1999, p. 27). However, time variation in this variance has mostly been neglected in both public discussions and academic research. This is surprising since recent research suggests that changes in uncertainty about macroeconomic variables may be an important factor in explaining business cycles of advanced (Bloom, 2009) and emerging countries (Fernández-Villaverde et al., 2011b). I attempt to close this gap by studying the effects of terms of trade uncertainty, i.e. the time-varying volatility of terms of trade shocks, on business cycles through the lens of a small open economy DSGE model à la Lubik (2003), Santacreu (2005), and Monacelli and Perotti (2010). In particular, I analyze the response of output and its components following an exogenous increase in terms of trade uncertainty. This exogenous increase in uncertainty is conceptualized as a mean-preserving spread to the shock distribution.\footnote{Note that, as explained in Chapter 2, this ex-ante effect of higher uncertainty about the future terms of trade is conceptually different from the ex-post effect of larger shock realizations.}

The empirical analysis in this study is based on quarterly Chilean aggregate data from 1996:Q2-2011:Q2. Chile is an interesting case to study the effects of terms of trade uncertainty for three reasons. First, although the Chilean economy is relatively diversified,\footnote{During the recent commodities price boom, copper exports increased in their importance due to a large rise in prices, somewhat decreasing the diversification. See Figure 3.9.} commodities compose a significant part of its exports. Hence, its terms of trade, although not entirely driven by commodities, exhibit significant and well-documented time-varying volatility. After average annual fluctuations in the terms of trade of $\pm 10\%$ from 1997 to 2003, they have almost doubled since (Desormeaux, García, and Soto, 2010). Second, Chile is small enough to plausibly assume that terms of trade variations are exogenous from its point of view. Of course, on the global level, changes in the relative price of exported to imported goods reflect changes in demand and supply conditions for the respective goods. But countries like Chile are small and do not have sufficient market power to affect prices.\footnote{Actually, the assumption of exogenous terms of trade might be valid for most countries. Supporting evidence comes from Mendoza (1995), who showed that, except for the U.S. and some fuel exporters, imports and exports do not Granger cause the terms of trade.} Hence, in the present study I do not attempt to identify the underlying shock processes driving the terms of trade, which may be “standard productivity or demand shocks at the global level or in large economies, but may also reflect rare events like the OPEC oil embargo.
3.1 Introduction

Figure 3.1: World Copper and Timber Prices and Their Monthly Changes

Notes: Price indices (2005=100) are measured in constant 2005 U.S. dollars; price changes are measured in percent.

the collapse of planned economies, or natural disasters” (Mendoza, 1995), or the rise of China over the last decade. Rather, I consider the terms of trade as a sufficient statistic for global demand and supply conditions faced by the small open economy. The third reason that Chile is a well suited case for my analysis is that Chile is a highly integrated open economy with a flexible exchange rate regime and good data availability.

My findings are threefold. First, constructing a monthly terms of trade data series from 1965-2010, I find that there is considerable evidence for time-varying terms of trade uncertainty in the Chilean data, with the variance of terms of trade shocks more than doubling in a short period of time. Second, I show that the ex-ante and the ex-post effects of increased terms of trade uncertainty in total can account for about one fifth of Chilean output fluctuations at business cycle frequencies. Third, I find that a two-standard deviation uncertainty shock, corresponding to a 54% increase
Chapter 3

in uncertainty about future terms of trade, leads to a 0.1% drop in output. This effect corresponds to more than 10% of the output effect of an average positive terms of trade level shock, which leads to a GDP increase of 0.9%. It is also three to four times larger than the effect found for political uncertainty in the U.S., another type of uncertainty that has gained a lot of attention recently (see Chapter 2 and Fernández-Villaverde et al., 2011a). Moreover, while the 54% increase in uncertainty is representative for the whole terms of trade sample ranging from 1965:1-2010:12, the fact that terms of trade uncertainty more than doubled during the recent commodities boom suggests that its actual contribution during this more recent period may have been substantial.

Regarding the transmission mechanism, the negative output response is mostly driven by firms in the non-tradable sector choosing higher markups over marginal costs to avoid being stuck with too low prices when large terms of trade shocks realize. In contrast, export good producers have less leverage in that the final price of their bundled good is given by the world market price. Increasing their price too much would result in the tradable good producer substituting import goods for domestic export goods. However, the negative effect on output is considerably dampened by two counteracting effects. First, the precautionary savings motive of the households leads them to increase their savings in foreign assets by increasing net exports. This buffer stock of foreign assets only slowly returns to its initial value as the increased uncertainty subsides. Second, the central bank reacts to the depressing output effects of increased terms of trade uncertainty and the corresponding deflationary response of consumer prices by lowering the domestic nominal interest rate. As a result, the nominal interest rate considerably falls with a peak response of about −1% in annualized terms.

The current chapter is related to two strands of the literature on terms of trade effects. The first strand considers the effects of level shocks to the terms of trade on the business cycle. The seminal work is Mendoza (1995), who analyzed business cycles through the lens of a multi-sector small open economy RBC model with terms

4To put this number into perspective, a GDP drop of 0.1% in the U.S. would correspond to a 0.5 percentage point increase in the Federal Funds Rate (Fernández-Villaverde et al., 2011a).
5It is also related to the literature on the business cycle effects of uncertainty reviewed in Chapter 2.
of trade shocks. He found that terms of trade shocks can account for \(45 - 60\%\) of GDP fluctuations. Kose (2002) extended Mendoza’s model to allow for more factor mobility between sectors and found that \(88\%\) of aggregate output fluctuations can be explained by world price shocks. In contrast, Lubik and Schorfheide (2007) and Lubik and Teo (2005) estimate New Keynesian open economy models featuring terms of trade shocks and find their business cycle contribution to be negligible.

The second strand of the literature from growth theory analyzes the effects of terms of trade uncertainty on economic growth using panel growth regressions.\(^6\) Apart from studying growth instead of business cycles, these studies differ from the present chapter in that they analyze cross-country variation in terms of trade uncertainty instead of time-variation within one country. Mendoza (1997) constructs an endogenous growth model of savings under uncertainty, where the mean and the variance of the terms of trade determine the savings rate and consumption growth. Depending on the parameter values, the model either generates positive or negative effects of terms of trade uncertainty on the consumption growth rate.\(^7\)

He then argues that the calibration generating negative effects on growth is the plausible one and uses the structure of his model to show that panel growth regressions indicate that countries with higher terms of trade risk have lower consumption growth. Bleaney and Greenaway (2001) show that the conclusions derived by Mendoza (1997) do not generalize to output growth and that the predicted relationship crucially depends on the degree of risk aversion. Studying a country sample in sub-Saharan Africa, which largely depends on commodity exports, they only find weak, marginally significant evidence for a negative effect of terms of trade uncertainty on output growth. Dehn (2000a,b) studies the effects of commodity price uncertainty - instead of the whole terms of trade - on economic growth. Using a similar distinction as in this chapter, separating “ex-post commodity” shocks from “ex-ante manifestation of commodity price uncertainty”, he finds that “ex-ante uncertainty” does not exert an influence on economic growth.

The chapter proceeds as follows. In Section 3.2, I create a monthly terms of trade

---

\(^6\)There is also a micro-econometric literature on the effects of exchange rate variability on investment in developing countries (see e.g. Servén, 2003).

\(^7\)The welfare effects of higher terms of trade uncertainty are always unambiguously negative and large, because uncertainty affects the trend growth rate. However, due to its restrictive assumptions, the model is not suited for business cycle analysis.
series for Chile and estimate a stochastic volatility process on this series to document that time-varying uncertainty at business cycle frequencies is an important stylized fact of the Chilean economy. I then integrate this terms of trade process into a New Keynesian multi-sector model calibrated to the Chilean economy in Section 3.3. Section 3.4 presents counterfactual experiments showing the importance of terms of trade risk for the Chilean business cycle. Finally, Section 3.5 concludes.

3.2 Terms of Trade Risk: Empirical Evidence

This section presents empirical evidence on the evolution of both the Chilean terms of trade and the associated terms of trade risk over time. For this purpose, I construct a monthly terms of trade series for the last four and a half decades. Fitting a stochastic volatility model to the cyclical component of the terms of trade, I show that they have been extremely volatile at business cycle frequencies and that this volatility has been changing considerably over time.

3.2.1 The Chilean Terms of Trade

To analyze the role of terms of trade risk shocks, I construct a monthly terms of trade series for Chile ranging from 1965:1 to 2010:12. The import price index is based on two categories: oil and other imports, where other imports are proxied by the world import unit values corrected for oil imports. The export price index is constructed from nine different world market price series: copper, metal prices, agricultural raw materials, food commodities, fish meal, beverages, timber, paper pulp, and industrial goods. These indices represent most of the Chilean exports,

\[ \text{import price index} = \text{oil} + \\text{other imports} \]

\[ \text{export price index} = \text{copper} + \text{metal prices} + \text{agricultural raw materials} + \text{food commodities} + \text{fish meal} + \text{beverages} + \text{timber} + \text{paper pulp} + \text{industrial goods} \]

\[ \text{construction of this series follows Bennett and Valdés (2001), who construct a chain-weighted Laspeyres-Index for Chilean import and export prices up to 1999. The importance of accounting for the changing composition of exports and imports can be seen in the varying export shares documented in Appendix 3.A. If trade shifts away from highly volatile commodities, using a fixed basket as in Dehn (2000a) would overstate the actual terms of trade volatility faced by Chile.} \]

\[ \text{unit values are real price measures obtained by dividing an index of current import or export values by a corresponding volume index, both constructed using balance of payments data. They are commonly used to measure terms of trade and are usually more reliable than national account-based price data (Mendoza, 1995).} \]
3.2 Terms of Trade Risk: Empirical Evidence

Figure 3.2: The Chilean Terms of Trade 1965-2010

Notes: Import and export price indices are in real terms (1977=100); the cyclical component is measured in percentage deviations from trend, extracted using an HP-filter with smoothing parameter $\lambda = 14,400$.

