Phenomenological Aspects of Mirage Mediation

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Abstract

We consider the possibility that string theory vacua with spontaneously broken supersymmetry and a small positive cosmological constant arise due to hidden sector matter interactions, known as $F$-uplifting/$F$-downlifting. We analyze this procedure in a model-independent way in the context of type IIB and heterotic string theory. Our investigation shows that the uplifting/downlifting sector has very important consequences for the resulting phenomenology. Not only does it adjust the vacuum energy, but it can also participate in the process of moduli stabilization. In addition, we find that this sector is the dominant source of supersymmetry breaking. It leads to a hybrid mediation scheme and its signature is a relaxed mirage pattern of the soft supersymmetry breaking terms. The low energy spectra exhibit distinct phenomenological properties and differ from conventional schemes considered so far.
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Chapter 1

Introduction

1.1 Motivation

The idea that our world is build up from indivisible constituents was formulated, for the first time, by Democritus around 400 B.C. in his famous atomic hypothesis. With John Dalton, about two thousand years later, the exploration of the composition of matter began and has grown ever since. Today, we believe that the fundamental constituents of matter are leptons and quarks and that there are four fundamental interactions between all elementary particles: electromagnetism, weak force, strong force and gravity.

Motivated by the desire to find a unified description of nature based on the smallest possible set of fundamental laws, the enormous progress in theoretical physics has led to the formulation of the Standard Model (SM) of particle physics [1–3]: a renormalizable quantum field theory that describes strong, weak and electromagnetic interactions in terms of the gauge group $SU(3)_{c} \times SU(2)_{L} \times U(1)_{Y}$. It successfully unifies weak and electromagnetic interactions within the electroweak (EW) theory which, below the EW scale $M_{EW} \sim 100 \text{ GeV}$, gets spontaneously broken to electromagnetism by the Higgs mechanism.

Although the predictions of the SM have been confirmed experimentally at very high precision, the Higgs boson has not been discovered yet. The EW precision data from the Large Electron Proton Collider (LEP2) suggests that the Higgs particle, if existent, has a mass between $114 \text{ GeV}$ and $200 \text{ GeV}$. This year, the Large Hadron Collider (LHC) at CERN (Geneva) will be brought into operation and there is a great deal of hope to discover the Higgs particle.

Despite its powerful predictivity, from the theoretical point of view, there are a number of pressing issues that cannot be addressed by the SM, if we consider it as a fundamental theory. For example, the SM only partially supports the concept of the unification of all fundamental forces. This is because on one hand the internal symmetries of the SM describing the EW and strong interactions are not interrelated, and, on the other hand, a quantum description of gravity can not be consistently accommodated within the SM. It also does not contain any candidates for the cold dark matter (DM), which constitutes a major part of the matter in the universe. Furthermore, the SM fails to explain the stability of the large "gap" between the EW scale and the (reduced) Planck scale $M_{P} \approx 2.4 \times 10^{18} \text{ GeV}$, known as the hierarchy problem [4].
Nevertheless, we can avoid running into troubles if we assume that the SM is not a fundamental but, instead, an effective theory, valid to a certain (cut-off) scale. Then, in order to address the issues above, one has to find suitable extensions of the SM.

**Supersymmetry**

In general, quantum corrections drive the masses of scalar particles to the largest available scale in the theory. On the other hand, the masses of chiral fermions and gauge bosons are protected against radiative corrections. This protection is provided by symmetries: chiral symmetry for fermions and gauge symmetry for gauge bosons.

In the SM this translates into understanding the stability of the weak scale against radiative corrections, i.e. the hierarchy between the weak and, say, the Planck scale. This problem arises from the fact that there is no symmetry protecting the masses of scalar particles.

For example, 1-loop corrections to the Higgs mass squared from (heavy) fermions $f$ are proportional to the square of the momentum cut-off scale $\Lambda_{\text{UV}}$. Although it is possible to keep the Higgs mass finite by performing unnatural fine-tuning, this procedure is not stable in perturbation theory and, from the theoretical vantage point, quite unattractive.

On the other hand, if we assume a scalar field $S$ coupling to the Higgs with the same coupling constant as $f$, the 1-loop correction to the Higgs mass squared from $S$, precisely cancels the quadratically divergent contribution from $f$. In this case, the complete correction to the Higgs mass, $\delta m_H^2 \sim |m_S^2 - m_f^2| \log \Lambda_{\text{UV}}$, is only logarithmically divergent. In addition, if we claim $m_S = m_f$, the 1-loop correction would neatly vanish. This illustrates the importance and power of the concept of symmetry in particle physics.

In fact, this has led to the invention of *supersymmetry* (SUSY) [5–13], a symmetry that relates bosons and fermions:

$$\mathcal{D}|\text{fermion}\rangle = |\text{boson}\rangle, \quad \mathcal{\bar{D}}|\text{boson}\rangle = |\text{fermion}\rangle,$$  \hspace{1cm} (1.1)

where the so-called *supercharge* $\mathcal{D}$ (transforming as a spinor) denotes the generator of SUSY transformations. The number of distinct pairs of $\mathcal{D}$ and $\mathcal{\bar{D}}$ determines the number $\mathcal{N}$ of supersymmetries.\(^1\) Infinitesimal SUSY transformations are described by an anti-commuting spinor parameter $\epsilon$ which is spacetime-independent in global SUSY.

What makes SUSY so unique is the fact that it is the only *graded* Lie algebra of symmetries of the $S$-matrix consistent with relativistic quantum field theory [14]. This symmetry guarantees the cancellation of quadratic divergences in perturbation theory. Moreover, the non-renormalization theorem [5, 15] ensures that

\(^1\)Theories with $\mathcal{N} > 1$ are called extended supersymmetries. In 4D only $\mathcal{N} = 1$ SUSY contains chiral fermions.
(supersymmetric) masses of scalar particles are not renormalized to any order in perturbation theory. Single particle states fall into irreducible representations of the SUSY algebra, called supermultiplets. Bosons and fermions within a supermultiplet are called superpartners of each other. Moreover, the generators of SUSY transformations commute with those of gauge transformations, thus the superpartners of a supermultiplet are in the same representation of the gauge group. Each supermultiplet contains the same number of fermionic and bosonic degrees of freedom (DOF).

In 4D \( N = 1 \) SUSY it is convenient to express the supermultiplets as superfields. These live in the superspace which is spanned by spacetime coordinates and by four anti-commuting coordinates \( \theta \) and \( \bar{\theta} \) that transform as spinors. The simplest superfield is the so-called chiral superfield \( \phi_i \), containing a Weyl fermion \( \psi_i \), a complex scalar \( \phi_i \) (sfermion) and a complex auxiliary field \( F_i \) (which is necessary to close the SUSY algebra off-shell). The next to simplest superfield is the so-called vector superfield \( V_a \) which, in the Wess–Zumino gauge, contains a vector boson \( A_\mu \), a Weyl fermion \( \lambda_a \) (gaugino) and a real auxiliary field \( D_a \). All couplings and masses are determined by the superpotential \( W \), a holomorphic function of chiral superfields. The tree-level scalar potential arises from the auxiliary fields

\[
V_{\text{SUSY}} = \sum_i |F_i|^2 + \sum_a g_a^2 D^a D^a ,
\]

where \( g_a \) denotes the gauge coupling. Observe that in global SUSY the scalar potential is always non-negative.

**Supergravity**

SUSY can be promoted to a local symmetry by making the infinitesimal transformation parameter spacetime dependent, \( \epsilon \rightarrow \epsilon(x) \). Then the product of two local SUSY transformations leads to local spacetime translations, i.e. a general coordinate transformation. To make the total Lagrangian density locally supersymmetric requires the gauge field of local SUSY transformations to be a spin-\(3/2\) fermion, called gravitino. Its superpartner is a spin-2 boson, the graviton, which is the messenger of gravitational force. Therefore, local SUSY is referred to as supergravity (SUGRA) \([7–13, 16]\). The graviton and the gravitino, together with an auxiliary field, form the so-called SUGRA multiplet.\(^2\)

SUGRA, however, is a non-renormalizable theory as the gravitational coupling is a dimensionful parameter. In the effective low-energy Lagrangian, non-renormalizable terms appear suppressed by inverse powers of \( M_P \). The scalar potential in 4D \( N = 1 \) SUGRA is given by

\[
V_{\text{SUGRA}} = K_{ij} \bar{F}_i \bar{F}_j - 3e^K \frac{\left( \phi^I \partial_I \bar{\phi}^J \partial_J \phi^K \right)}{M_P^2} \frac{\left| W(\phi) \right|^2}{M_P^2} + M_P^4 \sum |\text{Re} f_a(\phi_i)| \frac{D^a D^a}{2} ,
\]

\( ^2 \)The number of the gravitini in the SUGRA multiplet is equal to the number of supersymmetries.
where the Kähler potential $K(\phi, \overline{\phi})$ describes the kinetic terms of chiral superfields, $K_i$ is the Kähler metric and the gauge kinetic function $f_a(\phi)$ determines the kinetic terms of vector multiplets, and in particular the gauge coupling constants. Unlike global SUSY, the scalar potential in SUGRA can be negative.

**The Minimal Supersymmetric Standard Model (MSSM)**

The MSSM \cite{10,11} is the simplest supersymmetric extension of the SM. It contains one supercharge and a minimal particle content. Each lepton and quark is accompanied by a slepton or a squark, respectively (tab. 1.1). Fermions and sfermions reside in chiral superfields whereas gauge bosons and gauginos (bino, wino, gluino) form vector superfields. Unlike the SM, the MSSM requires two Higgs doublets so as to avoid gauge anomalies. The Higgs bosons and their superpartners, higgsinos, form chiral superfields. All superfields are labeled the same way as ordinary SM particles and the sparticles are denoted by a tilde.

In contrast to the SM, the MSSM does not automatically preserve baryon ($B$) and lepton ($L$) number. In order to to avoid $B$ and $L$ violation (in particular proton decay) one introduces the so-called $R$-parity, $P_R = (-1)^{3(B-L)+2s}$ ($s$ denotes the spin), a discrete symmetry that does not commute with SUSY. Particles have $R$-charge +1 whereas sparticles have −1. If one requires the MSSM to conserve $R$-parity, this leads to distinct phenomenological properties. In particular, the lightest supersymmetric particle (LSP) will be stable. In addition, if the LSP is neutral it can play the role of a cold DM candidate.
Breakdown of supersymmetry

If SUSY were an exact symmetry, particles and sparticles would have the same mass. Since sparticles with SM masses have never been observed, SUSY must be broken (at low energies) and the main problem is to understand the mechanism of SUSY breaking.

From the theoretical point of view, it is natural to consider a spontaneous breakdown of SUSY which occurs if the auxiliary fields of chiral superfields (F-terms) and/or vector superfields (D-terms) acquire non-zero VEVs. In analogy to ordinary symmetry, spontaneous breakdown of SUSY leads to the emergence of massless Weyl fermions, called goldstinos. In SUGRA, the gravitino “eats” the goldstino and becomes massive. This effect is called super-Higgs mechanism and is completely analogous to the ordinary Higgs mechanism.

In the context of global SUSY, eq. (1.2) implies that supersymmetric minima always have zero vacuum energy, i.e. they correspond to Minkowski vacua. Vacuum configurations with broken SUSY always have positive energy, yielding a de Sitter (dS) space. On the other hand, SUGRA yields the relation $D^a \sim F^i$. Thus, unless $W = 0$, a supersymmetric minimum has negative energy (eq. (1.3)), i.e. it corresponds to an anti de Sitter (AdS) vacuum. Furthermore, in SUGRA, the vacuum energy of non-supersymmetric minima can have a positive, negative or zero value, depending on the magnitude of $F^i$. This makes SUGRA models attractive for the discussion of the inflatory universe since recent observational data [17, 18] requires a cosmological constant (CC) of order $\Lambda_{cc} \sim 10^{-120} M^4_{pl}$. In SUGRA one can arbitrarily adjust the contribution from SUSY breaking to the CC.

Within the framework of the MSSM, however, the spontaneous breakdown of SUSY seems rather unfortunate as all members of a superfield carry the same quantum numbers and a non-zero VEV of an auxiliary field would immediately break various internal symmetries like color, electromagnetism, etc.