are disaggregated as far as data availability permits, and are deflated using the U.S. producer price index (PPI). Appendix 3.A details the construction of these indices. The top row of Figure 3.2 shows the evolution of the Chilean import and export price indices. Real import prices varied considerably over time, having their trough in 1970 at about 70 and, after the two clearly visible oil price shocks in 1973/4 and 1979, reach their peak in early 1980. Since then, import prices have still fluctuated considerably, but have come down from a persistently high level in 1985-1995 to a level of about 80 during the 2000s. In contrast, real export prices were high at the beginning and the end of the sample, reaching their peak at about the time of the first oil price crisis in early 1974, and were relatively low in the meantime. From 1965 to 1985 there seems to be a long-term downward trend in export prices that is masked by a sequence of short but pronounced spikes. In contrast, the period from 1980
until the mid-2000s was relatively tranquil at a low price level. At the end of the sample, starting in 2006, export prices picked up again and started fluctuating more, with the commodities boom and the subsequent financial crisis being clearly visible. It is important to note that the changes in import and export prices do not merely reflect oil price changes or changes in the U.S. PPI deflator. Dehn (2000a,b) estimates commodity price uncertainty indices using a GARCH model and finds that the “high incidence of shocks in particular years reflects instability in many commodities rather than oil shocks or deflator shocks” (Dehn, 2000b). While for example the oil price is directly responsible for the increase in import prices, the world-wide rise in export prices in 1973/74 visible in the Chilean terms of trade is driven by both adverse supply shocks and the strong demand from rapidly growing industrialized countries (Cashin, Liang, and McDermott, 2000).

The bottom row shows the resulting terms of trade series, defined as the relative price of exports over imports, and their HP-filtered cyclical component, where the low frequency movements visible in the left panel have been filtered out, because the analysis of this chapter is confined to business cycle frequencies. The cyclical component shows that during the 1960s and 1970s Chile faced a sequence of terms of trade shocks that lead to deviations from the long-term trend of more than ±40%. Fluctuations of the same magnitude again occurred from 2006 to 2010, with the Great Recession leading to the largest recorded drop in Chilean export prices during the considered sample. However, even during the more tranquil intermediate period, the terms of trade regularly fluctuated by about 15-20%. This substantial change in the volatility of the terms of trade during the sample period suggests that terms of trade risk may have potentially played a large role for Chile.

3.2.2 Terms of Trade Risk in the Data

To quantify the terms of trade risk present in the cyclical component, I fit an $AR(2)$-process\footnote{This was not a global phenomenon. For commodity exporters in general there is no consistent evidence for a lower degree of export price uncertainty compared to the previous period (Dehn, 2000b).} with first-order stochastic volatility $\sigma_t^{tot}$ (see e.g. Shephard, 2008):

\footnote{The sample partial autocorrelation function suggest the presence of two highly significant autoregressive roots. As lags 3, 4, and 12 are only marginally significant, I opt for a parsimonious}
3.2 Terms of Trade Risk: Empirical Evidence

\[
\log(tot_t) = \rho_1 \log(tot_{t-1}) + \rho_2 \log(tot_{t-2}) + e^{\sigma_{tot}^t} \nu_{tot}^t, \quad \nu_{tot}^t \sim \mathcal{N}(0, 1) \tag{3.1}
\]

\[
\sigma_{tot}^t = (1 - \rho_{\sigma_{tot}}) \bar{\sigma}_{tot} + \rho_{\sigma_{tot}} \sigma_{tot}^{t-1} + \xi_{tot} \varepsilon_{tot}^t, \quad \varepsilon_{tot}^t \sim \mathcal{N}(0, 1), \tag{3.2}
\]

where \(\bar{\sigma}_{tot}\) is the unconditional mean of \(\sigma_{tot}^t\). The shock to the volatility, \(\varepsilon_{tot}^t\), is assumed to be independent from the level shock, \(\nu_{tot}^t\).

Using a stochastic volatility process to model time-varying uncertainty instead of a GARCH process implies that uncertainty is exogenous in the sense that there is a separate stochastic volatility shock process, \(\varepsilon_{tot}^t\), that increases the variance of the error term independently of all other shocks. In contrast, in the GARCH framework, where the variance equation does not feature a separate shock term but only lagged level shocks, \(\nu_{tot}^{t-i}, i > 0\), uncertainty would be fully endogenous in the sense that a higher variance is always caused by past level shocks. Hence, to the degree that part of the time-varying uncertainty in the data is endogenous, using a stochastic volatility model will overstate the effect. The present chapter can thus be interpreted as a thought experiment: what is the effect of terms of trade risk if all the time-varying volatility is exogenous?

Estimation of (3.1)-(3.2) is performed using Bayesian likelihood-based techniques. Due to the non-linearity embedded in the stochastic volatility setup of the shocks, I use the Sequential Importance Resampling (SIR) particle filter to evaluate the likelihood.\(^{12}\) After obtaining the likelihood of the observables given the parameters, I use a Tailored Randomized Block Metropolis-Hastings (TuRB-MH) (Chib and Ramamurthy, 2010) to maximize the posterior likelihood. The prior distributions of the parameters, which are relatively weak, are given in Table 3.1.\(^{13}\) To back out the historical values of the

\(^{12}\)Technical details of the algorithms used in this subsection can be found in Appendix 2.B.

\(^{13}\)For the autoregressive parameters of the level equation, \(\rho_1\) and \(\rho_2\), I impose a uniform prior for each of the corresponding autoregressive roots over the stability region. The autoregressive parameter for the volatility equation \(\rho_{\sigma_{tot}}\) is assumed to be Beta-distributed with mean 0.9 and standard deviation 0.1. For the standard deviation of the terms of trade risk shock, \(\xi_{tot}\), a Gamma-distributed prior with mean 0.5 and standard deviation 0.1 was imposed. The unconditional mean of the log-volatility \(\bar{\sigma}_{tot}\) is assumed to be uniformly distributed with mean \(-7\) and standard deviation 5.3. The posterior distribution was computed from a 30,500 draw Monte Carlo Markov Chain using 3,000 particles, where the first 5,500 draws were discarded as burn-in draws. The acceptance rate was 38%.
latent state $\sigma_{t}^{tot}$ given the whole set of observations, the *backward-smoothing routine* (Godsill, Doucet, and West, 2004) was used. The smoothed values were computed at the mean of the posterior distribution using 10,000 particles.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior distribution</th>
<th>Posterior distribution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_1$</td>
<td>Uniform*</td>
<td>Mean 0.77 1.221 1.178 1.264</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>Uniform*</td>
<td>Mean 0.77 -0.348 -0.385 -0.314</td>
</tr>
<tr>
<td>$\rho^{tot}$</td>
<td>Beta*</td>
<td>Mean 1.00 0.929 0.913 0.944</td>
</tr>
<tr>
<td>$\xi_{tot}$</td>
<td>Gamma</td>
<td>Mean 0.267 0.239 0.297</td>
</tr>
<tr>
<td>$\bar{\sigma}_{tot}$</td>
<td>Uniform</td>
<td>Mean -7.00 -3.423 -3.588 -3.258</td>
</tr>
</tbody>
</table>

*Note:* Beta* indicates that the parameter divided by 0.999 follows a beta distribution. Uniform* indicates that the roots of the autoregressive process were estimated instead of the autoregressive coefficients and followed the specified prior distribution.

The estimation results are presented in Table 3.1. Detailed convergence diagnostics are shown in Appendix 3.B. In general, all parameters are precisely estimated as evidenced by the narrow percentiles. Considering that the data is monthly, the terms of trade show a moderate degree of persistence with the sum of the AR-coefficients being 0.873. Moreover, the estimated process shows considerable evidence of uncertainty with $\xi_{tot} = 0.27$. A one-standard deviation terms of trade risk shock increases the volatility of the terms of trade from 3.3 percent per month by 31 percent to 4.3 percent per month. With a point estimate of the autoregressive parameter of 0.93, such an increase in volatility has a half-life of about 9 months. Appendix 3.C shows the results of model misspecification tests applied to the SV model. The model fits the data well and cannot be rejected at conventional levels.\(^\text{14}\)

Figure 3.3 shows the historical evolution of the latent state $\sigma_{t}^{tot}$ derived from the particle smoother. Terms of trade risk considerably varied over the sample. Particularly the decade from 1965 to 1975 was plagued by high terms of trade volatility. At its peak in 1968, the average monthly terms of trade shock had a size of 17%. While this volatility subsequently decreased to a value below its unconditional

\(^\text{14}\)Details about the test can be found in Appendix 2.C. In contrast, a model without stochastic volatility, i.e. $\xi^{tot} = 0$ is clearly rejected by the data.
3.2 Terms of Trade Risk: Empirical Evidence

Figure 3.3: Historical Evolution of the Volatility of the Chilean Cyclical Terms of Trade Component

Notes: Gray shaded area: 95% confidence bands; red dotted line: unconditional mean.

mean in 1972, it again peaked at almost 15% about two years later, shortly before the first oil crisis. From 1979 until 2006 the volatility of the terms of trade shocks was mostly below its unconditional mean, but still experienced significant but smaller spikes during the second oil crisis in 1979, in 1987, and in 1997. Since 2006 terms of trade volatility increased again to an average of about 7.5%, but with temporary spikes of more than 10%.

To answer the question whether terms of trade risk was an important factor in Chilean business cycles, I integrate the estimated terms of trade process into a calibrated DSGE model. This allows me to conduct policy experiments and to

---

15 Figure 3.3 could also be interpreted as implying two distinct regime shifts around 1975 and 2006. In the absence of a clear interpretation of what these two distinct regimes could be and what might have caused the regime to shift, I treat the time series as originating from one stationary stochastic process. Tracing out the implications of potential regime shifts in e.g. a Markov-switching model is left for future research.
consider counterfactual economies that do not face terms of trade risk.