To avoid a phenomenological disaster, the breakdown of SUSY within the MSSM must be explicit, but cannot be arbitrary. In order for SUSY to remain a solution to the hierarchy problem (i.e. ensure the cancellation of quadratic divergences) the explicit breaking terms must contain couplings of positive mass dimension [19]. This so-called soft breaking can be parameterized by $\mathcal{L} = \mathcal{L}_{\text{SUSY}} + \mathcal{L}_{\text{SOFT}}$ where

$$\mathcal{L}_{\text{SOFT}} = \left[ -\frac{1}{2} M_a \bar{\lambda}^a \lambda^a - \frac{1}{6} A_{ijk} \psi^i \psi^j \psi^k - \frac{1}{2} B_{ij} \psi^i \psi^j + \text{h.c.} \right] - \frac{1}{2} m_{ij} \psi^i \psi^j$$

contains soft gaugino masses ($M_a$), soft scalar squared masses ($m^2$), soft bilinear ($B$) and trilinear ($A$) couplings. The scale of the soft parameters $m_{\text{SOFT}}$, characterizing the mass splitting in the supermultiplets, should be of order 1 TeV, otherwise the couplings in the Higgs sector would reach unnatural size [20].

In general, the soft terms in eq. (1.4) can introduce dangerous CP and flavor violations. These effects can be evaded if one assumes that SUSY breaking is suitably universal; e.g. if slepton and squark mass matrices are flavor-blind and the $A$ terms are proportional to the corresponding Yukawa couplings.
Interestingly enough, MSSM offers a convincing DM candidate through the weakly interacting neutralino LSP.

**Origins of supersymmetry breaking**

On general grounds, we expect SUSY to be spontaneously broken by auxiliary $(F$-term) components of (chiral) superfields which are singlets under the SM gauge group. Since such fields cannot be part of ordinary matter one assumes that SUSY breaking occurs in a hidden sector, which is only weakly coupled to the observable (MSSM) sector. Then in order to connect hidden and observable sector and mediated the breakdown of SUSY to the MSSM, new physical phenomena are required.

A natural choice to connect these two sectors would be gravitationally, through operators suppressed by inverse powers of $M_P$. The realization of this scenario can be achieved if one couples the MSSM to SUGRA. In this picture, the hidden sector $F$-terms acquire non-vanishing VEVs and induce spontaneous breakdown of (local) SUSY. Since the hidden sector fields communicate only gravitationally with the MSSM, below $M_P$ they decouple from the low energy theory and the only marks they leave are explicit soft breaking terms of order $m_{soft} \sim F^3/M_P$. These soft terms are renormalized at a scale $M_{IN} < M_P$ where gravitational DOF have been integrated out. For $m_{soft} = O(1 \text{ TeV})$ the scale of spontaneous SUSY breakdown is $\sqrt{F^3} \sim 10^{11} \text{ GeV}$. The gravitino mass is $m_{3/2} \sim F^3/M_P$ and thus sets the scale of the soft terms.

A dynamical mechanism to explain the hierarchy between the gravitino mass and the Planck scale is provided by hidden sector gaugino condensation [21–26]. It is specified by a non-zero VEV of a composite field made up of fermions charged under some non-Abelian gauge group. In a pure supersymmetric Yang Mills theory the only candidates for such condensates are gauginos. One expects a non-vanishing condensate $\langle \lambda \lambda \rangle$ to develop at the renormalization group (RG) invariant scale $\Lambda \sim M_P \exp(-1/(b_0 g^2)) \ll M_P$, where $b_0$ and $g$ are the $\beta$-function and the gauge coupling of the hidden sector confining gauge group, respectively. By dimensional analysis, $\langle \lambda \lambda \rangle \sim \Lambda^3$. The strong dynamics triggers a breakdown of local SUSY via $F \sim \Lambda^3/M_P$, leading to a gravitino mass $m_{3/2} \sim \Lambda^3/M_P^2$.

**Grand unification**

Even though we have no direct experimental sign for SUSY, electroweak precision data gives a good fit to the MSSM and supports gauge coupling unification. Consider for example the RG evolution of the three gauge couplings in the framework of the SM model, fig. 1.1.a. This does not reveal any (sign of) unification as the three curves fail to meet in one point. On the contrary, the MSSM seems to have just the appropriate particle content which modifies the $\beta$-functions such that the gauge couplings coincide at about $2 \times 10^{16} \text{ GeV} [27–30]$, fig. 1.1.b.

This coincidence may be a hint in favor of a grand unified theory (GUT) [31],
1.1 Motivation

Fig. 1.1: RG evolution of the inverse SM gauge couplings $\alpha_s^{-1} = 4\pi/g_s^2$. Panel (a) illustrates the situation in the SM. In the MSSM (panel (b)) the slopes are different due to the presence of sparticles. In panel (b) the sparticle thresholds are varied between 250 GeV and 1 TeV, and $\alpha_3(m_Z)$ between 0.113 and 0.123.

A particularly simple and predictive scheme is that of minimal supergravity (MSUGRA) [32], where one assumes universal soft breaking parameters of $O(m_{3/2})$ at the GUT scale. These include a universal gaugino mass $m_{1/2}$, a universal scalar mass $m_0$, a universal trilinear coupling $A_0$ and two other (model-dependent) parameters. In this way, the $O(100)$ parameters of the MSSM are described in terms of just five parameters. RG evolution of these parameters down to the TeV scale will allow one to predict the entire MSSM spectrum. The possible numerical values of these soft terms are only constrained by experimental bounds on sparticle masses and some indirect theoretical arguments.

However, since gravity is a non-renormalizable theory one might worry about possible higher dimensional terms appearing in schemes of gravity mediated SUSY breaking. In order to have control over such terms one needs a meaningful underlying theory which also justifies the presence of SUSY and hidden sectors.

String theory

Unlike ordinary quantum field theory, the fundamental building blocks in string theory [33–36] are given by extended one-dimensional oscillating objects, called strings. Different excitations of strings at their characteristic scale $M_{\text{STR}}$ (which is expected to be large) can be identified with particles at low(er) energies. Strings can be open and closed and during their propagation they can split and merge. Open strings end on spatially extended objects of dimension $p$, called Dirichlet branes ($Dp$ branes). One of the remarkable properties is that the spectrum of
string excitations contains a spin-2 particle, corresponding to the graviton. This offers a quantized description of gravity [37–39] and, most notably, a unified description of all fundamental forces. In particular, at low energies string theory reduces to Einstein’s theory of gravity. Consistency conditions require string theories to have ten spacetime dimensions as well as to include SUSY at the string scale [40,41]. There are only five consistent (super)string theories in 10D: type I, type IIA, type IIB, heterotic SO(32) and heterotic $E_8 \times E_8$. It has been realized that these five string theories are related among each other by a web of dualities and that they seem to be different perturbative limits of an underlying 11D theory called $M$-theory [42,43].

If our intention is to relate string theory to the observable world, we have to find a way to reduce the number of spacetime dimension from 10 to 4. One possibility is to confine the six extra dimensions to some compact manifold $M^6$ such that the 10D spacetime of the string is given by the direct product $M^{10} = M^{3,1} \times M^6$, where $M^{3,1}$ denotes our 4D Minkowski spacetime. This procedure is called compactification. If the length scales associated with $M^6$ correspond to very high energies the extra spatial dimensions appear unobservable from the 4D perspective. Since we are aiming at a chiral low energy theory only manifolds ensuring $N = 1$ SUSY in 4D come into consideration. A possible class of such manifolds are the so-called Calabi–Yau (CY) manifold [44]. Unfortunately, the metric for (almost all) CY manifolds is still unknown. Therefore in such compactifications one is led to consider a low energy approximation of the 10D string theories below $M_{str}$. This approximation can be described by an effective 10D $N = 1$ SUGRA. After the compactification of the six extra dimensions one obtains an effective 4D $N = 1$ SUGRA, valid at energies below the scale of compactification $M_{comp}$.

The low energy effective 4D SUGRA contains only massless excitations of strings. The geometry of the compact space (i.e. CY manifolds) is parameterized by moduli. These are massless, gauge singlet scalar fields and interact only gravitationally with ordinary matter. In other words, a hidden sector is naturally built-in in string theory models. The most important moduli are the Kähler moduli $T_i$ describing the volume of the compact space, the complex structure moduli (CSM) $Z_i$ parameterizing the shape of CY manifolds and the dilaton $S$.

Moduli can be considered as the “engine” that drives any string-derived 4D low energy Lagrangian, as their VEVs determine all couplings. However, moduli do not have a scalar potential at the perturbative level, i.e. they are flat directions in the potential.

**String phenomenology**

If we wish to understand the phenomenology of any string-derived theory, the important questions we have to address include:

1) *What is the dynamics of moduli stabilization?*

2) *How is SUSY broken and communicated to the observable sector?*
3) Does the vacuum energy take a small positive value consistent with observation?
4) What is the pattern of the soft breaking terms?

To find and, in particular, to understand the answers to these questions is the heart of string phenomenology.

To generate a potential for the moduli, one has to rely on fluxes [45,46] and non-perturbative effects such as gaugino condensation. On the other hand, a hierarchically small scale of SUSY breaking can be reliably explained by dimensional transmutation, which again, can be realized by gaugino condensation. Once SUSY is broken, the moduli get a non-trivial potential which might result in their stabilization. The auxiliary (F-term) components of moduli will generically take non-vanishing values, thereby indicating in which “direction” SUSY is broken. Furthermore, in such a scheme one would usually encounter a situation where the mass of (most of) the moduli is of order the gravitino mass: \( m_{\text{MODULI}} \sim m_3/2 \). Since gravity connects the moduli (hidden) and the MSSM (observable) sector it will be, predominantly, the mediator of SUSY breakdown.

Notice that in non-stringy models, the gauge singlet fields \( X \) responsible for SUSY breaking were introduced “ad hoc”. In string-derived models, however, these gauge singlet fields are well-motivated through moduli, hence gravity mediation is usually referred to as modulus mediation.

In this picture, the tree-level contribution from modulus mediation is usually of order \( m_{\text{SOFT}} \sim m_3/2 \). Hence moduli mediation is the dominant source of SUSY breaking, causing the mass pattern \( m_{\text{MODULI}} \sim m_3/2 \sim m_{\text{SOFT}} \).

Generically, string-derived models lead to dS and AdS vacua where the contribution to the CC exceed the observed value by orders of magnitude. In order to obtain a reasonable vacuum energy \( V_0 \), one needs an additional sector which, in case of an AdS minimum provides an “uplifting”, and, in case of a dS minimum a “downlifting” of \( V_0 \) to the desired value.

It is very important to stress, that the soft breaking terms can only be reliably computed after all moduli have been stabilized and the vacuum energy has been adjusted properly. Soft terms obtained at an intermediate stage might (and usually will) drastically change through the stabilization of the remaining moduli [47]. Any (additional) source of vacuum energy density generically affects the soft scalar masses and, therefore, must be taken into account.

This shows how close the three questions are related to each other.

**Mirage mediation**

The importance of the mechanism of adjusting the vacuum energy has only been appreciated recently [47,48]. In its simplest form it has been discovered in the toy model of Kachru, Kallosh, Linde and Trivedi (KKLT) [49], constructed in the framework of type IIB string theory.

Although KKLT (for the first time) achieve the stabilization of all moduli they obtain a deep AdS minimum. To render the vacuum realistic, KKLT introduce...
an "uplifting" sector. However, it turns out that apart from adjusting (uplifting) the vacuum energy, the uplifting sector has far-reaching consequences for the explicit pattern of SUSY breaking. In particular, the uplifting sector provides the dominant source of SUSY breaking and leads to the appearance of the so-called little hierarchy \([50]\), characterized by the factor \(\log(M_p/m_{3/2}) \sim 4\pi^2\).

The tree-level contribution from modulus mediation becomes suppressed by this factor such that radiative contributions from the SUSY breakdown in the uplifting sector become competitive. One possible scheme of mediation via radiative corrections is the scheme of anomaly mediation \([51, 52]\). In this case, instead of pure modulus mediation one is led to a mixed modulus-anomaly mediation, known as mirage mediation \([47, 53, 54]\). This scheme exhibits very distinct phenomenological properties \([55, 56]\) as it allows to retain the attractive features of the respective mediation mechanisms while discarding the problematic aspects. Here, the general mass pattern is \(m_{\text{SOFT}} \ll m_{3/2} \ll m_{\text{MODULI}}\).

Based on the publications \([57–59]\) we investigate the role of the matter sector in the context of type IIB and heterotic string theory. As we shall see this leads to very interesting phenomenological implications.

### 1.2 Outline

In the following we provide some details about the contents of each chapter.