### 3.3 A Small Open Economy Model of the Chilean Economy

I use a small open economy framework with a non-tradable sector similar to Lubik (2003), Santacreu (2005), and Monacelli and Perotti (2010). An overview about the structure is given in Figure 3.4. The model economy is populated by a representative household, who owns the monopolistically competitive firms in the domestic intermediate tradable goods sector (indexed by superscript h) and in the intermediate non-tradable goods sector (indexed by superscript N). Moreover, the domestic economy features a final good firm producing a consumption/investment good out of the domestic intermediate non-tradable goods, a homogenous import good, and a homogenous, domestically produced bundled tradable good. The latter is produced by a tradable good bundler, which sells its output to both the domestic final good producer and to the world market. Foreign variables are denoted with an asterisk. In contrast to e.g. Galí and Monacelli (2005), who consider a semi-small open economy where the export firms have pricing power in the foreign market, I consider the case of a small open economy. The domestic economy sells a homogenous bundled tradable good to the world market and has no pricing power. Hence, it takes the terms of trade, i.e. the price of exports relative to its imports, as given. Modeling the terms of trade as an exogenous process is similar to Lubik and Schorfheide (2007). In their model, firms have pricing power in foreign markets but the terms of trade are nevertheless specified as being exogenous as a way to deal with model misspecification. In my framework, only a homogenous export good is exported to competitive world markets, meaning that the terms of trade are truly exogenous and not a modeling short-cut. That the assumption of exogenous terms of trade may not be too restrictive in the Chilean case is suggested by Del Negro and Schorfheide (2009), who compare a DSGE model to a DSGE-VAR and conclude that modeling the terms of trade as exogenous is not at odds with the data.\(^{16}\)

\(^{16}\)Medina and Naudon (2011) use the same exogeneity assumption in their assessment of the consequences of terms of trade shocks on labor market outcomes.
3.3 A Small Open Economy Model of the Chilean Economy

Figure 3.4: Structure of the Model Economy

3.3.1 Household Sector

The representative agent derives utility from consumption $C_t$ and leisure $1 - L_t$, where labor $L_t = L_t^h + L_t^N$ is supplied to the intermediate tradable goods sector and the intermediate non-tradable goods sector, respectively. I assume the preference specification of Jaimovich and Rebelo (2009) and allow for habits in consumption:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t \frac{(C_t - \phi_c C_{t-1} - \psi \frac{(L_t^h + L_t^N)^{1+\sigma_l}}{1+\sigma_l} S_t)^{1-\sigma_c}}{1-\sigma_c},$$

where $E_0$ is the mathematical expectations operator, $\phi_c \in [0, 1]$ indexes the degree of internal habit persistence, $\psi \geq 0$ scales the disutility of labor, $\sigma_c$ is related to the intertemporal elasticity of substitution and the household’s risk aversion, and $\sigma_l$ is a parameter governing the Frisch elasticity of labor supply. The strength of the wealth effect on the labor supply is parametrized by the parameter $\sigma_G$ in the law of motion for $S_t$

$$S_t = (C_t - \phi_c C_{t-1})^{\sigma_G} S_{t-1}^{1-\sigma_G}.$$  

(3.3)
which makes utility non-separable in leisure and consumption.\textsuperscript{17}

The household faces the following budget constraint in real terms:

\[ C_t + I^h_t + I^N_t + \frac{B_t}{P_{CPI,t}} + RER_t \frac{B^f_t}{P^*_t} + \frac{\Phi_D}{2} RER_t \left( \frac{B^f_t}{P^*_t} - \frac{B^f_t}{P^*_t} \right)^2 = \]  

\[ W^h_t L^h_t + W^N_t L^N_t + R^h_t K^h_{t-1} + R^N_t K^N_{t-1} + \frac{B_{t-1}}{P_{CPI,t-1}} R_{t-1} \frac{R^f_{t-1}}{P^*_t} + \Xi_t. \]  

It uses its income for consumption \( C_t \), investment in the tradable intermediate good and the non-tradable good sector, \( I^h_t \) and \( I^N_t \), and to invest in financial assets. Markets are incomplete and the household has access to domestic and foreign bonds, \( B_t \) and \( B^f_t \), denominated in domestic and foreign currency, respectively, which pay a gross nominal risk-free rate of \( R_t \) and \( R^*_t \). \( P_{CPI,t} \) and \( P^*_t \) are the foreign and domestic price levels and \( \Pi_{CPI,t} \) and \( \Pi^*_t \) the corresponding consumer price gross inflation rates. \( RER_t = \frac{S_{nomex}^t P^*_t}{P_t} \) denotes the real exchange rate with \( S_{nomex}^t \) being the nominal exchange rate. The last term on the left hand side represents the costs of holding a net foreign asset position, where \( \frac{B^f_t}{P_{CPI}} \) determines the foreign bond holdings in steady state and \( \Phi_D \) controls the size of the costs. Following Fernández-Villaverde et al. (2011b) these costs are assumed to be paid to some foreign international institution that handles the portfolio for the household.

The household receives income from supplying labor \( L^h_t \) at real wage \( W^h_t \) to the intermediate tradable good sector and \( L^N_t \) at real wage \( W^N_t \) to the non-tradable intermediate good sector. Moreover, it owns the firms in the economy and receives their profits \( \Xi_t \). The household is assumed to own the capital stock in both sectors, \( K^h_t \) and \( K^N_t \), which it rents out to firms at the rental rates \( R^h_t \) and \( R^N_t \). The law of motion for the capital stock is given by

\[ K^m_t = (1 - \delta) \left( K^m_{t-1} \right) + I^m_{t-3} - \frac{\phi_k}{2} \left( \frac{I^m_{t-3}}{K^m_{t-4}} - \delta \right)^2 K^m_{t-4}, \forall m = h, N, \]  

\textsuperscript{17}\textit{In the absence of habits, with } \sigma_G = 0 \textit{ one obtains the preference specification of Greenwood, Hercowitz, and Huffman (1988), where the wealth effect on the labor supply is completely shut off, while with } \sigma_G = 1 \textit{ the preference specification is identical to the King, Plosser, and Rebelo (1988)-preferences.}
where $0 < \delta < 1$ is the depreciation rate. Both sectors have distinct capital stocks with three periods time to build (Kydland and Prescott, 1982) and Hayashi (1982)-capital adjustment costs, where $\phi_k$ is a parameter governing the costs of adjustment.\footnote{Regarding the choice of Hayashi (1982)-capital adjustment costs, see the discussion in Chapter 2.} I introduce time to build, because the model will be calibrated to monthly frequency and I want to avoid capital freely relocating from one sector to the other within one month.\footnote{Casares (2007) is an earlier paper also using a combination of time to build and a different type of quadratic adjustment cost. Using only time to build is insufficient to match the typical hump shaped impulse responses after monetary policy shocks (Casares, 2006).}

### 3.3.2 Final Good Sector

A competitive final good firm produces a final good, $F_t$, from composite tradable goods, $X^T_t$, and composite non-tradable goods, $X^N_t$, using a CES production function with substitution elasticity $\eta$

$$ F_t = \left[ (1 - \omega_N)^{\frac{1}{\eta}} \left( X^T_t \right)^{\frac{\eta-1}{\eta}} + \omega_N \left( X^N_t \right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, $$  \hspace{1cm} \text{(3.6)}

where $\omega_N$ is the share of non-traded goods in the final good. From the zero profit condition follows the definition of the final goods price index

$$ P_{CPI,t} = \left[ (1 - \omega_N) \left( P^T_t \right)^{1-\eta} + \omega_N \left( P^N_t \right)^{1-\eta} \right]^{\frac{1}{1-\eta}}, $$

where $P^T_t$ and $P^N_t$ are the domestic currency prices of the tradable and the non-tradable good. The composite non-tradable good $X^N_t$ is bundled from a set of $i, i \in [0, 1]$ differentiated intermediate non-tradable goods $X^N_t (i)$ using a Dixit-Stiglitz aggregator $X^N_t = \left( \int_0^1 (X^N_t (i))^{\frac{\varepsilon-1}{\varepsilon}} \, di \right)^{\frac{\varepsilon}{\varepsilon-1}}$ with substitution elasticity $\varepsilon$. Expenditure minimization yields the optimal demand for variety $i$

$$ X^N_t (i) = \left( \frac{P^N_t (i)}{P^N_t} \right)^{-\varepsilon} X^N_t, $$  \hspace{1cm} \text{(3.7)}
where \( P_{Nt} = \left( \int_0^1 \left( P_{Nt}(i) \right)^{1-\epsilon} \, di \right)^{\frac{1}{1-\epsilon}} \) is the aggregate price index for the non-tradable good.

The tradable good \( X_{Tt} \) is a composite of the domestic bundled intermediate good \( X_{ht} \) and the import good \( X_{ft} \), produced using a CES production function with substitution elasticity \( \nu \)

\[
X_{Tt} = \left[ (1 - \omega) \frac{1}{\nu} \left( X_{ht}^{\frac{\nu-1}{\nu}} + \omega \left( X_{ft}^{\frac{\nu-1}{\nu}} \right) \right) \right]^{\frac{1}{1-\nu}}, \quad (3.8)
\]

where \( \omega \) is the share of the import good in the tradable good. Cost minimization yields the optimal demand functions

\[
X_{ht} = (1 - \omega) \left( \frac{P_{ht}}{P_{Tt}} \right)^{-\nu} X_{Tt} \quad (3.9)
\]

\[
X_{ft} = \omega \left( \frac{P_{ft}}{P_{Tt}} \right)^{-\nu} X_{Tt} \quad (3.10)
\]

where the composite tradable goods price index is given by

\[
P_{Tt} = \left[ (1 - \omega) \left( \frac{P_{ht}}{P_{Tt}} \right)^{1-\nu} + \omega \left( \frac{P_{ft}}{P_{Tt}} \right)^{1-\nu} \right]^{\frac{1}{1-\nu}} \quad (3.11)
\]

and \( P_{ht} \) and \( P_{ft} \) are the domestic currency prices of the domestic intermediate good and the import good, respectively. This implies that the relative price of tradable goods to import goods is a function of the exogenous terms of trade:

\[
\frac{P_{Tt}}{P_{ft}} = \left[ (1 - \omega) \left( \frac{P_{ht}}{P_{ft}} \right)^{1-\nu} + \omega \right]^{\frac{1}{1-\nu}} \quad . \quad (3.12)
\]

3.3.3 Non-tradable Intermediate Good Producers

There is a continuum of monopolistically competitive non-tradable good producers \( i, i \in [0, 1] \), that produce differentiated goods \( X_{Nt}^i \) from capital \( K_{Nt}^i \) and labor \( L_{Nt}^i \) using a Cobb-Douglas production function
3.3 A Small Open Economy Model of the Chilean Economy

\[ X^N_t(i) = \begin{cases} 
  z^N_t \left( K^N_{t-1}(i) \right)^{\alpha} \left( L^N_t(i) \right)^{1-\alpha} - \Psi^N, \text{ if } z^N_t \left( K^N_{t-1}(i) \right)^{\alpha} \left( L^N_t(i) \right)^{1-\alpha} > \Psi^N, \\
  0, \text{ otherwise} 
\end{cases} \]

where \( z^N_t \) is a sector-specific technology shock, \( \alpha \) is the capital share in the production function, and \( \Psi^N \) is a parameter determining the fixed costs of production. I assume staggered price setting à la Calvo (1983)/Yun (1996): each period, a fraction \( 1 - \theta_N \), \( \theta_N \in [0, 1] \), of firms is able to reset their price. Firms maximize the discounted sum of profits subject to the demand for their variety \( i \) from the final good producer, equation (3.7).

### 3.3.4 Tradable Good Bundler

There is a competitive tradable good bundler which bundles the domestic tradable good \( D^h_t \) from a continuum \( j \) of differentiated intermediate tradable goods \( X^h_t(j) \) using a Dixit-Stiglitz aggregator \( D^h_t = \left( \int_0^1 (X^h_t(j))^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \) with substitution elasticity \( \varepsilon \). Expenditure minimization yields the optimal demand for variety \( j \) of the domestic intermediate good

\[ X^h_t(j) = \left( \frac{P^h_t(j)}{P^h_t} \right)^{-\varepsilon} D^h_t, \quad (3.13) \]

where \( P^h_t = \left( \int_0^1 (P^h_t(j))^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}} \) is the producer price index in the domestic tradable good sector. The tradable good bundler subsequently sells the bundled good \( D^h_t \) to the domestic final good producer, which demands \( X^h_t \), and to the rest of the world, which demands \( X^h^{*t} \):

\[ D^h_t = X_t^{h*} + X^h_t. \]

### 3.3.5 Intermediate Tradable Good Producers

The differentiated intermediate tradable goods, \( X^h_t(j) \), are produced by a continuum of monopolistically competitive producers \( j, j \in [0, 1] \), from capital \( K^h_{t-1}(j) \) and
Chapter 3

labor $L^h_t (j)$ using a Cobb-Douglas production function

$$X^h_t (j) = \begin{cases} 
z^h_t \left( K^h_{t-1} (j) \right)^\alpha \left( L^h_t (j) \right)^{1-\alpha} - \Psi^h, & \text{if } z^h_t \left( K^h_{t-1} (j) \right)^\alpha \left( L^h_t (j) \right)^{1-\alpha} > \Psi^h, \\
0, & \text{otherwise}
\end{cases}$$

where $z^h_t$ is a sector-specific technology shock, $\alpha$ is the capital share in the production function, and $\Psi^T$ is a parameter determining the fixed costs of production. As in the non-tradable sector, each period a fraction $1 - \theta_h$, $\theta_h \in [0, 1]$, of firms may reset their price. Firms able to reset their price do so to maximize their discounted sum of profits subject to the demand function (3.13).

3.3.6 Market Clearing and Definitions

Market clearing in the final good market and the domestic bundled tradable good market implies

$$F_t = C_t + I^h_t + I^N_t, \quad (3.14)$$

and

$$X^h_t + X^{h^*}_t = \frac{z^h_t \left( K^h_{t-1} \right)^\alpha \left( L^h_t \right)^{1-\alpha} - \Psi^h}{O^h_t}, \quad (3.15)$$

while market clearing in the non-tradable sector requires

$$X^N_t = \frac{z^N_t \left( K^N_{t-1} \right)^\alpha \left( L^N_t \right)^{1-\alpha} - \Psi^N}{O^N_t}, \quad (3.16)$$

where the $O^m_t$ with $m = h, N$ measure the price dispersion introduced in the respective sectors by staggered price setting. These terms follow the laws of motion

$$O^m_t = \theta_m \Pi^\varepsilon_{m,t} O^m_{t-1} + (1 - \theta_m) \left( \Pi^{opt}_{m,t} \right)^{-\varepsilon} \forall m = h, N. \quad (3.17)$$

The law of motion for producer price inflation (PPI) in the respective sectors is
3.3 A Small Open Economy Model of the Chilean Economy

given by

\[ 1 = \theta_m \Pi_{m,t}^{\varepsilon-1} + (1 - \theta_m) \left( \Pi_{m,t}^{opt} \right)^{1-\varepsilon} \forall m = h, N. \tag{3.18} \]

The consumer price index, \( \Pi_{CPI,t} \), is linked to the tradable price inflation and the sectoral relative price between tradables and non-tradables via:

\[ \Pi_{CPI,t} = \left( \Pi_{T,t} \right)^{1-\eta} \left( 1 - \omega_N \right) + \omega_N \left( \frac{P_N^t}{P_T^t} \right)^{1-\eta}, \tag{3.19} \]

while tradable inflation, \( \Pi_{T,t} \), is linked to the domestic intermediate goods PPI and the terms of trade through

\[ \Pi_{T,t} = \left( \Pi_{h,t} \right)^{1-\nu} \frac{1 - \omega}{1 - \omega + \omega (tot_t)^{1-\nu}}. \tag{3.20} \]

The terms of trade, \( tot_t \), are defined as

\[ tot_t = \frac{P_h^t}{P_f^t}. \tag{3.21} \]

The balance of payments implies that the current account equals the change in international net asset position:

\[ RER_t \frac{B_t^f}{P_t^*} = X_t^h P_t^h - RER_t X_t^f + RER_t \frac{B_t^{f-1}}{P_t^{f-1}} \Pi_t^f - RER_t \frac{\phi_D}{2} \left( \frac{B_t^f}{P_t^*} - \frac{B_t^*}{P_t^*} \right)^2. \tag{3.22} \]

Domestic bonds, \( B_t \), are in zero net supply. I assume that the law of one price holds, i.e. \( P_t^f = S_t P_t^* \), and for simplicity that \( P_t^{f,t} = P_{CPI,t}^{opt} \) so that I don’t need to specify an exogenous law of motion for non-tradables prices in the rest of the world.

Domestic total output \( Y_t \) is given as

\[ Y_t = P_t^N X_t^N + P_t^h D_t^h, \tag{3.23} \]
while total domestic investment is defined as 

\[ I_t = I_t^N + I_t^h. \]  

(3.24)

Prices relative to the domestic CPI measured in local currency are denoted with small letters, i.e.

\[ p_t^N = \frac{P_t^N}{P_{CPI,t}}, p_t^T = \frac{P_t^T}{P_{CPI,t}}, p_t^h = \frac{P_t^h}{P_{CPI,t}}, p_t^f = \frac{P_t^f}{P_{CPI,t}}. \]

Finally, for the impulse response analysis in Section 3.4.2 it is convenient to define imports, \( I_{mt} \), and exports, \( E_{xt} \), in terms of CPI prices:

\[ I_{mt} = X_t^f p_t^f, \quad E_{xt} = X_t^h p_t^h. \]

### 3.3.7 Monetary Policy and Exogenous Processes

Monetary policy is conducted according to a Taylor rule that responds to inflation, output growth, and the real exchange rate

\[
\frac{R_t}{R} = \left( \frac{R_{t-1}}{R} \right)^{\rho_R} \left( \frac{\Pi_{CPI,t}}{\Pi_{CPI}} \right)^{\phi_{RR}} \left( \frac{Y_t}{Y_{t-1}} \right)^{\phi_{RY}} \left( \frac{RER_t}{RER} \right)^{\phi_{RER}} \left( \frac{1}{1 - \rho_R} \right). \]  

(3.25)

This specification is similar to Lubik and Schorfheide (2007) in that the government reacts to changes in output growth rather than potential output, which typically cannot be observed. It differs from their specification in that the central bank is assumed to react to movements in the real exchange rate instead of the nominal exchange rate to allow for the central bank leaning against deviations of the real exchange rate from its long-run equilibrium level. This is consistent with Chilean policy for at least most of the 1990s (Frankel and Rapetti, 2010; Ilzetzki, Reinhart, and Rogoff, 2008).\(^{20}\) The domestic technology processes and the foreign variables are assumed to follow exogenous AR(1)-processes

\(^{20}\)Medina and Soto (2007) also include this term in their specification of the Chilean central bank’s monetary policy reaction function. De Gregorio and Labbé (2011) analyze such a rule as it particularly fits the behavior of the Chilean central bank during the 1990s.
3.3 A Small Open Economy Model of the Chilean Economy

\[ \log \left( z_t^h \right) = \rho_2 \log \left( z_{t-1}^h \right) + \varepsilon^h_{z,t}, \varepsilon^h_{z,t} \sim N \left( 0, \sigma^2_z \right) \]  

(3.26)

\[ \log \left( z_t^N \right) = \rho_2 \log \left( z_{t-1}^N \right) + \varepsilon^N_{z,t}, \varepsilon^N_{z,t} \sim N \left( 0, \sigma^2_z \right) \]  

(3.27)

\[ \log \left( \frac{R_t^*}{R^*} \right) = \rho_R \log \left( \frac{R_{t-1}^*}{R^*} \right) + \varepsilon^R, \varepsilon^N_{z,t} \sim N \left( 0, \sigma^2_R \right) \]  

(3.28)

\[ \log \left( \frac{\Pi_t^*}{\Pi^*} \right) = \rho_{\Pi} \log \left( \frac{\Pi_{t-1}^*}{\Pi^*} \right) + \varepsilon^\Pi, \varepsilon^N_{z,t} \sim N \left( 0, \sigma^2_\Pi \right) \]  

(3.29)

Finally, the terms of trade are assumed to follow an exogenous stochastic volatility process as discussed in Section 3.2. The equations are repeated for convenience:

\[ \log \left( \text{tot}_t \right) = \rho_1 \log \left( \text{tot}_{t-1} \right) + \rho_2 \log \left( \text{tot}_{t-2} \right) + \varepsilon^{\text{tot}}_t \nu^{\text{tot}}_t, \nu^{\text{tot}}_t \sim N \left( 0, 1 \right) \]

\[ \sigma^{\text{tot}}_t = \left( 1 - \rho^{\text{tot}}_\sigma \right) \tilde{\sigma}^{\text{tot}} + \rho^{\text{tot}}_\sigma \sigma^{\text{tot}}_{t-1} + \xi^{\text{tot}}_t \varepsilon^{\text{tot}}_t, \varepsilon^{\text{tot}}_t \sim N \left( 0, 1 \right) \].