**Chapter 2** In this chapter we present the origin of mirage mediation. First we review the original construction of KKLT which we subsequently generalize. This class of models, obtained in the framework of type IIB string theory compactifications, uses 3-form fluxes to stabilize the CSM and the dilaton. Non-perturbative corrections to the superpotential are then used to fix the remaining Kähler moduli. The process of moduli stabilization leaves the ground state of the theory supersymmetric with a large negative energy. An "ad hoc" uplifting sector is introduced to break SUSY and fine-tune the vacuum energy to a desired value. We review the implications of this uplifting sector for the low energy phenomenology. Under rather general circumstances one is led to a scenario in which the mass scales in the low energy theory are endowed with a moderate hierarchy specified by the logarithm of the large hierarchy between the Planck scale and the scale of the soft masses. This exhibits a new mediation scheme which, as we will explain in detail, is called mirage mediation.

**Chapter 3** The main drawback of KKLT-type models is that the "ad hoc" uplifting sector explicitly breaks SUSY. In such a situation the effective 4D theory cannot be put into the standard \(\mathcal{N} = 1\) SUGRA form, which considerably complicates the analysis. Therefore it is desirable to obtain dS/Minkowski vacua in the framework of spontaneously broken SUSY. This can be achieved by changing the uplifting sector. As was pointed out in \([60]\), uplifting within the SUGRA framework requires additional fields in the system which are necessary to provide
the goldstino which is necessary to make the gravitino heavy. In string theory models, matter fields are as common as moduli and thus can in principle be used to uplift supersymmetric AdS minima. In this case, dS/Minkowski vacua with spontaneously broken SUSY can be obtained due to $F$-terms of hidden sector matter fields $[57,60]$. This mechanisms is known as $F$-uplifting. In this chapter we show that this scheme leads to the appearance of the little hierarchy as well. The matter uplifting sector provides the dominant source of SUSY breakdown and, unlike KKLT-type models, also affects the soft breaking terms, leading to a so-called relaxed mirage mediation scenario. Still, the “pure” mirage pattern is possible as well, but only for certain values of the parameters.

Chapter 4 With the recent success of model building in the framework of heterotic orbifold compactifications $[61–63]$, it is important to reconsider the question of moduli stabilization. One of the main difficulties of moduli stabilization in heterotic string theory is the appearance of only one type of fluxes, while two of them are available in the type IIB theory. Publication $[58]$ investigated for the first time the role of the uplifting sector (composed of matter fields) in the context of heterotic orbifold compactifications. To illustrate the importance of this sector we consider a simple example: a gaugino condensate in the absence of a flux background. This is known to lead to a run-away scalar potential for the dilaton with a large positive vacuum energy. A closer inspection of the interactions between the moduli and matter fields reveals the surprising fact that the uplifting sector alone is responsible for both moduli stabilization and “downlifting” the large positive vacuum energy to a smaller value. Thus in context of the heterotic string theory the uplifting sector turns out to be a “downlifting” sector. We refer to this mechanism as $F$-downlifting. As we shall see it also plays an important role in SUSY breaking and its mediation. For this class of models we are again led to a kind of mirage pattern as previously identified in the context of type IIB string theory. The soft breaking terms, although very similar to those of type IIB models, exhibit some quantitative differences that will be discussed in detail. At the end of this chapter we consider one possible application of the downlifting procedure.

Chapter 5 In this chapter we present a detailed discussion of the low energy spectra emerging in the scheme of $F$-uplifting $[57]$ and $F$-downlifting $[58]$. We find that, even though the effective theory contains several parameters, the low energy spectra is described by just two continuous parameters: the ratio of moduli to anomaly mediation, $\varrho$, and the gravitino mass, $m_{3/2}$, which sets the scale of the soft terms. In this schemes, the soft gaugino masses and the soft $A$-terms receive comparable contributions from modulus and anomaly mediation. However, the soft scalar masses can additionally receive a contribution from the matter sector. This results in a quite distinctive low energy phenomenology, which is different from that of pure modulus or pure anomaly mediation. We

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3More fluxes can appear if we go beyond CY compactifications $[64–66]$. 

impose several phenomenological constraints and find that they are satisfied in considerable regions of the parameter space. In particular, we can avoid tachyonic boundary conditions (which usually are provided by anomaly mediation) and find allowed regions in the parameter space with a higgsino dominated LSP.

Chapter 6 concludes this work with a summary and a brief outlook. Afterwards, some technicalities are presented in the appendices.

1.3 Publications

Parts of this work have been published in scientific journals:

- O. Lebedev, V. Löwen, H. P. Nilles, Y. Mambrini and M. Ratz
  “Metastable vacua in flux compactifications and their phenomenology”
  JHEP 0702, 063 (2007)

- V. Löwen and H. P. Nilles,
  “Mirage pattern from the heterotic string”

- V. Löwen, H. P. Nilles and A. Zanzi
  “Gaugino condensation with a doubly suppressed gravitino mass”
  arXiv:0804.3913 [hep-th]
Chapter 2

Road to mirage mediation

After a short introduction to the original KKLT construction we present a generalization of this scheme which captures a large class of string theory models. The analysis is carried out in the framework of a low energy SUGRA approximation of type IIB string theory. We identify the source of the SUSY breakdown by computing the moduli F-terms. The properties of the emerging soft gaugino and scalar masses as well as their RG evolution are examined. We identify criteria for the appearance of the mirage pattern and discuss the implications for the low energy phenomenology.

2.1 The model of KKLT

The starting point of the construction is the 10D SUGRA expansion (to leading order in \( \alpha' \) and \( g_{\text{STR}} \)) of type IIB string theory \([33–36]\) compactified on CY orientifolds \([67]\) with fluxes\(^1\) \([45,46]\). From here, the analysis of the model is carried out in the framework of a 4D \( \mathcal{N}=1 \) low energy SUGRA approximation just below the compactification scale \( M_{\text{COMP}} \). We will work with a universal Kähler modulus \( T \) as the generalization to multi Kähler moduli is straightforward.

In the standard \( \mathcal{N}=1 \) SUGRA language \([7–13]\) it is convenient to express the scalar potential and the auxiliary fields in terms of the real Kähler function

\[
G = K + \log\left(W \overline{W}\right),
\]

with \( K \) and \( W \) being the Kähler potential and the superpotential, respectively, as

\[
V = \phi^G \left[K^{-1}_{IJ} G_I G_J - 3\right],
\]

where the subscripts \( I, J \) denote differentiation with respect to the moduli (and other hidden sector fields) and \( K^{-1}_{IJ} \) is the inverse Kähler metric. Here and from now on (unless stated otherwise), we work in SUGRA units where \( M_P = (8\pi G)^{-1/2} \equiv 1 \). The SUSY breaking F-terms are found from

\[
F^I = \phi^{G/2} K^{-1}_{IJ} G_J,
\]

\(^1\)Fluxes are vacuum expectation values of certain field strengths in the compact space.
Road to mirage mediation

and the gravitino mass is

\[ m_{3/2} = \frac{G}{2}, \tag{2.4} \]

both evaluated at the minimum. SUSY is broken spontaneously if one of the \( F^I \) is non-zero, which in turn depends on whether \( G_I \) is non-zero or not. Furthermore, the gauge kinetic function \( f_a \) determines the gauge coupling constants \( g_a \) of the theory through

\[ \frac{1}{g_a^2} = \Re f_a. \tag{2.5} \]

Due to the presence of fluxes a non-trivial superpotential \( W(S, U) \) for the dilaton \( S \) and the CSM \( U \) is generated [68,69]. The corresponding Kähler potential at tree-level is given by [48,70–72]

\[ K = - \log \left(S + \bar{S}\right) - 3 \log \left(T + \bar{T}\right) + \mathcal{K}(U, \bar{U}). \tag{2.6} \]

This leads to the scalar potential

\[ V = e^G K^{-1} \alpha^a \beta G^a_{\alpha} G^\beta_{\beta} \geq 0, \tag{2.7} \]

where the subscripts \( \alpha, \beta \) run over all moduli except \( T \). As the superpotential is independent of \( T \), the contribution form eq. (2.6) cancels the \(-3\) in eq. (2.2) resulting in the no-scale structure [73,74] scalar potential eq. (2.7). Since \( T \) is not stabilized by the flux dynamics it has a flat potential.

The dilaton and the CSM are stabilized and acquire certain masses. Their masses as well as their VEVs depend on the choice of fluxes [47]. At this stage SUSY is broken, since generically \( \langle G_I \rangle \neq 0 \) in the minimum. The flux induced gravitino mass \( (m_{3/2})_{\text{flux}} \) depends on the alignment of flux vacua [47]. For a specific alignment of flux vacua, which corresponds to a fine-tuning, one may achieve that \( S \) and \( U \) acquire Planckian VEVs and masses of order of the compactification scale \( M_{\text{comp}} \) which is rather close to \( M_P \). At the same time such alignment of fluxes makes the gravitino much lighter than the dilaton and the CSM, i.e. \( (m_{3/2})_{\text{flux}} \ll m_S, m_U \). This is a welcome feature for the realization of low scale SUSY. In addition to the stabilization of \( S \) and \( U \), fluxes also generate a warped geometry [69]. In the most of the CY the warping is not significant except for a small region containing the so called Klebanov–Strassler throat [75] where the warping becomes exponentially large (see fig. 2.1). This ensures that the dynamics of the moduli (and other bulk degrees of freedom) are not significantly affected by the warping.

Since \( T \) is not stabilized by perturbative dynamics, KKLT introduce non-perturbative effects in order to violate the no-scale structure. These effects can originate from gaugino condensates [21–26,76] and/or instantons [77] on D7 and/or D3 branes. Both of these effects generate an exponential superpotential for the Kähler modulus. Furthermore, KKLT assume that \( T \) is much lighter than
the super-heavy $S$ and $U$. This allows one to formulate an effective SUGRA theory just below $m_S$, $m_U$ by integrating out $S$ and $U$. For concreteness, consider gaugino condensation on D7 branes. This will introduce a superpotential of the form $W = W_{\text{flux}} - A e^{-aT}$, (2.8)

where the quantized constant $W_{\text{flux}} = \langle W(S, U) \rangle$ is obtained in the process of integrating out $S$ and $U$, $A = O(1)$ and $a = 8\pi^2/N$ for a SU($N$) gauge group. The effective Kähler potential is now given by

$$K = -3 \log (T + \bar{T}).$$

(2.9)

The effective theory described by eqs. (2.8) and (2.9) stabilizes $T$ in an AdS vacuum (cf. fig. 2.2). Moreover, SUSY is restored by this procedure since in the minimum $G_T(T_0) = 0$ and $W(T_0) \neq 0$ with $T_0$ denoting the position of the minimum. This $\mathcal{N} = 1$ AdS vacuum exhibits some unique features [47,48]:

$$a T_0 \approx \log \left( \frac{M_p}{m_{3/2}} \right),$$

(2.10)

$$\left( m_{3/2} \right)_{\text{flux}} \approx \frac{W_{\text{flux}}}{M_p^2},$$

(2.11)

$$m_T \approx (a T_0) m_{3/2},$$

(2.12)

$$\langle V_{\mathcal{N}=1} \rangle = -3 m_{3/2}^2 M_p^2,$$

(2.13)

where $T$ is assumed real. Since we are aiming at low scale SUSY we assume to have the appropriate flux configuration realizing $W_{\text{flux}} = O(10^{-14} M_p^3)$. Then eq. (2.10) implies $a T_0 = O(4\pi^2)$ and the mass of the $T$ modulus appears to be enhanced by this moderately large quantity relative to the gravitino mass parameter.

Although KKLT manage to stabilize all moduli under suitable theoretical assumptions, one hurdle is still to overcome: the construction of a phenomenologically desirable SUSY breaking dS/Minkowski vacuum [17,18]. To this end KKLT introduce anti D3 branes ($\overline{D3}$) [78] which favor [53] to be stabilized at the tip of the Klebanov–Strassler throat where the geometry is highly warped (see fig. 2.1). On the other hand, SM fields are assumed to live on D7 and/or D3 branes in a region with negligible warping. On the $\overline{D3}$ the $\mathcal{N} = 1$ SUSY, which is preserved by the combined dynamics of fluxes and gaugino condensation, is broken explicitly [79].

In order to describe the couplings between the $\mathcal{N} = 0$ sector on the $\overline{D3}$ and the $\mathcal{N} = 1$ sector on the D7/D3 a superconformal (off-shell) formulation [8] of supergravity is required. Then, the effective action of the $\mathcal{N} = 1$ supersymmetric

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2Note that $W_{\text{flux}}$ is quantized as it originates from fluxes which are quantized objects.
Road to mirage mediation

**Fig. 2.1:** The KKLT setup with visible fields on D7 branes. The warping along the compact dimension $y$ is described by the factor $e^{2A(y)}$. The D3 brane is stabilized at the tip of the KS throat.

part is described by [47]

\[
\mathcal{S}_{N=1} = - \int d^4x \sqrt{g^C} \left[ \int d^4 \theta \bar{C}C 3^{\frac{1}{3}} K_{\text{EFF}}^{\frac{1}{3}} \right. \\
\left. - \left\{ \int d^2 \theta \left( \frac{1}{4} f^a_{\alpha \beta} \Xi^a_{\alpha \beta} + C^3 W_{\text{EFF}} \right) + \text{h.c.} \right\} \right],
\]

where $g^C_{\mu \nu}$ is the 4D metric in the superconformal frame which is related to the metric in the Einstein frame via $g^C_{\mu \nu} = (\mathcal{C}C)^{-1} g_{\mu \nu}^{\text{E}}$, $C = C_0 + \theta^2 F_C$ is the chiral conformal compensator superfield of 4D $\mathcal{N} = 1$ SUGRA and $\Xi^a_{\alpha \beta}$ is the spinorial gauge field strength. The effective Kähler potential and superpotential, to leading order, are given by

\[
K_{\text{EFF}} = K(T, \bar{T}) + Q_i \bar{Q}^i Z_i(T, \bar{T}), \tag{2.15}
\]

\[
W_{\text{EFF}} = W(T) + \lambda_{ijk} Q^i Q^j Q^k, \tag{2.16}
\]

where $Q^i$ denote visible matter superfields, $\lambda_{ijk}$ are the holomorphic Yukawa couplings assumed to be moduli independent, $Z_i$ describes the Kähler metric of the visible fields and

\[
K(T, \bar{T}) = -3 \log \left( T + \bar{T} \right), \tag{2.17}
\]

\[
W(T) = W_{\text{FLUX}} - A \psi^{-a} T. \tag{2.18}
\]

The impact of the D3 branes on the low energy dynamics of the (light) moduli and visible fields can be described in 4D SUGRA by spurion operators [47]. These are non-dynamical fields parameterizing the explicit breakdown of SUSY. Then to leading order

\[
\mathcal{S}_{\text{D3}} = \int d^4x \sqrt{g^C} \int d^4 \theta \left[ - C^2 \bar{C}^2 \theta^2 \bar{\theta}^2 P_{\text{D3}} + C^3 \bar{\theta} R_{\text{D3}} + \text{h.c.} \right], \tag{2.19}
\]

where $P_{\text{D3}}$ and $R_{\text{D3}}$ denote the (model-dependent) spurion operators. The spurion $R_{\text{D3}}$ is suppressed with respect to $P_{\text{D3}}$ in terms of the warp factor which at
the location of the $\mathcal{D}3$ branes is supposed to be exponentially small. As far as $\mathcal{D}3$ branes are concerned we have [47]

\[
\mathcal{P}_{\mathcal{D}3} = \kappa, \\
\mathcal{R}_{\mathcal{D}3} = \varsigma,
\]

where $0 < \varsigma < \kappa$ are related to the warp factor. Including the spurions, the low energy effective action of KKLT is given by $\mathcal{S}_{\text{tot}} = \mathcal{S}_{N=1} + \mathcal{S}_{\mathcal{D}3}$. The scalar potential induced by the $\mathcal{D}3$ branes is given by [47,78]

\[
V_{\mathcal{D}3} = \frac{\kappa^{2k/3}}{(T + T_0)^2}, \tag{2.22}
\]

implying that the presence of $\mathcal{D}3$ branes provides a positive contribution to the total scalar potential. Adding the energy from the $\mathcal{D}3$ branes to the negative vacuum energy induced by gaugino condensation eq. (2.13) thus leads to an uplifting of the AdS minimum. The adjustment of the vacuum energy to the observed value of the CC, i.e. $\langle V_{\text{tot}} \rangle \approx 0$, is achieved with $\kappa = \mathcal{O}(m_{3/2}^2 M_P^2)$.\footnote{Even though the CC is not zero but very close to it, for our purposes this “slightly” dS vacuum can be very well approximated by a Minkowski vacuum.} Fig. 2.2 illustrates the relevant scales. One should, however, note that such an adjustment requires a very careful fine-tuning of $\kappa$. Since $\kappa$ is connected to the warp factor, constraints will be posed on the warping [47].

After uplifting the AdS minimum we obtain a dS/Minkowski vacuum with broken SUSY. The breaking of SUSY can be understood as follows. Before uplifting $T$ was stabilized at $T_0$ in a supersymmetric AdS minimum, thus $G_T(T_0) = 0$. The $N = 1$ SUGRA potential is exponentially steep around $T_0$ unlike the uplifting potential (cf. fig. 2.2). As a consequence, the addition of the uplifting potential only slightly moves the position of the $T$ modulus to $T_0 + \delta T$. At this new
minimum $G_T$ does no longer vanish, $G_T(T_0 + \delta T) \neq 0$. Therefore $F^T$ is non-zero at the new minimum, signaling the breakdown of SUSY. The mass of the gravitino, however, is essentially left intact in going from the AdS minimum to the new dS minimum. The same applies to the mass of $T$ [47, 50]. The only effect of the uplifting sector is to change (uplift) the vacuum energy to a small positive or zero value resulting in a SUSY breaking vacuum shift. Moduli stabilization is not affected by this procedure.

The sources of SUSY breakdown are VEVs of the auxiliary components of the moduli. In case of KKLT we have $F^S, F^U$ and $F^T$. In addition to the moduli, the 4D SUGRA multiplet (parameterized by the conformal compensator $C$) provides a model-independent source of SUSY breaking. The auxiliary component $F^C$ induces soft terms only at loop level by the mechanism of anomaly mediation [51, 52, 80]. In order to identify the dominant sources of the soft breaking terms one has to solve the equations of motion for the auxiliary fields in the total $\mathcal{S}_{TOT} = \mathcal{S}_{N=1} + \mathcal{S}_{D3}$ system. Since $S$ and $U$ acquire huge masses compared to $T$, $m_S, m_U \gg m_T$, their contribution to SUSY breakdown remains negligible as can be read off from $F ^2 \sim m_T^2 / m_I$. Thus we only have to deal with $F^T$ and $F^C$.

In the Einstein frame the auxiliary components take the general form [47]

\[
\frac{F^C}{C_0} = e^{\tilde{G}/2} + \frac{1}{3} F^I \partial_I K, \tag{2.24}
\]

\[
F^m = e^{\tilde{G}/2} K^{-1}_I \tilde{G}_I, \tag{2.25}
\]

and the (moduli) scalar potential becomes

\[
V_{TOT} = e^{\tilde{G}} \left( K^{-1}_I \tilde{G}_I \tilde{G}_I - 3 \right) + e^{2k/3} \mathcal{P}_{D3}, \tag{2.26}
\]

with the modified Kähler function

\[
\tilde{G} = K + \log |W + R_{D3}|^2. \tag{2.27}
\]

As noticed above, in order to obtain the desired value of the CC a very special warp factor has to be chosen in the process of fine-tuning $\kappa$. As studied in [47], in a Minkowski/dS vacuum the warp factor will make the $R_{D3}$ contribution to eqs. (2.24)–(2.27) negligible. In this case $\tilde{G} \approx G$ and the SUSY breaking $F$-terms are well approximated by the standard $\mathcal{N} = 1$ SUGRA expressions. The scalar potential takes the simple structure

\[
V_{TOT} \approx V_{N=1} + V_{D3}. \tag{2.28}
\]

### 2.2 Generalization of the KKLT model

It has been observed [47, 48, 53] that the model of KKLT represents a specific example of a more general scenario with all moduli fixed, realizing low energy SUSY at the TeV scale with a phenomenologically viable value of the CC. The construction contains three building blocks:
Most of the moduli are stabilized by high scale dynamics and acquire super-heavy masses. The $\mathcal{N} = 1$ SUSY is broken.

The remaining few light moduli are stabilized by invoking non-perturbative effects. These non-perturbative effects dynamically cancel the previously induced SUSY breaking, yielding a $\mathcal{N} = 1$ supersymmetric AdS vacuum.

This supersymmetric AdS vacuum is lifted to a SUSY breaking dS/Minkowski vacuum by an appropriate uplifting mechanism which is assumed to be sequestered from the visible sector.

For models constructed in the framework of type IIB string theory high scale dynamics (e.g. fluxes) typically generate a potential for the dilaton and the CSM but not for the Kähler moduli. Under suitable assumptions one can arrange for a decoupling of $S$ and $U$ and formulate an effective theory for the (light) Kähler moduli $T_i$ [48,81,82]. For simplicity, we consider one single Kähler modulus $T$. This class of models exhibits some interesting features:

- The non-perturbative superpotential providing the stabilization of the light moduli has the model independent structure [50]

$$W = W_0 - \frac{A}{x^{aT}},$$

where $a$ is moderately large, $A = O(1)$ and $W_0$ is the remnant from the stabilization of heavy moduli. Its value has to be chosen small compared to $M_P$ so as to provide low scale SUSY. The parameters $A$ and $W_0$ can in general be complex and thus present a potential source of dangerous CP violation. However, the superconformal formulation of the 4D $\mathcal{N} = 1$ SUGRA possesses a $U(1)_R$ as well as an axionic shift symmetry which can be used to make $A$ and $W_0$ real and positive [47,83].

- The supersymmetric AdS minimum appears at

$$\text{Re} T_0^{\text{AdS}} \approx -\frac{1}{a} \log \frac{W_0}{aA},$$

with the property [47,48]

$$a \text{Re} T_0^{\text{AdS}} \approx \log \left(\frac{M_P}{m_{3/2}}\right),$$

The mass of $T$ is enhanced by this factor with respect to $m_{3/2}$. This enhancement of moduli masses is known to be a rather generic feature of non-perturbative moduli stabilization [84,85]. In particular it occurs when the moduli dependence is logarithmic in the Kähler potential and exponential in the superpotential [50].
Sequestering\textsuperscript{4} \cite{51,88,89} of the uplifting sector means that the communication of the SUSY breakdown is more suppressed than by inverse powers of $M_P$. The low energy consequence of the sequestered SUSY breaking can be represented by a spurion operator $P_{\text{LIFT}}$ \cite{47} which is a model dependent object. In the effective action $P_{\text{LIFT}}$ mimics explicit SUSY breaking in a sequestered sector and provides a model dependent uplifting scalar potential

\[ V_{\text{LIFT}} = \kappa^{2K/3} P_{\text{LIFT}}. \]  

(2.32)

As pointed out in \cite{53} the spurion operator may be parameterized by

\[ P_{\text{LIFT}} = \kappa \left( T + \overline{T} \right)^{n_{\mu}}, \]  

(2.33)

where $\kappa$ is a positive constant and $n_{\mu}$ is a rational number. With the standard tree-level Kähler potential eq. (2.17) one has

\[ V_{\text{LIFT}} = \kappa \left( T + \overline{T} \right)^{2-n_{\mu}}. \]  

(2.34)

In order to describe the present stage of the acceleration of the universe \cite{17,18} the vacuum energy of the dS minimum needs to be fine-tuned to $\langle V_{\text{TOT}} \rangle = V_0 \sim 10^{-120} M_P^4$. This can be achieved by fine-tuning $\kappa$ in eq. (2.34). The obtained dS vacuum (at finite volume) is separated from the Minkowski vacuum (at infinite volume of the internal space) by a barrier which is approximately given by the depth of the AdS minimum\textsuperscript{5} (see fig. 2.2). Thus, in principle, the dS vacuum can be destabilized by tunneling effects. Given the height of the barrier $O(m_3^2 M_P^2)$ the lifetime of the dS vacuum was shown to be about $10^{10^{120}}$ years \cite{90}, thus for all practical purposes it can be considered as completely stable. This generalizes to any construction where a dS minimum is separated from the run-away vacuum by a potential which remains positive.

The $F$-terms of the heavy moduli are practically negligible as can be seen from $F_{\text{HEAVY}} \sim m_{\text{3/2}}^2/m_{\text{HEAVY}} \ll m_{\text{3/2}}$. Since SUSY breaking fields in the uplifting sector are (assumed to be) sequestered, they do not have cross-couplings with any other sector fields. Hence their $F$-terms are irrelevant to visible soft terms. The only relevant effect of sequestered SUSY breaking is to provide a positive contribution, eq. (2.34), which serves to uplift the

\textsuperscript{4}In general, the SUSY breaking in the uplifting sector would give rise to soft SUSY breaking terms through effective couplings between visible and uplifting sector fields. Soft terms can involve flavor violation \cite{86,87} which is restricted by experiments. Hence, in order to avoid an additional source of flavor violation, these couplings should be strongly suppressed. In the KKLT setup sequestering is introduced through warping.