3.3.8 Model Calibration

The model is calibrated to Chilean data from 1996:Q2-2011:Q2, because this is the longest sample for which a consistent National Accounts series of nominal private consumption is available. Unfortunately, in September 1999 the official IMF exchange rate regime classification changed from managed floating to independently floating.

Regarding Chile’s de facto exchange rate regime, Ilzetzki, Reinhart, and Rogoff (2008), classify Chile’s regime as a “De facto crawling band that is narrower than or equal to +/-5%” (category 10) from 1992:2 to 2007:12 with a short intermediate period of “Pre-announced crawling band that is wider than or equal to +/-2% “ (category 9) from 1998:9-1999:9 and “Managed floating” (category 12) from 1999:9-2001:12.\(^{21}\)

\(^{21}\)In particular, from 1992:1 to 1998:6, there was a PPP rule with a de facto band of ±5% to the dollar. It was replaced by a ±8% preannounced crawling band in December 1998 until a unified exchange market with a de facto band of ±5% around the U.S. dollar was implemented in 1999:9 (Ilzetzki, Reinhart, and Rogoff, 2008). In the latter period, exchange rate interventions seem to have been mostly confined to short periods of turbulence in financial markets: in late 2001 during the Argentinean convertibility crisis, in late 2002 near the presidential elections in Brazil, during the 2008 financial crisis, and in 2011 to replenish foreign reserves relative to GDP, which had decreased due to the increase in the denominator (De Gregorio, 2011).
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Target/Source</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady state inflation rate</td>
<td>Π_{CPI}</td>
<td>1</td>
<td>Steady state inflation</td>
</tr>
<tr>
<td>Foreign inflation</td>
<td>Π^*_{CPI}</td>
<td>1</td>
<td>Steady state inflation</td>
</tr>
<tr>
<td>Discount factor</td>
<td>β</td>
<td>0.995</td>
<td>Annual risk free rate</td>
</tr>
<tr>
<td>Depreciation rate</td>
<td>δ</td>
<td>0.0052</td>
<td>I/Y</td>
</tr>
<tr>
<td>Curvature of labor</td>
<td>σ_l</td>
<td>1.6</td>
<td>Neumeyer/Perri (2005)</td>
</tr>
<tr>
<td>Jaimovich/Rebelo preferences</td>
<td>σ_G</td>
<td>0.001</td>
<td>Jaimovich/Rebelo (2009)</td>
</tr>
<tr>
<td>Risk aversion</td>
<td>σ_c</td>
<td>5</td>
<td>Neumeyer/Perri (2005)</td>
</tr>
<tr>
<td>Consumption habits</td>
<td>φ_c</td>
<td>0.7</td>
<td>Standard value</td>
</tr>
<tr>
<td>Labor disutility</td>
<td>ψ</td>
<td>14.2757</td>
<td>Hours worked steady state</td>
</tr>
<tr>
<td>Fixed costs tradable sector</td>
<td>Ψ^h</td>
<td>0.1136</td>
<td>Steady state profits</td>
</tr>
<tr>
<td>Fixed costs non-tradable sector</td>
<td>Ψ^N</td>
<td>0.0742</td>
<td>Steady state profits</td>
</tr>
<tr>
<td>Price elasticity</td>
<td>ε</td>
<td>11</td>
<td>Markup</td>
</tr>
<tr>
<td>Capital share</td>
<td>α</td>
<td>0.33</td>
<td>Labor share</td>
</tr>
<tr>
<td>Price rigidities tradables</td>
<td>θ_h</td>
<td>6/9</td>
<td>Price duration</td>
</tr>
<tr>
<td>Price rigidities non-tradables</td>
<td>θ_N</td>
<td>8/9</td>
<td>Price duration</td>
</tr>
<tr>
<td>Foreign Debt</td>
<td>B^<em>/P^</em>_{CPI}</td>
<td>-9.02</td>
<td>B^*/Y^{annual}</td>
</tr>
<tr>
<td>Capital adjustment costs</td>
<td>φ_k</td>
<td>800</td>
<td>Relative volatility I/Y</td>
</tr>
<tr>
<td>Debt adjustment costs</td>
<td>φ_D</td>
<td>1</td>
<td>Relative volatility Im/Y</td>
</tr>
<tr>
<td>Trade price elasticity</td>
<td>η</td>
<td>1.1</td>
<td>Relative volatility Ex/Im</td>
</tr>
<tr>
<td>Price elasticity non-tradables</td>
<td>ν</td>
<td>0.44</td>
<td>Stockman/Tesar (1995)</td>
</tr>
<tr>
<td>Weight domestic goods</td>
<td>ω</td>
<td>0.3</td>
<td>Total trade share</td>
</tr>
<tr>
<td>Weight tradable goods</td>
<td>ω_N</td>
<td>0.4</td>
<td>Non-tradables in final good</td>
</tr>
<tr>
<td>Taylor rule inflation</td>
<td>φ_{RR}</td>
<td>2.85</td>
<td>Del Negro/Schorfheide (2009)</td>
</tr>
<tr>
<td>Taylor rule output growth</td>
<td>φ_{B_y}</td>
<td>0.16</td>
<td>Del Negro/Schorfheide (2009)</td>
</tr>
<tr>
<td>Taylor rule real exchange rate</td>
<td>φ_{RRER}</td>
<td>0.3</td>
<td>Covariance Π_{CPLA,Y}</td>
</tr>
<tr>
<td>Taylor rule interest smoothing</td>
<td>ρ_R</td>
<td>0.8</td>
<td>Del Negro/Schorfheide (2009)</td>
</tr>
<tr>
<td>Autocorr. intern. interest rate</td>
<td>ρ_{R*}</td>
<td>0.99</td>
<td>Sample autocorrelation</td>
</tr>
<tr>
<td>St.dev. intern. interest rate</td>
<td>σ_{R*}</td>
<td>1.83e-004</td>
<td>Sample standard deviation</td>
</tr>
<tr>
<td>Autocorr. foreign inflation</td>
<td>ρ_{W}</td>
<td>0.4136</td>
<td>Sample autocorrelation</td>
</tr>
<tr>
<td>St.dev. foreign inflation</td>
<td>σ_{W}</td>
<td>0.0030</td>
<td>Sample average</td>
</tr>
<tr>
<td>Autocorr. technology shock</td>
<td>ρ_z</td>
<td>0.95</td>
<td>Fernández-Villaverde et al. (2011b)</td>
</tr>
<tr>
<td>St.dev. technology shock</td>
<td>σ_z</td>
<td>0.02</td>
<td>Output volatility</td>
</tr>
</tbody>
</table>
Despite these slight changes in the de facto exchange rate regime, Chile’s exchange rate regime nevertheless can be broadly categorized as a flexible one for the whole sample period.\textsuperscript{22}

I calibrate the model to a zero inflation steady state and set both domestic gross inflation, $\Pi_{CPI}$, and foreign gross inflation, $\Pi^*_{CPI}$, to 1 in steady state. The discount factor is set to generate a 3% risk free rate in steady state as in Medina and Soto (2007).\textsuperscript{23}

The depreciation rate, $\delta = 0.0052$, is chosen to match the sample mean investment to output-ratio of 22.3%. Following Neumeyer and Perri (2005), the curvature parameter governing the Frisch elasticity of labor supply is set to 1.6. The wealth elasticity of labor supply is chosen to be 0.001 (Jaimovich and Rebelo, 2009), while the risk aversion parameter is assumed to be 5 as in Neumeyer and Perri (2005). The habit persistence parameter, $\phi_c = 0.7$, is set to an intermediate value taken from the literature. It corresponds to the average estimates typically found in business cycle studies for a variety of countries like e.g. Sweden (Adolfson et al., 2007, 2008) and the U.S. (Smets and Wouters, 2007). The labor disutility parameter, $\psi$, pins the ratio of hours worked to total hours to one third. Fixed costs in both sectors, $\Psi^m$, are chosen to set profits in steady state to 0 to rule out entry/exit (see Christiano, Eichenbaum, and Evans, 2005). The price elasticity parameter, $\varepsilon = 11$, corresponds to a steady state markup of 10%. The capital share parameter, $\alpha = 0.33$, targets a labor share of 2/3. The Calvo parameter in the non-tradable sector, $\theta_N = 8/9$, targets a price duration of 3 quarters. For the tradable sector, which is more exposed to the terms of trade shocks and would thus adjust prices more frequently in a model of state-dependent pricing decisions, I assume a price duration of only 2 quarters,

\textsuperscript{22}For a similar categorization of Chile, see Ilzetzki, Mendoza, and Végh (2010). I opt to not start the sample in 1999Q3 because this would leave only 48 quarters of data and increase the risk of the HP-filter introducing significant artifacts at the beginning and the end of the data set.