\textsuperscript{5}This is because the uplifting potential is a slowly decreasing function whereas the $N = 1$ potential exponentially approaches zero at large $T$. 
minimum of the potential to a dS/Minkowski vacuum. The SUSY breaking vacuum shift induced by $V_{\text{1st}}$ is small \[50\]

$$\delta T \sim \frac{1}{a^2} = O\left(\frac{1}{(4\pi^2)^2}\right),$$  \hspace{1cm} (2.35)

implying that the minimum of the total scalar potential is at

$$T_0 \equiv T_0^{\text{SUSY}} = T_0^{\text{AdS}} + \delta T \approx T_0^{\text{AdS}},$$  \hspace{1cm} (2.36)

such that $T$ is stabilized close to a supersymmetric point.

- Minimization of the total potential under the fine-tuning for $\langle V_{\text{TOT}} \rangle \approx 0$ straightforwardly yields \[53\]

$$\frac{F^T}{T_0 + \bar{T}_0} \approx \frac{2 - n_p}{2\ a \Re T_0} m_{3/2},$$ \hspace{1cm} (2.37)

$$m_T \approx (a \Re T_0) m_{3/2},$$ \hspace{1cm} (2.38)

$$\frac{F^C}{C_0} \approx m_{3/2} \left(1 - \frac{2 - n_p}{2\ a \Re T_0} \frac{1}{m_{3/2}}\right),$$ \hspace{1cm} (2.39)

with

$$a \Re T_0 \sim \log\left(\frac{A}{W_0}\right) \sim \log\left(\frac{M_P}{m_{3/2}}\right).$$ \hspace{1cm} (2.40)

Thus, the mass scales in the low energy effective theory exhibit a moderate hierarchy. This so called little hierarchy \[47,48,50\] is characterized by the logarithm of the large hierarchy between $M_P$ and $m_{3/2} = O(\text{TeV})$ and is numerically $O(4\pi^2)$. The $F$-term of the light modulus is suppressed by $a \Re T_0$ while its mass is enhanced by the same factor with respect to the gravitino mass. Note that the $F$-terms of the light modulus and the 4D SUGRA compensator possess model-dependence through $n_p$.

- This scheme, although quite general, leads to a specific pattern of mass scales \[47\]

$$M_{\text{STR}} \approx 5 \times 10^{17} \text{ GeV},$$ \hspace{1cm} (2.41)

$$M_{\text{COMP}} \approx 10^{17} \text{ GeV},$$ \hspace{1cm} (2.42)

$$m_S, m_U \approx 10^{16} \text{ GeV},$$ \hspace{1cm} (2.43)

$$m_T \approx 10^6 \text{ GeV},$$ \hspace{1cm} (2.44)

$$m_{3/2} \approx 10^4 \text{ GeV}.$$ \hspace{1cm} (2.45)

Given the idiosyncrasy of this scheme we would like to analyze the pattern of the emerging soft terms and discuss its consequences for phenomenology.
2.3 Soft masses in the KKLT scheme

The soft terms induced just below the compactification scale $M_{\text{COMP}}$ receive contributions from modulus mediation, superconformal anomaly mediation as well as loop contributions coming from Kähler anomalies and string threshold corrections. In principle also field theoretic gauge threshold corrections below $M_{\text{COMP}}$ are possible. In this work we neglect them by assuming that there are no thresholds between $M_{\text{COMP}}$ and the TeV scale. In what follows we denote the energy range around 1 TeV symbolically by $M_{\text{TeV}}$.

2.3.1 Soft gaugino masses

Let us begin with the soft gaugino masses. Just below the compactification scale one has

$$M_a = M_{\text{MODULUS}}^a + M_{\text{ANOMALY}}^a + M_{\text{KÄHLER}}^a + M_{\text{STRING}}^a$$

$$= \frac{1}{2 \text{Re} f_a} F^T \partial_T f_a + \frac{b_a g_a^2}{4} \frac{F^C}{C_0} + O\left(\frac{F^T}{4\pi^2}\right), \quad (2.46)$$

where $f_a$ are the gauge kinetic functions of the visible fields, $b_a$ are the 1-loop $\beta$-function coefficients at $M_{\text{TeV}}$ and $g_a$ denotes the gauge coupling at $M_{\text{COMP}}$. The modulus/gravity mediated contribution (first term in eq. (2.46)) depends on the structure of the gauge kinetic functions and can be present at tree and/or loop level or even be absent. The anomaly mediated part (second term in eq. (2.46)) is always present and contributes at loop level [51, 52, 80]. Furthermore, $M_{\text{ANOMALY}}^a$ is determined by the matter content at the TeV scale.

To proceed further one needs to specify the location of the visible sector. At tree-level the gauge kinetic functions of the visible gauge fields generically are given by

$$f_a = k_a T^{l_a}, \quad (2.47)$$

and are related to the gauge coupling constants at the compactification scale via

$$\text{Re} f_a = \frac{1}{G_a^2 (M_{\text{COMP}})}, \quad (2.48)$$

where $a$ labels the gauge group, $k_a$ are integers of order unity and $l_a$ depends on the location of the visible gauge fields. If the visible gauge fields originate from D3 branes $l_a = 0$ whereas $l_a = 1$ for gauge fields on D7 branes [96–98]. From this one arrives at

$$M_a = \frac{l_a}{T_0 + \overline{T}_0} F^T + \frac{b_a g_a^2}{4} \frac{F^C}{4\pi^2 C_0} + O\left(\frac{F^T}{4\pi^2}\right). \quad (2.49)$$

For matter fields on D3 branes two difficulties arise. First, the resulting soft terms will be dominated by anomaly mediation which is plagued by tachyonic
sleptons \([99, 100]\). Second, in the KKLT framework the stabilization of the position of D3 branes appears to be problematic \([53]\). In view of these difficulties it is more appealing to consider visible sector fields living on D7 branes \((l_\alpha = 1)\) where both \(F_T\) and \(F_C\) contribute to the soft terms.

Even though \(g_a^2(M_{\text{COMP}})\) are in general non-universal, the modulus mediated part at leading order provides a universal contribution to the gauginos \([92]\). On the other hand, anomaly mediation is non-universal and requires a specification of the matter content at the TeV scale. As is well known, the extrapolation of the low energy data within the MSSM yields an almost perfect unification of the SM gauge couplings at \(M_{\text{GUT}} \approx 2 \times 10^{16} \text{GeV}\) with \(g_1 = g_2 = g_3 = 1/2\) \([27-30]\). Furthermore, for \(\Theta T = O(1)\) \(M_{\text{COMP}}\) is close to \(M_{\text{GUT}}\) \([101-103]\). Therefore it is reasonable to adopt the MSSM particle content and assume that \(g_a\) are unified at \(M_{\text{COMP}} \approx M_{\text{GUT}}\) with

\[
  g_a^{-2}(M_{\text{COMP}}) \approx g_a^{-2}(M_{\text{GUT}}) = g_{\text{GUT}}^{-2} \approx 2. \quad (2.50)
\]

Now that we have specified the matter content at \(M_{\text{TeV}}\) we can continue the discussion of the gaugino masses, eq. \((2.49)\). The SUSY breaking parameters in the KKLT setup are controlled by the little hierarchy eq. \((2.40)\). If \(m_{3/2}\) is of order of the TeV scale, \(\Theta T_0 = O(4\pi^2)\) is comparable to a 1-loop suppression factor. In this case modulus mediated contribution \(F_T/(T_0 + \bar{T}_0)\) is suppressed by this (loop like) factor against \(m_{3/2}\). On the other hand, since anomaly mediation enters the soft terms at loop level, the contribution from the SUGRA compensator is suppressed by a 1-loop factor. From eq. \((2.39)\) we also know \(F_C/C_0 \sim m_{3/2}\) and thus the contribution from anomaly mediation becomes equally important to the tree level modulus mediation.

Since the contributions from Kähler anomalies and string threshold corrections involve \(F_T\) at loop level they will be doubly suppressed and thus sub-leading. As a consequence, the soft gaugino masses at \(M_{\text{GUT}}\) are dominated by comparable contributions from modulus and anomaly mediations. However, the balance between these two contributions to \(M_a\) will in general depend on further details of the particular model.

For the study of the mixed modulus-anomaly mediation it is convenient to introduce the following parameterization \([55]\)

\[
  \varrho \equiv \frac{F_T}{T_0 + \bar{T}_0} \frac{C_0}{F_C}, \quad (2.51)
\]

\[
  M_0 \equiv \frac{m_{3/2}}{16\pi^2}, \quad (2.52)
\]

which by applying eqs. \((2.37)\) and \((2.39)\) becomes

\[
  \varrho \equiv 2 - n_p - \frac{16\pi^2}{2 \log(M_P/m_{3/2})}, \quad (2.53)
\]
The parameter $\rho$ measures the ratio between modulus and anomaly mediation and $M_0$ denotes the characteristic scale of the gaugino/soft masses. Given this parameterization eq. (2.49) can be recast as

$$M_a = M_0 \left[ \rho + b_a g^2_{\text{GUT}} \right],$$

with $b_a = (33/5, 1, -3)$ for the MSSM matter content (cf. appendix B). Note that $\rho \to 0$ corresponds to pure anomaly mediation whereas for $\rho \gg 1$ modulus mediation is dominating.

The values of the gaugino masses at the TeV scale are obtained via RG evolution of the boundary condition eq. (2.56) given just below $M_{\text{GUT}}$. Taking into account 1-loop RG running\(^6\) (c.f. eqs. (C.1) and (C.2)) the gaugino masses have a simple relation to the RG of the SM gauge couplings, namely the quantity $M_a^2/g_a^2$ does not run at 1-loop. Hence, at the renormalization point $M_{\text{TeV}} \leq \mu \leq M_{\text{GUT}}$ one obtains [53, 55]

$$M_a(\mu) = \rho M_0 \left[ 1 + \frac{b_a g_a^2(\mu)}{8\pi^2} \log \left( \frac{\mu}{M_{\text{GUT}}} \right) \right],$$

where $g_a^2(\mu)$ are the running gauge couplings at the scale $\mu$. This result allows one to draw an immediate conclusion: the gaugino masses unify. However, this does not happen at $M_{\text{GUT}}$ but at the intermediate scale [53, 55]

$$M_{\text{MIR}} = M_{\text{GUT}} e^{-\frac{b_a}{\sigma}}.$$

At this scale the RG evolution of the gaugino masses is canceled by the anomaly mediated part, leading to a unification of the gaugino masses at $M_{\text{MIR}}$. As there is no physical threshold associated with this scale, it is called mirage scale [50, 53].

From eq. (2.56) we see that at the GUT scale gaugino masses receive a universal contribution from modulus mediation and a non-universal one from anomaly mediation which is specified by the respective (1-loop) $\beta$-function coefficients $b_a$. The (1-loop) RG running eq. (2.57) is governed by the same $\beta$-functions. Thus, at an intermediate scale the splitting disappears yielding a mirage unification of the gaugino masses. This leads to the conclusion that the low energy gaugino masses in the mixed modulus-anomaly mediation with messenger scale $M_{\text{GUT}}$ are (approximately) the same [53, 55] as those of pure modulus mediation with the intermediate messenger scale $M_{\text{MIR}}$. In order for the mirage scale to be “truly”

\(^6\) We assume there are no thresholds between the TeV and GUT scale. Evolution of the soft parameters below $M_{\text{low}}$ requires threshold corrections from heavy states being integrated out at the TeV scale.
mirage $\varrho = O(1)$ otherwise, if $\varrho \gg 1$ $M_{\text{mir}}$ coincides with $M_{\text{GUT}}$ (pure modulus mediation) and for $\varrho \to 0$ (pure anomaly mediation) there is no unification at all. To sum up, in a mixed modulus-anomaly mediation the parameter $\varrho$ tells us where the gaugino masses coincide and thus the size of $M_{\text{mir}}$. The precise value of $\varrho$ is of course model-dependent. In particular, it will depend on the shape of the uplifting potential and on the values of $M_{\text{str}}$ and $m_{3/2}$ [48, 53, 55]. The original KKLT setup with $n_p = 2$ predicts

$$\varrho^{(\text{KKLT})} \approx 4.8 \ldots 6,$$

$$M_{\text{mir}}^{(\text{KKLT})} \approx 10^9 \ldots 10^{11} \text{GeV}. \quad (2.59)$$

This particular case is illustrated in figs. 2.3.b and 2.3.c.

Mixed modulus-anomaly mediation exhibits some unique features different from other mediation schemes. Here, we shall point out two of them by considering gaugino mass ratios at the GUT and the TeV scales:

@ $M_{\text{GUT}}$ :: $M_1 \div M_2 \div M_3 \approx |\varrho + 3.3| \div |\varrho + 0.5| \div |\varrho - 1.5|$, \quad (2.61)

@ $M_{\text{GUT}}$ :: $M_1 \div M_2 \div M_3 \approx |\varrho + 3.3| \div |2\varrho + 1| \div |6\varrho - 9|$, \quad (2.62)

where $M_1$ is bino, $M_2$ is wino and $M_3$ is gluino. As already stated above, at the GUT scale all gaugino masses receive the same contribution from modulus mediation but different one from anomaly. Due to the large negative $\beta$-function coefficients of the SU(3)$_c$, the gluino is the lightest gaugino at the GUT scale. Clearly, this does not hold in the limiting cases $\varrho \to 0$ and $\varrho \gg 1$. For $\varrho^{(\text{KKLT})}$ eq. (2.59) one typically has $M_1 \div M_2 \div M_3 \approx 2.4 \div 1.6 \div 1$ at $M_{\text{GUT}}$. Due to the large negative $b_3$ the RG evolution will make the gluino the heaviest gaugino around the TeV scale. The opposite applies to the bino. It receives a large positive contribution from anomaly mediation and RG makes it the lightest gaugino around the TeV scale. Hence, the ratio of the gauginos at the TeV scale will be inverted with respect to the ratio at the GUT scale. For KKLT this gives $M_1 \div M_2 \div M_3 \approx 1 \div 1.3 \div 2.5$. Another interesting aspect of the scheme is that for $\varrho \approx 2.6$ the mirage unification of the gauginos occurs at the TeV scale providing a striking pattern of the soft parameters$^7$ [53]. It is important to emphasize that the appearance of mirage unification does not require gauge coupling unification. Mirage unification of the gaugino masses results from the RG evolution of the boundary conditions in the mixed modulus-anomaly mediation where both mediations are of comparable magnitude. Of course one can include a really unified theory like a SU(5) GUT [31]. In such a case there will be a unified gauge coupling and the unified soft masses above the GUT scale. Just below the GUT scale the boundary conditions eq. (2.56) split the gaugino masses. These two limits are reconciled via threshold effects at the GUT scale. As illustrated in fig. 2.3.b gaugino masses experience two unifications: the true unification at/above the GUT scale and a mirage unification at $M_{\text{mir}}$ [104].

---

$^7$Such value of $\varrho$ could originate from an uplifting potential eq. (2.34) with $n_p = 1$. 
2.3.2 Soft scalar squared masses

The structure of the soft scalar squared masses induced just below the GUT scale is in general more complicated compared to the gauginos and also involves a stronger model-dependence [47, 91, 92]. Nonetheless, the contributions from Kähler anomalies and string threshold corrections are sub-leading due to the suppression of $F^T$. Thus, the soft scalar squared masses are dominated by pure modulus contribution at tree-level and pure anomaly as well as mixed modulus-anomaly contribution at 1-loop and 2-loop levels. Moreover, the soft scalar squared masses depend on the location of the visible matter fields through the Kähler metric $Z_i$ which in general depends on the moduli through [47, 53]

$$Z_i = (T + ar{T})^{-n_i},$$

(2.63)
where \( n_i \) are the so-called effective modular weights. With matter fields on D7 branes \( n_i = 0 \) and the soft scalar squared masses are given by [47,48]

\[
m_i^2 = (m_i^{\text{MOMULUS}})^2 + (m_i^{\text{ANOMALY}})^2 + (m_i^{\text{MIXED}})^2
\]

\[
= \left( \frac{F^T}{T_0 + \overline{T}_0} \right)^2 - \left( \frac{g^4_{\text{GUT}}}{8} \sum_a b_a C_i^a - \frac{1}{16} \sum_{jk} |y_{ijk}|^2 b_{y_{ijk}} \right) \left( \frac{1}{4\pi^2 C_0} \right)^2
\]

\[
+ \left( -g^2_{\text{GUT}} \sum_a C_i^a + \frac{3}{2} \sum_{jk} |y_{ijk}|^2 \right) \left( \frac{F^T}{4\pi^2} \frac{F^C}{4\pi^2 C_0} \right), \quad (2.64)
\]

where \( C_i^a \) are the quadratic Casimirs of the matter representation and \( b_{y_{ijk}} \) are the \( \beta \)-functions of the Yukawa couplings \( y_{ijk} \) eqs. (C.3) – (C.5). Again, from eqs. (2.37) and (2.39) we know that the tree-level modulus contribution is suppressed against \( m_3/2 \) by the little hierarchy which is \( \mathcal{O}(4\pi^2) \) and \( F^C \sim m_3/2 \). Hence, all terms in eq. (2.64) are of comparable size indicating that also the soft scalar masses experience a mixed modulus-anomaly mediation. Using our parameter-ization eqs. (2.51) and (2.52) we recover

\[
m_i^2 = M_0^2 \left[ g^2 - \left( 2g^4_{\text{GUT}} \sum_a C_i^a - \sum_{jk} |y_{ijk}|^2 b_{y_{ijk}} \right) \right]
\]

\[
+ 2g \left( T_0 + \overline{T}_0 \right) \left( -g^2_{\text{GUT}} \sum_a C_i^a + 3 \sum_{jk} |y_{ijk}|^2 \right). \quad (2.65)
\]

As far as the sfermions are concerned we have to distinguish between the first two generations and the third generation scalars. For the first two generations the effect of the Yukawa couplings can be neglected\(^6\) in eqs. (2.64) and (2.65). In this approximation the 1-loop RG yields [53,55]

\[
m_i^2(\mu) \equiv g^2 M_0^2 \left[ 1 + \frac{2C_i^a}{b_a} - \frac{2C_i^a}{b_a} M_2^2(\mu) \right]
\]

\[
= g^2 M_0^2 \left[ 1 - \frac{C_i^a g^2(\mu)}{4\pi^2} \log \left( \frac{\mu}{M_{\text{GUT}}} \right) \frac{\pi}{\sqrt{g}} \right], \quad (2.66)
\]

\[
\text{for } M_{\text{UV}} \leq \mu \leq M_{\text{GUT}}. \quad \text{From eq.}(2.67)\text{ we can conclude that at 1-loop RG in the limit of vanishing Yukawa couplings the sfermion masses of the first two generations unify at the same intermediate scale } M_{\text{Mir}} \text{ eq.}(2.58) \text{ as the gaugino masses do. Moreover, due to } M_{\text{q}}(M_{\text{Mir}}) = \varrho M_0 \text{ and } m_i^{(1)(2)} \approx \varrho M_0 \text{ the masses of the gauginos and the fermions of the first two generations are approximately the same (cf. fig. 2.3.c)}.
\]

\(^6\)This is obviously a good approximation within the MSSM. See e. g. [10,11].
For the third generation sfermions and the Higgses we cannot neglect the Yukawa couplings. In this case (even) the 1-loop RG becomes quite complicated yielding \( m_i^{(3)}(M_{\text{Mir}}) \neq 0 M_0 \neq m_i^{\text{Higgs}}(M_{\text{Mir}}) \). Thus, whenever a scalar feels the effect of Yukawa couplings mirage unification does not hold anymore (cf. fig. 2.3.d). Same applies to the soft trilinear couplings [53, 55].

### 2.4 Mirage mediation

A generic feature of string theory models with non-perturbative moduli stabilization and sequestered SUSY breaking is the appearance of the little hierarchy eq. (2.40) which relates the SUSY breaking \( F \)-terms as [47, 48, 50]

\[
\frac{F_T}{T_0 + \overline{T}_0} \approx \frac{1}{\log \left( M_\text{P}/m_{3/2} \right)} \frac{F^C}{C_0} \approx \frac{1}{4\pi^2} \frac{F^C}{C_0} \approx \frac{m_{3/2}}{4\pi^2}.
\] (2.68)

In KKLT-type models with visible fields on D7 branes eq. (2.68) guarantees that the soft breaking terms are dominated by the tree-level modulus and the loop-level anomaly mediations. Other (loop) contributions due to Kähler anomalies and string threshold corrections are negligible.

The contribution from modulus mediation is suppressed by the little hierarchy, which in case of a TeV gravitino, is of order of a loop suppression factor. As a consequence the soft breaking terms are determined by a specific mixed modulus-anomaly mediation in which the two mediation mechanisms are of comparable strength. In the framework of the MSSM, the soft terms just below the GUT scale receive a non-universal contribution from anomaly mediation in terms of the MSSM \( \beta \)- and \( \gamma \)-functions and a universal contribution from modulus mediation. In case of the gaugino masses the splitting just below the GUT scale is provided by the respective \( \beta \)-function coefficients. Since RG evolution of the gaugino masses is governed by the same \( \beta \)-function coefficients, the splitting disappears at an intermediate scale eq. (2.58), known as the mirage scale.

Eq. (2.68) also ensures that the soft scalar masses and trilinear couplings are dominated by contributions from modulus and anomaly mediations. The soft scalar masses of the first two generations exhibit a similar RG structure as the soft gaugino masses. Consequently they do (approximately) mirage unify at the same intermediate scale eq. (2.58) and have (approximately) the same values as the gaugino masses (cf. fig. 2.3.c). The soft scalar masses of the third generation and the Higgses as well as the soft trilinear couplings do not share the mirage unification feature. In addition, soft scalar masses and trilinear couplings show a stronger model-dependence such that mirage unification for these parameters is in general unlikely to occur. Thus we are led to the following definitions.

**Definition** Consider a string theory inspired scheme of SUSY breaking where little hierarchy \( \log(M_\text{P}/m_{3/2}) \) emerges and the soft breaking parameters receive contributions from modulus and anomaly mediations as well as from other sources of SUSY breaking.
2.5 General properties of mirage mediation

A scheme in which all soft terms are dominated by equally important modulus and anomaly mediation is called mirage mediation. Gaugino masses as well as scalar masses of the first two generations unify at the mirage scale eq. (2.58). The general mass pattern is determined by the little hierarchy and in case of $m_{3/2} = O(\text{TeV})$ it is

$$m_{\text{MODULI}} \sim \log \left( \frac{M_p}{m_{3/2}} \right) m_{3/2} \sim \log^2 \left( \frac{M_p}{m_{3/2}} \right) m_{\text{SOFT}},$$

$$m_{\text{MODULI}} \sim (4\pi^2) m_{3/2} \sim (4\pi^2)^2 m_{\text{SOFT}}. \quad (2.69)$$

A scheme in which at least the gaugino soft masses are dominated by equally important modulus and anomaly mediations is called relaxed mirage mediation. In this case only the gaugino masses show mirage unification and the general mass pattern for $m_{3/2} = O(\text{TeV})$ is

$$m_{\text{MODULI}} \sim \log (4\pi^2) m_{3/2} \sim (4\pi^2)^2 M_a,$$

$$m_i \geq M_a. \quad (2.70)$$

### 2.5 General properties of mirage mediation

As we have seen above, mixed modulus-anomaly mediation can lead to the phenomenon of mirage mediation. One of the interesting features of this mediation scheme is that the soft terms experience modulus and anomaly mediations at the same strength. Specifically, in the MSSM, the soft breaking terms just below the GUT scale can be parameterized by (cf. appendix A.3)

$$M_a = M_0 \left[ \frac{1}{2} g^2 \right],$$

$$A_{ijk} = M_0 \left[ (-3 \varrho + n_i + n_j + n_k) \gamma_i + \gamma_j + \gamma_k \right],$$

$$m_i^2 = M_0^2 \left[ (1 - n_i) \varrho^2 - \gamma_i + 2 \varrho \Psi_i^T \right], \quad (2.73)$$

where $\varrho = O(1)$ measures the balance between modulus and anomaly mediation, $M_0$ sets the scale of the soft terms, $b_a$ are the $\beta$-function coefficients, $\gamma_i$ are the anomalous dimensions, $\tilde{\gamma}_i$ denotes the running of the anomalous dimensions and $\Psi_i^T$ describes the $T$ dependence of the anomalous dimension. The parameters $n_i$ are the so-called effective modular weights and depend on the location of the visible matter fields. For matter fields on D7 branes, $n_i = 1$, whereas $n_i = 0$ for matter on D3 branes. In case the matter fields live on brane intersections, $n_i$ takes fractional values $n_i \in (0, 1)$.