\textsuperscript{23}For my sample, constructing the international interest rate as in Fernández-Villaverde et al. (2011b) and Neumeyer and Perri (2005) as the average real interest rate on three month T-bills plus the sovereign spread for Chile from the global Emerging Market Bond Index results in a yearly international interest rate of 1.57%. This low number largely reflects the extended period of negative real interest rates in the U.S.. Calibrating the model to this interest rate would imply an unrealistically high discount factor, an annual depreciation rate of less than 4%, and an unrealistic degree of interest sensitivity. In contrast, using an annual international interest rate of 3% results in a depreciation rate of 6.24%, which is more consistent with other studies of Chile that assume 6% (see e.g. Medina and Naudon, 2011; Medina and Soto, 2007).
corresponding to \( \theta_h = 6/9 \). I set the steady state level of real foreign bond holdings to correspond to an average external debt level of 40% of GDP, the sample average from 1999 to 2009 (Banco Central de Chile, 2010). The capital adjustment cost parameter, \( \phi_k \), and the portfolio adjustment cost parameter, \( \phi_D \), are chosen to target the investment volatility relative to output and the volatility of imports/exports relative to GDP. The portfolio adjustment cost parameter, \( \phi_D \), can be interpreted as a shortcut for a financial friction faced by the domestic economy as in García-Cicco, Pancrazi, and Uribe (2010). There is a large debate about the correct value of the trade price elasticity, \( \eta \), with estimates ranging from 0.9 (Heathcote and Perri, 2002) up to 2 (Backus, Kehoe, and Kydland, 1994). Hence, I choose the value to match the relative volatility of imports to exports. Following Stockman and Tesar (1995), the non-traded goods price elasticity is \( \nu = 0.44 \), which was their cross-sectional average for 30 countries. This implies that traded and non-traded goods are complements. The weight of the bundled domestic tradable goods in the composite tradable good is \( \omega = 0.3 \) to generate a steady state ratio of total trade to output of 60%. I choose the weight of non-tradable goods in the final good to be \( \omega_N = 0.4 \), the middle of the range found for the share of non-traded goods in final consumption in Stockman and Tesar (1995).

Regarding the conduct of monetary policy, the Taylor rule parameters for interest

---

24 Micro estimates typically tend to be smaller. For the Chilean economy, the average price duration in micro-data is estimated to be around one quarter (Medina, Rappoport, and Soto, 2007). However, Medina, Rappoport, and Soto (2007) acknowledge that their data contains sales, which may be responsible for the discrepancy between micro and macro estimates (Nakamura and Steinsson, 2008). Note that the period for which the model is calibrated is a relatively low inflation environment, with inflation expectations ranging around the Chilean target rate of 3% (Desormeaux, García, and Soto, 2010). In this environment, the assumption of non-state-dependent pricing may be a good approximation to state-dependent pricing decisions (Burstein, 2006).

25 This sample is restricted by data availability.

26 The resulting persistence parameters, which appear quite large even after considering that the model is calibrated at monthly frequency, reflect the general problem of matching the volatility in the data given the large variance of the underlying shocks. Medina and Naudon (2011) study the effect of terms of trade shocks on the Chilean labor market. Their model, using lower adjustment costs, leads to large fluctuations after a non-mining terms of trade shock that are off by a factor of four.

27 Following Stockman and Tesar (1995), most studies use a larger value of 0.5. But as noted in their paper, there is evidence for an increase in services trade relative to the service share in output. For example, finance and insurance were counted as non-tradable goods but have since arguably experienced a large internationalization.
smoothing, $\rho_R = 0.8$, for inflation feedback, $\phi_{R\pi} = 2.85$, and output feedback, $\phi_{Ry} = 0.16$, are taken from the DSGE-VAR estimates of Del Negro and Schorfheide (2009). However, their sample ran from 1999:Q1 to 2006:Q4, while my sample also includes the earlier crawling-band exchange rate period and the two most recent central bank interventions in the foreign exchange market. Hence, I choose the reaction parameter to deviations from the long-run equilibrium real exchange rate, $\phi_{RRER}$, to match the covariance of inflation with output. The autocorrelation, $\rho_{R^*}$, and the standard deviation, $\sigma_{R^*}$, of the international nominal interest rate are computed as the sample standard deviation and autocorrelation of the 3 month T-Bill rate plus the EMBI sovereign spread for Chile. Similarly, the autoregressive coefficients for foreign inflation, $\rho_{\Pi^*}$, and its volatility, $\sigma_{\Pi^*}$, are chosen to match the autocorrelation and standard deviation of U.S. monthly consumer price inflation. The autoregressive coefficient, $\rho_z$, of the technology processes is set to 0.95 as in Fernández-Villaverde et al. (2011b) and its standard deviation is set to match output volatility. Finally, I assume that $P_{CPI,0} = P_{CPI,0}^*$, i.e. that the steady state price levels in the beginning of time were the same, which together with the normalization of the terms of trade implies $RER = 1$ in steady state.

3.4 The Aggregate Effects of Terms of Trade Uncertainty

Due to the inherent nonlinearity embedded in the stochastic volatility process, the terms of trade volatility shocks only enter the model’s policy functions independently from the level shocks at third order. Hence, the model is solved using a third order perturbation to the policy function. Given the solution to the model, I simulate the model in Section 3.4.1 in order to compare empirical and model moments and

---

28This concept of the international interest rate available to the small open economy is the same as in Fernández-Villaverde et al. (2011b), except that I consider nominal interest rates. Data availability for the EMBI spread limits the sample for this series from 1999:5 to 2011:9.

29For details see Appendix 2.B. Using a non-linear solution to the model of the previous section is also compatible with the finding of De Gregorio and Labbé (2011) that the relationship between copper price volatility and GDP growth volatility fluctuated over the past 30 years. Due to the nonlinearity of the model, all exercises in this section are conducted starting at the mean of the ergodic distribution, which is approximated by the mean over 2000 simulation periods.
to quantify the importance of time-varying terms of trade volatility on the Chilean business cycles. In Section 3.4.2 I then conduct an impulse response function analysis and conduct policy experiments to trace out the role of the ex-ante terms of trade uncertainty effect and its transmission in the economy.

3.4.1 The Effects of Time-Varying Volatility

Table 3.3 compares the empirical data moments for the Chilean economy over the sample from 1996:Q2 to 2011:Q2 with the moments generated by the model under its baseline parametrization. Model moments are computed from the quarterly aggregates of the model’s monthly variables over a time series 10 times the length of the empirical data, i.e. 620 quarters. Following the convention in the terms of trade literature (see e.g. Mendoza, 1995), all components of GDP are measured in import prices. The model fits the data quite well. Consumption is a bit too volatile, but is still the least volatile component of output, although it was not explicitly targeted. In contrast, net exports are not volatile enough. The reason is that imports in the model are about as pro-cyclical as exports, leading to a-cyclical net exports. In contrast, Chilean net exports are positively correlated with output measured in import prices. This mirrors the too pro-cyclical behavior of imports. The model actually shares this weakness with the models of Mendoza (1995) and Kose (2002), which also generate consistently too low correlations between the trade balance and GDP measured in import prices.

The volatility of CPI inflation, which was also not explicitly targeted, is at 1.66% a bit above the value in the data, suggesting that higher nominal rigidities like sticky wages might be required to better match inflation volatility. However, the autocorrelation of inflation is almost exactly on target. The covariance of the other variables with output is better matched, although the model generates somewhat

---

30 This convention together with the terms of trade fluctuations explains for example the high quarterly volatility of output. For comparison, Mendoza (1995) reported an annual cyclical volatility of output measured in import prices of 24.18% for Chile from 1965-1990 and of 9.61% for the U.S. during the same sample.

31 A well-known stylized fact of international business cycles is that net exports, measured as nominal exports minus nominal imports over nominal GDP, tend to vary counter-cyclically with real GDP for most countries and time periods (see e.g. Backus and Kehoe, 1992; Neumeyer and Perri, 2005). In Chile this correlation is a-cyclical with ~0.07.
3.4 The Aggregate Effects of Terms of Trade Uncertainty

too much co-movement. In contrast, the persistence of the cyclical component of the individual variables, except for net exports, is a bit too low. Finally, the simulated model generates a net export share of 4.2% compared to 4.7% in the data (not shown in the table).

Table 3.3: Model and Empirical Moments: Benchmark Calibration

<table>
<thead>
<tr>
<th></th>
<th>Model</th>
<th>Data</th>
<th>Model</th>
<th>Data</th>
<th>Model</th>
<th>Data</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\sigma(x_t))</td>
<td>(\sigma_{y_t}/\sigma_{y_t})</td>
<td>(\rho(x_t, y_t))</td>
<td>(\rho(x_t, x_{t-1}))</td>
<td>(\sigma(x_t))</td>
<td>(\sigma_{y_t}/\sigma_{y_t})</td>
<td>(\rho(x_t, y_t))</td>
<td>(\rho(x_t, x_{t-1}))</td>
</tr>
<tr>
<td>(Y)</td>
<td>7.80%</td>
<td>7.42%</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.53</td>
<td>0.77</td>
</tr>
<tr>
<td>(C)</td>
<td>7.13%</td>
<td>5.59%</td>
<td>0.91</td>
<td>0.75</td>
<td>0.99</td>
<td>0.82</td>
<td>0.56</td>
<td>0.69</td>
</tr>
<tr>
<td>(I)</td>
<td>9.45%</td>
<td>8.22%</td>
<td>1.21</td>
<td>1.11</td>
<td>0.96</td>
<td>0.57</td>
<td>0.58</td>
<td>0.80</td>
</tr>
<tr>
<td>(Ex)</td>
<td>10.04%</td>
<td>9.34%</td>
<td>1.29</td>
<td>1.26</td>
<td>0.90</td>
<td>0.86</td>
<td>0.35</td>
<td>0.82</td>
</tr>
<tr>
<td>(Im)</td>
<td>7.57%</td>
<td>7.38%</td>
<td>0.97</td>
<td>0.99</td>
<td>0.97</td>
<td>0.49</td>
<td>0.59</td>
<td>0.77</td>
</tr>
<tr>
<td>(NX)</td>
<td>2.43%</td>
<td>2.99%</td>
<td>0.31</td>
<td>0.40</td>
<td>-0.01</td>
<td>0.60</td>
<td>0.99</td>
<td>0.72</td>
</tr>
<tr>
<td>(\Pi_{CPI})</td>
<td>1.66%</td>
<td>1.30%</td>
<td>0.21</td>
<td>0.18</td>
<td>0.22</td>
<td>0.17</td>
<td>0.04</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Notes: Time Series \(x_t\) are output (\(Y_t\)), consumption (\(C_t\)), investment (\(I_t\)), exports (\(X_f^t\)), imports (\(X_i^t\)), and net exports (\(NX_t\)), all measured in import prices, and CPI inflation \(\Pi_{CPI}\). All variables are logged (except for \(NX\)) and detrended using a HP-filter with smoothing parameter \(\lambda = 1600\).