The low energy sparticle spectrum in mirage mediation differs from other SUSY breaking scenarii. This is mainly due to the peculiar correlation between anomaly mediated contributions and the RG evolution of the soft parameters.
The former significantly cancels the latter, giving rise to a rather compressed low energy sparticle spectrum at the TeV. Also the phenomenology of mirage mediation [53–56, 105] differs from pure modulus (gravity) [106–109] and pure anomaly mediation [99, 100]. It seems to retain the attractive features of the particular mediation mechanisms while alleviating the problematic ones. This leads to quite distinctive properties which we summarize below.

**Tachyons ::** Pure anomaly mediation suffers from tachyonic sleptons [51, 52, 80]. In mirage mediation, scalar squared soft masses receive a positive contribution from modulus mediation which can cancel the tachyons. One should, however, note that due to the mixed modulus-anomaly term in eq. (2.73) also squarks might become tachyonic. Thus, absence of tachyonic fields sets a lower bound on the parameter $\varrho$ which in turn defines a constraint for model building (e.g. uplifting potential). Moreover, as evident from eq. (2.73), non-zero $n_i$ would enlarge the tachyonic regions. In this regard, matter fields on D7 branes are favored by phenomenology.

**MSSM fine-tuning ::** In supersymmetric models a certain fine-tuning is required to obtain the EW scale from the scale of the soft masses. Due to its specific structure, mirage mediation provides a possibility to reduce the MSSM fine-tuning, though it might require an extension to resolve it completely. Anyway this seems to be a model-dependent feature [105, 110].

**Flavor problem ::** Modular weights $n_i$ are in general generation-dependent. If, however, matter fields with common gauge charge originate from the same geometrical structure, the modular weighs will be flavor-independent. Since anomaly mediation is flavor-blind [51, 52, 80], the soft terms in mirage mediation preserve the lepton and quark flavors, provided that $n_i$ are flavor-independent.

**CP problem ::** Soft terms in mirage mediation also preserve CP since the relative CP phase between $F^T$ and $F^C$ can in principle be canceled by $U(1)_R$ and $U(1)_{PQ}$ rotations [83]. As a result the CP phase of the gaugino masses is aligned with the universal CP phase of the $A$-terms. However, the extreme smallness of various electric dipole moments might require further alignment of the phases [55].

**LSP ::** The LSP is the lightest neutralino and is mostly dominated by the bino component [54–56, 105]. The scheme offers an interesting scenario to produce a correct amount of neutralino dark matter consistent with data [111].

**Cosmology ::** SUGRA theories are often in conflict with cosmology as they predict long-lived particles. Late decays of such particles would spoil the standard nucleosynthesis [112], which has proven to be very successful. In string inspired models these long-lived particles are (usually) moduli and gravitini and the associated problems are known as the cosmological gravitino and moduli problems [113]. One way to avoid or at least alleviate these
problems is to make gravitini and moduli sufficiently heavy in order to enhance their decay rate. This is exactly what happens in the scheme of (pure) mirage mediation. The little hierarchy among the soft, the gravitino and moduli masses, eq. (2.69), implies that a TeV sparticle spectrum requires a $O(30 \text{ TeV})$ gravitino mass and $O(10^3 \text{ TeV})$ moduli masses. Such mass scales are enough for the gravitini and moduli to decay before nucleosynthesis and to not affect the abundances of light elements [54]. Nevertheless, there are other challenges from the cosmological point of view which may require further ingredients
Chapter 3

Uplifting in Type IIB string theory

In the previous chapter we have studied a method for constructing dS/Minkowski vacua in type IIB string theory through a combination of high scale dynamics (e.g. fluxes), D-branes and non-perturbative effects (e.g. gaugino condensation). Although this scheme is able to provide realistic vacua with quite distinct low energy phenomenology, it also contains a number of problematic features, one of the most important being the explicit breaking of SUSY. In this chapter we first show that in SUGRA theories with a single modulus dS/Minkowski vacua are not possible. We then review the difficulties of D-term uplifting. Afterwards, we study the possibility that dS/Minkowski vacua arise due to superpotential interactions of hidden matter fields, known as F-uplifting, and analyze the pattern of SUSY breaking.

3.1 No-go theorem

It has been pointed out [60, 114, 115] that dS/Minkowski vacua in the framework of spontaneously broken $\mathcal{N} = 1$ SUGRA are not possible for models with a single modulus $X$ as long as the Kähler potential takes on its tree-level form

$$K = -n \log (X + \bar{X}),$$

with $1 \leq n \leq 3$ depending on the nature of the modulus. Using the standard $\mathcal{N} = 1$ SUGRA formalism [7–13] one obtains

$$V = \phi^G \left[ K^{-1} G_X G_{\bar{X}} - 3 \right]$$

$$= \frac{1}{(X + \bar{X})^n} \left[ \frac{1}{n} W_X (X + \bar{X}) - n |W|^2 - 3 |W|^2 \right],$$

where we leave the superpotential undetermined. The stationary point condition

$$V_X = G_X V + \phi^G \left[ \left( K^{-1} \right)_X G_X G_{\bar{X}} + K^{-1} G_{XX} G_{\bar{X}} + G_X \right] \equiv 0$$
is trivially solved for a supersymmetric configuration $G_X = 0$. Imposing eq. (3.3) for a non-supersymmetric point $G_X \neq 0$ leads to

$$0 = \left( \overline{W}_X (X + \overline{X}) - n \overline{W} \right) \left( W_{XX} (X + \overline{X}) + (1 - n) W_X \right) \frac{X + \overline{X}}{n}$$

$$- \left( W_X (X + \overline{X}) - n W \right) \left( \overline{W}_X (X + \overline{X}) \frac{n - 1}{n} + (3 - n) \overline{W} \right).$$  (3.4)

In order to analyze the stability of the stationary point we have to consider the second derivatives of the scalar potential. Using eqs. (3.2) – (3.4) we can recast $\partial^2 V / \partial X \partial \overline{X}$ at the minimum $X_0$ as

$$\frac{\partial^2 V}{\partial X \partial \overline{X}} = - \frac{2}{(X_0 + \overline{X}_0)^2} \left( V_0 + \frac{3 - n}{(X_0 + \overline{X}_0)^n} |W(X_0)|^2 \right),$$  (3.5)

where $V_0$ denotes the vacuum energy. Clearly, for $n \leq 3$ and $V_0 \geq 0$ eq. (3.5) is non-positive, implying that at least one of the eigenvalues of the Hessian is negative or zero. Thus we can summarize this obstacle as a [57]

**No-go theorem** If a modulus $X$ is the only light field in the theory and its Kähler potential $K = -n \log (X + \overline{X})$ with $1 \leq n \leq 3$, dS or Minkowski vacua with spontaneously broken SUSY are not possible for any superpotential.

According to this observation our conclusion is twofold. On the one hand, with $X$ being the only (light) DOF, corrections to the Kähler potential are necessary so as to allow for dS/Minkowski vacua [116–118]. On the other hand, with the classical Kähler potential the existence of dS/Minkowski vacua requires additional DOF to be implemented within the 4D $\mathcal{N} = 1$ SUGRA [60]. In this work we are going to analyze the second option.

### 3.2 $D$-Uplifting

In our discussion so far we have focused our attention only on the $F$-term part of the scalar potential. There is also a contribution coming from the $D$-terms. The full 4D $\mathcal{N} = 1$ SUGRA potential in the Einstein frame is given by [7–13]

$$V_{\mathcal{N}=1} = V_F + V_D,$$  (3.6)

$$V_F = e^G \left[ G^{-1}_{ij} G_I G_T - 3 \right],$$  (3.7)

$$V_D = \frac{1}{2} \Re f_a D^i D^a$$

$$= \frac{1}{2} \Re f_a \left( i \eta^i_a \partial_i K - 3 i r_a \right)^2,$$  (3.8)
3.2 D-Uplifting

where I, J label the fields, $\eta^I_a$ denotes the gauge transformation of the chiral superfields under the gauge group factor $G_a$, while $r_a$ is determined by the transformation properties of the superpotential under $G_a$,

$$\delta_a W = \eta^I_a \partial_I W = -3r_a W.$$  \hfill (3.9)

From this equation and for $W \neq 0$ one can rewrite the D-terms as [47]

$$D^a = \frac{i}{\Re f_a} \frac{1}{W} \eta^I_a (\partial_I W + W \partial_I K) = \frac{i}{\Re f_a} \eta^I_a G_I.$$  \hfill (3.10)

A supersymmetric configuration with $\langle G_m \rangle = 0$, if allowed, is always a stationary point of $V_F$ as can be read off from eq. (3.3). Obviously, if $V_F$ admits a supersymmetric AdS vacuum, i.e. a stable solution with $\langle G_m \rangle = 0$ but $W \neq 0$, as is the case in the KKLMT scheme, the D-terms do necessarily vanish. Thus, supersymmetric minima cannot be uplifted by the D-terms [47]. $V_D$ merely improves the stability of the supersymmetric AdS solution of $V_F$. This solution remains a good solution of the complete scalar potential. Hence we conclude that only non-supersymmetric vacua can be uplifted by the D-terms [119, 120].

Let us briefly discuss the D-term uplifting of non-supersymmetric minima. For concreteness consider the D-term coming from an anomalous U(1) [79]

$$D \sim \frac{E}{\Re f_a} + \sum_i q_i \phi^*_i \phi_i,$$  \hfill (3.11)

where $E$ is a constant related to the trace of the anomalous U(1), $\phi_i$ are matter fields carrying the anomalous charges $q_i$ and for gauge fields on D7 branes we have $f_a = T$. At the minimum of the complete scalar potential the stationary point condition $\partial_T(V_F + V_D) = 0$ together with eq. (3.6) implies symbolically [57]

$$m_{3/2}^2 + D^2 + D \approx 0,$$  \hfill (3.12)

where we have neglected all coefficients and assumed that there are no very large/small factors in this equation. Using $m_{3/2} \sim 10^{-14} M_P$, as favored by phenomenology, this equation is solved by

$$|D| \sim m_{3/2}^2,$$  \hfill (3.13)

whereas

$$|F| \sim m_{3/2} M_P.$$  \hfill (3.14)

Thus we see that for a hierarchically small gravitino mass [121] D-terms are much smaller than the F-terms and consequently $D^2 = O(m_{3/2}^4)$ cannot uplift a AdS minimum with $V_{0,AdS} = O(-m_{3/2}^2 M_P^4)$. This mechanism can only work for a heavy (Planckian) gravitino mass [122–124].

In this work we are interested in a phenomenologically viable gravitino mass being in the TeV domain in order to provide a TeV sparticle spectrum. Therefore we can safely neglect the contribution from $V_D$ and study the F-term potential.
3.3 F-Uplifting

From the restrictions of the no-go theorem and due to the shortcomings of the D-term uplifting, it appears that uplifting of (supersymmetric) AdS vacua within the standard SUGRA framework requires extra DOFs in addition to a light Kähler modulus. Since matter fields are as generic as moduli in string theory constructions, they are well equipped to play the role of these additional DOFs we are looking for. In this section we study the possibility that dS/Minkowski vacua arise due to the F-terms of hidden sector matter fields. This procedure was shown to be viable and works for a hierarchically small gravitino mass \([60]\). Our theoretical framework is 4D \(\mathcal{N} = 1\) SUGRA with two sectors: moduli sector and matter/uplifting sector where SUSY is broken spontaneously in a dS/Minkowski vacuum. First we study these subsectors separately and analyze thereafter properties of their combination.