Given the calibrated model at hand, one can now quantify the contribution of terms of trade risk to the business cycle. Table 3.4 shows the moments from a counterfactual model economy where the stochastic volatility in the terms of trade has been shut off, i.e. \(\xi_{tot} = 0\). In this case of no ex-ante terms of trade uncertainty and no ex-post realizations of larger shocks, output volatility drops by two percentage points, mostly driven by a drop in the investment volatility. In contrast, export volatility decreases relatively less compared to the other GDP components, with the result that the relative volatility of net exports increases. This suggests that agents in an economy facing large terms of trade risk use exports to insulate themselves against these movements.

Note: To some degree, this reflects the general problem of DSGE-models mostly driven by a single exogenous shock process to generate the correct idiosyncratic movement of variables beyond their co-movement with output (Christiano and Eichenbaum, 1992; Nakamura, 2009).
Table 3.4: Model and Empirical Moments: No TOT Risk

<table>
<thead>
<tr>
<th></th>
<th>Model Data</th>
<th>Model Data</th>
<th>Model Data</th>
<th>Model Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(x_t)$</td>
<td>5.96% 7.42%</td>
<td>1.00 1.00</td>
<td>1.00 1.00</td>
<td>0.52 0.77</td>
</tr>
<tr>
<td>$\sigma_{x_t}/\sigma_{y_t}$</td>
<td>0.91 0.75</td>
<td>0.94 0.67</td>
<td>0.52 0.57</td>
<td>0.56 0.69</td>
</tr>
<tr>
<td>$\rho(x_t, y_t)$</td>
<td>1.22 1.11</td>
<td>0.86 0.86</td>
<td>0.32 0.82</td>
<td></td>
</tr>
<tr>
<td>$\rho(x_t, x_{t-1})$</td>
<td>0.97 0.99</td>
<td>0.96 0.49</td>
<td>0.58 0.77</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Time Series $x_t$ are output ($Y_t$), consumption ($C_t$), investment ($I_t$), exports ($X^f_t$), imports ($X^h_t$), and net exports ($NX_t$), all measured in import prices, and CPI inflation $\Pi_{CPI}$. All variables are logged (except for NX) and detrended using a HP-filter with smoothing parameter $\lambda = 1600$.

3.4.2 Impulse Response Analysis

Figure 3.5 depicts the impulse response functions after a two standard deviation terms of trade uncertainty shock.\textsuperscript{33} As can be seen in the bottom row, the standard deviation of the terms of trade shock increases by about 54%, while the level of the terms of trade stays constant. Hence, the response of the other variables is solely due to the ex-ante effect of a wider shock distribution from which future shocks are drawn. The top row shows that such an increase in terms of trade uncertainty leads to a 0.1% drop in output. Initially, this drop is mostly driven by an immediate drop in investment, which then recovers over the following year, followed by a period of overshooting. This behavior of investment is similar to the one reported by Bloom (2009) for the case of an uncertainty shock to idiosyncratic productivity in the United States. The investment response pattern in the two sectors (not shown here) is about the same as the aggregate response, given the symmetric technology, the complementarity between traded and non-traded goods and the constant relative price between export and import goods. At the same time, consumption also drops and reaches its trough at more than $-0.1\%$ after about 8 months. The drop in output affects both tradable and non-tradable goods due to the complementarity between

\textsuperscript{33}Appendix 2.B describes the construction of the impulse response functions.
3.4 The Aggregate Effects of Terms of Trade Uncertainty

both.

Looking at the domestic use of tradable and non-tradable goods, \( X^T_t \) and \( X^N_t \), we see an expenditure switching effect with domestic use of the non-tradable good \( X^N_t \) falling more than the use of the tradable good \( X^T_t \). This expenditure shift reflects an increase in the relative price of non-tradables \( p^N_t \), while the price of tradable \( p^T_t \) drops. The reason for non-tradable goods becoming more expensive compared to tradable goods is that the domestic producers of non-tradable goods have more leverage in setting higher markups to self-insure against future demand changes. For them, it is better to increase their markup when facing large uncertainty about future terms-of-trade, because a too low price set today would be associated with potentially selling a higher amount of goods at a loss tomorrow. In contrast, setting too high a price only results in selling fewer goods at a price still well above marginal costs. This is also the mechanism responsible for the negative effect of fiscal uncertainty in Fernández-Villaverde et al. (2011b).\(^{34}\) In contrast, export good producers have less leverage in that the final price of their bundled good is given by the world market price. Increasing their price too much would result in the tradable good producer substituting import goods for domestic export goods.

Moreover, agents on impact decrease domestic absorption through importing less of the foreign import good \( X^f_t \) and exporting more of the domestic tradable good \( X^h_t \). They do this to immediately build up a buffer stock of foreign bonds by increasing net exports. This buffer stock of about 0.2% of GDP only slowly returns to its initial value as the increased uncertainty subsides. Hence, the precautionary motive to self-insure against increased uncertainty dampens its negative output effects by necessitating an increase in domestic production of the export good. It also assures that production of the export good cannot fall too much as otherwise the buffer stock of foreign capital would be drawn down too fast. Labor in both sectors (omitted for brevity) reflects this differential change in production, with \( l^N \) falling by \(-0.15\%\) at its trough after 7 months and \( l^h \) initially increasing by 0.16% to produce the increased amount of exports.

At the same time the terms of trade uncertainty shock acts deflationary, with

---

\(^{34}\)It is also related to the work of Basu and Bundick (2011), who show that time-varying markups are key to generating negative responses to uncertainty shocks in closed economy DSGE-models with convex adjustment costs.
Figure 3.5: Impulse Responses to a Two-standard Deviation Terms of Trade Uncertainty Shock

Notes: Level shocks are held constant. All responses are in percent, except for \( \Pi_{CPI} \) and \( R \), which are in percentage points.

monthly inflation decreasing by 0.04% or about 0.5% percentage points on an annualized basis. The central bank reacts to these depressing output effects of increased terms of trade uncertainty and the corresponding deflationary response of consumer prices by lowering the domestic nominal interest rate, which considerably falls. The peak response after 8 months is almost \(-0.08\%\) or about one percentage point in annualized terms.

Hence, the central bank mitigates the negative effects of terms of trade uncertainty by expansionary monetary policy, in whose absence the output drop would be much larger. This can be seen in Figure 3.6, where the central bank’s response to output
3.4 The Aggregate Effects of Terms of Trade Uncertainty

growth and real exchange rate deviations from the long run-equilibrium has been shut off, i.e. $\phi_{Ry} = \phi_{RRER} = 0$. In general, the shape of the impulse responses is the same as in the baseline case. However, the magnitude of the responses is considerably larger. Output drops by almost 0.3\%, driven by significant decreases in consumption and investment. The larger response of the aggregate variables is driven by large changes in relative prices. In contrast, the nominal interest rate and inflation responses have about the same magnitude as before. This reflects the fact that the Taylor rule describes the off-equilibrium response of the central bank. Rational economic agents anticipate this behavior and choose their actions accordingly. As a result, in equilibrium interest rates and inflation may be observed to be (almost) the same as under an alternative policy rule.\footnote{This result underlies Cochrane (2011a), who criticizes single-equation estimation and identification of Taylor rules based on this potential observational equivalence in equilibrium. As discussed in the online appendix to his paper, one way to circumvent the identification issue is to look at the equilibrium response of variables other than inflation and the nominal interest rate, because their behavior may be different under alternative policy rules. This phenomenon is clearly visible in Figure 3.6.}

The dampening effect of monetary policy observed in the baseline case in Figure 3.5 is an example of the result shown in Bachmann and Bayer (2011) that general equilibrium responses of wages and interest rates often considerably attenuate the output effects of uncertainty shocks. This is the same effect that was also shown to be at work in Chapter 2, where is was responsible for the small effects of policy risk. An output response of 0.1\% for about 1 year might seem small. However, to put this number into perspective, it is three to four times the effect of a joint policy risk shock in the United States (see Chapter 2) and comparable to a 50 basis points increase in the Federal Funds Rate or twice the effect of Quantitative Easing (Fernández-Villaverde et al., 2011a). Moreover, Figure 3.3 in Section 3.2 suggests that there may be periods in which uncertainty about terms of trade can be a lot more important. The simulated increase in uncertainty by 54\% is rather representative for the time in the middle of the sample, where volatility fluctuations were relatively subdued. In contrast, at the beginning and the end of the sample, the volatility more than doubled in a short period of time, suggesting that the importance of terms of trade uncertainty may have been a lot larger during these periods.

The impulse responses to the level shocks show that the model behaves in the
Figure 3.6: Impulse Responses to a Two-standard Deviation Terms of Trade Uncertainty Shock with $\phi_{Ry} = \phi_{RRE} = 0$

Notes: Level shocks are held constant. All responses are in percent, except for $\Pi_{CPI}$ and $R$, which are in percentage points.

expected way consistent with the previous literature. Figure 3.7 depicts a one standard deviation terms of trade level shock, corresponding to an increase of the price of export goods relative to import goods by 4%. All aggregates measured in import prices (top row) increase significantly. The aggregate GDP components are plotted in import prices for better comparability to the results in the literature on terms of trade shocks like e.g. Mendoza (1995). With a value of about 1, the “output multiplier” of the terms of trade shock is somewhat larger in my model than the 0.6 in Mendoza’s model. His lower terms of trade multiplier reflects i) the higher elasticity of substitution between tradables and non-tradables, ii) the higher share
3.4 The Aggregate Effects of Terms of Trade Uncertainty

Figure 3.7: Impulse Responses to a One-standard Deviation Terms of Trade Level Shock

Notes: All responses are in percent, except for $\Pi_{CPI}$ and $R$, which are in percentage points.

of non-tradables assumed in his model which tends to dampen the role of the terms of trade, and iii) the accommodating response of monetary policy that lowers the nominal interest rate when the inflation rate and the real exchange rate drop.

In terms of CPI prices, the GDP response depicted in the left panel of the second row is 0.9% at its peak, showing that much of the change in output at import prices reflects the higher purchasing power of domestic export goods. Net exports as a share of GDP initially drop due to an increase in the denominator that is stronger than the numerator as exports react sluggishly. But subsequently, the higher purchasing power of exports dominates and the trade balance turns positive with the agents building up a higher net foreign asset position (omitted for brevity) on which they draw upon in the following periods when the shock subsides to fund part of their increased imports. Due to complementarity between domestic and foreign goods, domestic use of both tradable and non-tradable goods increases. The use of tradable goods rises relatively more due to the fall in the relative price of tradables brought
about by the positive terms of trade shock as the cost of imports decreases.

Figure 3.8: Impulse Responses to a One-standard Deviation Technology Shock in the Non-tradable Sector

Notes: All responses are in percent, except for $\Pi_{CPI}$ and $R$, which are in percentage points.

Finally, Figure 3.8 shows the impulse responses to a sectoral TFP shock in the non-tradable sector. As a result, output, consumption, and investment increase. Consistent with Galí (1999), the correlation between technology and labor in the sector affected by the TFP shock is negative. Due to price rigidities, labor partially reallocates to the tradable sector, whose goods are in relatively high demand as evidenced by the increase in the relative price of tradables. In contrast, the relative price of non-tradables, $p^N_t$, falls. The move of labor from the production of non-tradables to tradables leads to a substitution of domestic export goods for import goods in the production of tradable goods, which is reflected in an initial fall of both
imports and exports. Because imports fall relatively more than exports, the net result is an increase in net exports. The impulse responses to a technology shock in the tradable sector are similar and omitted for brevity.

3.5 Conclusion

The current chapter has shown that terms of trade uncertainty is an important, yet underappreciated factor for explaining business cycles in small open economies. For the case of Chile, I have presented evidence for considerable time-variation in the volatility of terms of trade shocks, with the variance more than doubling during the recent commodities boom of 2006-2008. Using a calibrated open economy DSGE-model I have shown that the ex-ante and ex-post effects of time-varying terms of trade uncertainty can account for about 20% of business cycle fluctuations. An average exogenous increase in uncertainty of 54% leads to a decrease in output of $-0.1\%$, a magnitude comparable to an exogenous 50 basis points increase in the Federal Funds rate for the United States.

The negative output effect after such an exogenous uncertainty shock was shown to be driven by the price-setting behavior of firms, who increase their markups. The reason for the relatively mild recession generated by an increase in terms of trade uncertainty was a dampening effect due to both the household’s precautionary motive and the central bank’s interest rate response. The fact that the terms of trade volatility more than doubled during the recent commodities boom suggests that terms of trade uncertainty may have been an important factor during this period.

The present study was concerned with the positive analysis of terms of trade uncertainty effects. The normative analysis whether terms of trade uncertainty leads to significant welfare losses or is associated with welfare gains and how the optimal policy response function of the central bank should look like is left to future work. Dib (2008) is a first study in this direction, showing in an estimated model of the Canadian economy that permanently higher terms of trade uncertainty under flexible exchange rates may be welfare increasing due to positive Hartman-Abel effects. Moreover, future work should explore the consequences of terms of trade uncertainty in a production structure of the Melitz (2003) and Ghironi and Melitz (2005)-type.
Chapter 3

Here, increased uncertainty about export prices in combination with entry costs may lead to compositional changes in production and thereby affect aggregate productivity. This might lead to additional output effects not considered in the present chapter.
Appendix to Chapter 3

3.A Data Appendix

This section details the construction of the Chilean terms of trade index and the data sources for computing the business cycle moments.

3.A.1 Terms of Trade Index

Data series for the terms of trade construction were taken from Datastream, except. The prices for oil, fish meal and wood pulp were downloaded from the Chilean central bank at: http://www.bcentral.cl/eng/economic-statistics/series-indicators/xls/Precio_Cobre__HPescado_Petrol_Celulosa%20.xls. Datastream mnemonics are provided in brackets.

Import Price Index

For the Import Price Index, the following series were used:


2. World import unit values, US$, 2005=100: WD IMPORT UNIT VALUES (IN US$ TERMS) (740010147)


The import price series is generated in several steps. First, an oil price series, $P_{oil}^t$, is constructed through splicing the two oil price series. For this purpose, the second series measured in U.S. $ is rebased to 2005=100 and then concatenated. Second, the World import unit values series, $P_{WorldIm}^t$, is purged of the effect of oil prices by subtracting the Share of oil imports in total imports, $s_t$, for the respective month times the price of oil. As the Share of oil imports in total imports is only available at
Chapter 3

annual frequency, $s_t$ is linearly interpolated. The final nominal import price index, $P_{f^*}$, is constructed by assuming a log linear specification with the weights given by the share of oil imports in total world imports:

$$
\log \left( P_{f^*} \right) = s_t \log (P^{oil}_t) + (1 - s_t) \log \left( P^{WorldIm}_t \right)
$$

Export Price Index

For the Export Price Index, the following series were used:

1. Copper price, US$/pound (Source: Chilean Central Bank): BML price of refined copper (dollars/pound)
### 3.A Data Appendix

8. Wood pulp/cellulose price index, US$, 2005=100 (Source: IMF IFS): FN EXPORT PRICE - NEWSPRINT UNIT VALUE (FNI74ULDF)\(^{36}\)


All series that need to be concatenated are spliced in the way described for the import price index. The export shares are based on the linear interpolation of annual export shares, computed as the fraction of the export value of the respective category in the value of all categories. Data is taken from the annual nominal national accounts. Computation of the Laspeyeres-Index follows Bennett and Valdés (2001). Finally, the nominal indices are deflated using the U.S. Producer Price Index (2005=100, Source: IMF IFS, US PPI (USI63...F)).

#### 3.A.2 Moment Comparison

The nominal National Accounts data series for Chile were taken from the Statistics Database → Section National Accounts → GDP expenditure and income → GDP expenditure, at current prices, spliced series, 2003 base (millions of pesos).

The real National Accounts data series were taken from the Statistics Database → Section National Accounts → GDP expenditure and income → GDP expenditure, at constant prices, spliced series, 2003 base (millions of pesos).

\(^{36}\)Due to the non-availability of the Merrill Lynch price index used in Bennett and Valdés (2001) for the period 1970-1986, I use the series provided by the IMF until 1986.
Figure 3.9: Export Price Index – Price Components and Basket Shares

(a) Individual export price index components, deflated with U.S. PPI (2005=100)

(b) Export price index components: basket shares
3.B Convergence Diagnostics

A description of the convergence diagnostics can be found in Appendix 2.C.

Table 3.5: Convergence Diagnostics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>4% taper</th>
<th>8% taper</th>
<th>15% taper</th>
</tr>
</thead>
<tbody>
<tr>
<td>Government Spending</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>0.377</td>
<td>0.449</td>
<td>0.494</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>0.597</td>
<td>0.658</td>
<td>0.695</td>
</tr>
<tr>
<td>$\rho_{\text{tot}}$</td>
<td>0.850</td>
<td>0.863</td>
<td>0.866</td>
</tr>
<tr>
<td>$\xi_{\text{tot}}$</td>
<td>0.737</td>
<td>0.759</td>
<td>0.767</td>
</tr>
<tr>
<td>$\sigma_{\text{tot}}$</td>
<td>0.937</td>
<td>0.938</td>
<td>0.930</td>
</tr>
</tbody>
</table>

Notes: Numbers are p-values of the $\chi^2$-test for equal means of the first 20% of draws and the last 50% of the draws (after the first 5500 draws are discarded as burn-in).
Figure 3.10: Evolution of MCMC Sampler over Time.

(a) MCMC draws

(b) Mean of the parameters over time
3.C Misspecification Tests

A description of the misspecification test performed can be found in Appendix 2.C.

Table 3.6: Tests for Model Misspecification

<table>
<thead>
<tr>
<th></th>
<th>JB</th>
<th>KS</th>
<th>SW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stochastic Volatility Model</td>
<td>0.500</td>
<td>0.274</td>
<td>0.940</td>
</tr>
<tr>
<td>OLS, $\xi_{tot} = 0$</td>
<td><strong>0.001</strong></td>
<td><strong>0.000</strong></td>
<td><strong>0.000</strong></td>
</tr>
</tbody>
</table>

Note: Bold number indicate significance at the 5% level. JB refers to the Jarque and Bera (1987)-test, KS refers to the Kolmogorov (1933)/Smirnov (1948)-test, and SW refers to the Shapiro and Wilk (1965)-test.

Figure 3.11: QQ-plot for Model Misspecification

Notes: Top panel: Model without stochastic volatility, i.e. $\xi_{tot} = 0$; bottom panel: stochastic volatility model.
Bibliography


Bibliography


Banco Central de Chile (2010). *Chilean External Debt*. Banco Central de Chile.


Bibliography


Bibliography


Bibliography


Bibliography


Bibliography


Bibliography