3.3.1 Moduli sector

This sector is responsible for the stabilization of all moduli. In particular, it represents the first two steps of the KKLT construction (cf. section 2.2). We assume that the dilaton and the CSM are stabilized by fluxes and acquire huge masses such that they can be integrated out. The effective theory for the remaining (light) Kähler modulus is described by the classical Kähler potential and a superpotential induced by fluxes and gaugino condensation

\[
K_{\text{MOD}} = -3 \log\left(T + T^\star\right),
\]

\[
W_{\text{MOD}} = W_0 - A e^{-aT},
\]

with \(A = O(1), a \gg 1\) and \(W_0\) originates from fluxes. The scalar potential is

\[
V_{\text{MOD}} = e^G\left[K_{TT}^{-1} G_T G_T - 3\right].
\]

Assuming a real \(T = O(1)\) the supersymmetric minimum \(G_T(T_0) = 0\) appears at

\[
T_0 = -\frac{1}{a} \log \left(\frac{W_0}{A a}\right),
\]

with \(a T_0 \approx \log(M_P/m_{3/2})\) and the corresponding vacuum energy

\[
\langle V_{\text{MOD}} \rangle = -3e^G \sim | W_{\text{MOD}}(T_0) |^2.
\]

3.3.2 Matter sector

Since the moduli stabilization mechanism outlined above does not break SUSY we introduce matter fields for this purpose. Consider a hidden sector matter field \(\phi\) with canonical Kähler potential and a generic superpotential

\[
K_{\text{MAT}} = \phi \phi^\star,
\]

\[
W_{\text{MAT}} = W_{\text{MAT}}(\phi).
\]
3.3 F-Uplifting

The corresponding scalar potential

\[ V_{\text{MAT}} = e^G \left[ G_\phi G_\phi - 3 \right], \quad (3.22) \]

admits supersymmetric as well as non-supersymmetric solutions. Suppose for simplicity that the minimum of the scalar potential is at real \( \phi \). Then a non-supersymmetric extremum is found from

\[ \partial_\phi V_{\text{MAT}} = e^G G_\phi \left[ G_\phi + G_{\phi\phi} - 2 \right] \frac{1}{2} = 0, \quad (3.23) \]

with \( G_\phi(\phi_0) \neq 0 \). From stability considerations we obtain

\[ \partial_{\phi\phi} V_{\text{MAT}} = e^G \left[ G_\phi^2 + 2G_\phi G_{\phi\phi} + G_{\phi\phi}^2 - 2 \right] = 2e^G > 0, \quad (3.24) \]

which confirms that the non-supersymmetric solution is a minimum. Moreover, the mass of the hidden matter field is of order of the gravitino mass \( m_\phi \sim m_{3/2} \).

By adjusting the parameters of \( W_{\text{MAT}} \) the vacuum energy can be chosen positive and arbitrarily small

\[ \langle V_{\text{MAT}} \rangle = V_{\text{MAT}}(\phi_0) \geq 0. \quad (3.25) \]

In this minimum SUSY is broken by \( F_\phi = e^{G/2} G_\phi \). In a nearly Minkowski vacuum \( G_\phi = \mathcal{O}(1) \) and consequently

\[ F_\phi \approx e^{G/2} \sim |W_{\text{MAT}}(\phi_0)|. \quad (3.26) \]

### 3.3.3 The uplifting

Now we combine the two sectors. That is, we suppose that the low energy theory involves a single Kähler modulus \( T \) and a hidden sector matter field \( \phi \). The corresponding Kähler potential is given by\(^1\)

\[ K = -3 \log \left( T + \bar{T} \right) + \phi \bar{\phi} + \phi \bar{\phi} \left( T + \bar{T} \right)^{-n_\phi}, \quad (3.27) \]

where \( n_\phi \) denotes the modular weight for the matter field \( \phi \). For definiteness, we choose \( n_\phi = 0 \). The superpotential of the combined system takes the form

\[ W(T, \phi) = W_{\text{MOD}}(T) + W_{\text{MAT}}(\phi) = W_0 - A e^{-a T} + W_{\text{MAT}}(\phi). \quad (3.28) \]

The two subsystems have their minima at \( T_0 \) and \( \phi_0 \), respectively. The question is now, how much does the minimum of the combined system deviate from the individual minima?

\(^1\) A realization of this setup can be found in \([125]\).
Consider the system in the vicinity of the reference point \((T_0, \phi_0)\). At \(\phi = \phi_0\) the superpotential for \(T\) is
\[
W = W_{\text{mod}}(T) + W_{\text{mat}}(\phi_0),
\]
whereas the superpotential for \(\phi\) at \(T = T_0\) is
\[
W = W_{\text{mod}}(T_0) + W_{\text{mat}}(\phi) .
\]

Thus the constant terms in the superpotential shift relative to those of the original subsectors. Therefore it makes sense to define \(T_0\) as the minimum of the moduli subsector with the superpotential eq. (3.29) and similarly \(\phi_0\) as the minimum of the matter subsector with the superpotential eq. (3.30).

The scalar potential of the combined system is given by
\[
V = e^G [K_T^{-1} G_T G_{\overline{\phi}} + G_\phi G_{\overline{\phi}} - 3].
\]

We would like to see whether the minima of the separate subsectors \((T_0, \phi_0)\) represent a stationary point of the combined system. The stationary point equations read
\[
V_T = G_T V + e^G \frac{\partial}{\partial T} \left( K_T^{-1} G_T G_{\overline{\phi}} \right) + e^G \frac{\partial}{\partial T} \left( G_\phi G_{\overline{\phi}} \right),
\]
\[
V_\phi = G_\phi V + e^G \frac{\partial}{\partial \phi} \left( G_\phi G_{\overline{\phi}} \right) + e^G K_T^{-1} \frac{\partial}{\partial \phi} \left( G_T G_{\overline{\phi}} \right).
\]

Consider \(V_\phi\). It vanishes at \((T_0, \phi_0)\) because the first two terms represent the equations of motion for the \(\phi\)-subsector eq. (3.23) and the third term is proportional to \(G_T\) which is zero at \((T_0, \phi_0)\). Consider now \(V_T\). The first two terms are zero as they represent the equations of motion for the \(T\)-subsector. The last term, however, does not vanish. To estimate it we recall that a small vacuum energy ensures \(G_\phi = O(1)\) and \(G_T = 0\) provides \(W_T/W = O(1)\). Hence
\[
e^G \frac{\partial}{\partial T} \left( G_\phi G_{\overline{\phi}} \right) = e^G G_T G_\phi \sim e^G \sim m_{3/2}^2.
\]

Finally, the vacuum energy at \((T_0, \phi_0)\) equals that of the \(\phi\)-subsector, eq. (3.25).

This shows that the stationary point conditions are “almost” satisfied at \((T_0, \phi_0)\). Let us now estimate how much the true minimum is shifted compared to \((T_0, \phi_0)\). Suppose the true minimum is at \((T_0 + \delta T, \phi_0 + \delta \phi)\). At this point
\[
V_T(T_0 + \delta T, \phi_0 + \delta \phi) = 0,
\]
\[
V_\phi(T_0 + \delta T, \phi_0 + \delta \phi) = 0.
\]

Assume, for simplicity, that the minimum occurs at real \(T\) and \(\phi\). Expanding eqs. (3.35) and (3.36) to first order in \(\delta T\) and \(\delta \phi\) gives
\[
V_T + V_{TT} \delta T + V_{T\phi} \delta \phi = 0,
\]
\[
V_\phi + V_{\phi\phi} \delta \phi + V_{\phi T} \delta T = 0,
\]
where \( V_\phi(T_0, \phi_0) = 0 \) as explained above. The solution is

\[
\delta T = \frac{V_T}{V_{T\phi}/V_{\phi\phi} - V_{TT}},
\]

\[
\delta \phi = -\frac{V_{T\phi}}{V_{\phi\phi}} \delta T. \tag{3.40}
\]

In the large \( a \) limit one obtains \([57,60]\)

\[
\delta T \sim \frac{1}{a^2}, \tag{3.41}
\]

\[
\delta \phi \sim \frac{1}{a}. \tag{3.42}
\]

Consequently the \( T \) modulus only slightly shifts from its previous position \( T_0 \). Note that the shift in \( T \) is of the same order as in the KKLT case, eq. (2.35).

### 3.4 The pattern of SUSY breaking in \( F \)-uplifting

Let us examine the pattern of SUSY breaking. At the true minimum we have

\[
G_T(T_0 + \delta T, \phi_0 + \delta \phi) = G_T + G_{TT} \delta T + G_{T\phi} \delta \phi, \tag{3.43}
\]

\[
G_\phi(T_0 + \delta T, \phi_0 + \delta \phi) = G_\phi + G_{\phi\phi} \delta \phi + G_{\phi T} \delta T, \tag{3.44}
\]

where again \( G_T(T_0, \phi_0) = 0 \) as explained above. Using \( G_\phi = O(1) \), \( W_T/W = O(1) \) and \( |W_{TT}| = a |W_T| \) we can estimate

\[
G_T(T_0 + \delta T, \phi_0 + \delta \phi) \sim 1/a, \tag{3.45}
\]

\[
G_\phi(T_0 + \delta T, \phi_0 + \delta \phi) \sim 1. \tag{3.46}
\]

This highlights that the SUSY breakdown is now triggered both by \( F^T \) and \( F^\phi \) with the latter providing the dominant contribution,

\[
F^T = a \sqrt{G/2} K_{TT}^{-1} G_T \sim \frac{m_{3/2}}{a}, \tag{3.47}
\]

\[
F^\phi = a \sqrt{G/2} G_{T\phi} \sim m_{3/2}. \tag{3.48}
\]

The masses of the modulus and the matter fields can be estimated in a similar way \([104]\)

\[
m_T \sim (a T_0) m_{3/2}, \tag{3.49}
\]

\[
m_\phi \sim m_{3/2}. \tag{3.50}
\]

indicating that \( T \) is heavy compared to the gravitino and the matter field. It is important to note that eqs. (3.47) – (3.50) have been obtained in the limit of a small vacuum energy in the \( \phi \)-subsector. In this case, due to the suppression of \( F^T \) (eq. (3.47)) the first term in eq. (3.31) is \( O(1/a^2) \). Thus, the vacuum energy of the combined system is well approximated by the vacuum energy of the \( \phi \)-subsector, eq. (3.22). By adjusting the parameters of the \( \phi \)-subsector the vacuum energy can be made positive and arbitrarily small.
3.4.1 The little hierarchy

This uplifting procedure causes $T$ to change its position only slightly, such that $T$ is stabilized close to a supersymmetric point. Hence, up to a correction $O(1/a^2)$ the true minimum is well approximated by eq. (3.18), namely

$$T_0 \sim - \frac{1}{a} \log \left( \frac{W_{\text{MAT}}(\phi_0)}{a A} \right),$$

(3.51)

implying

$$W_{\text{MAT}}(\phi_0) \sim a \left( A e^{-a T_0} \right),$$

(3.52)

in the minimum. Consequently, the gravitino mass

$$m_{3/2} = \langle \alpha^{G/2} \rangle \sim |W_{\text{MAT}}(\phi_0)| \sim a \left( A e^{-a T_0} \right)$$

(3.53)

originates from gaugino condensation. Moreover, as $T$ is close to $T_0$ the vacuum exhibits the unique property

$$a T_0 \sim \log \left( \frac{A}{W_{\text{MAT}}(\phi_0)} \right) \sim \log \left( \frac{M_P}{m_{3/2}} \right),$$

(3.54)

which is known as the little hierarchy [47]. For $m_{3/2}$ lying in the TeV range $a T_0 = O(4\pi^2)$ is comparable to a loop suppression factor. In this case the general mass pattern is given by

$$m_{3/2} \sim m_\phi \ll m_T.$$  

(3.55)

3.4.2 Comparison with KKLT

Both KKLT and matter uplifting schemes exhibit the appearance of the little hierarchy $a T_0 \sim \log(M_P/m_{3/2})$ which suppresses the $F$-term of the modulus and enhances its mass with respect to the gravitino mass. However, there is one essential difference between these two schemes: in the matter uplifting scenario the SUSY breaking sector is not (necessarily) sequestered from the $T$ modulus and the visible matter fields. In this case matter fields in a hidden sector break SUSY spontaneously. Due to the absence of sequestering the couplings of the hidden matter to the modulus and the visible matter are “only” suppressed by $M_P$. This adds authority to the hidden matter sector as it provides the dominant source of SUSY breaking, $F^\phi \gg F^T$. Rewriting the scalar potential eq. (3.31) at the minimum as

$$V_0 = K_{TT} \left| F^T \right|^2 + \left| F^\phi \right|^2 - 3 m_{3/2}^2,$$

(3.56)

shows that the $F$-term of the hidden matter field is responsible for uplifting the deep AdS minimum to a small and positive value, hence the name $F$-uplifting.
3.4 The pattern of SUSY breaking in F-uplifting

3.4.3 An example

As an illustrative example consider a combination of the moduli sector with the Polonyi model \([9, 126]\). The Kähler potential and the superpotential are given by

\[
K = -3 \log (T + \overline{T}) + \phi \overline{\phi}, \quad (3.57)
\]

\[
W = c + \mu^2 \phi - A e^{-a T}, \quad (3.58)
\]

where \(c\) and \(\mu^2\) are constants and \(a\) is related to the \(\beta\)-function of the condensing gauge group. If observable gauge fields originate from D7 branes the SM gauge couplings eq. (2.50) require \(T_0 \approx 2\) at the minimum. A non-supersymmetric Polonyi vacuum is determined by

\[
G_\phi^2 + G_{\phi\phi} - 2 = 0. \quad (3.59)
\]

Choosing

\[
G_\phi^2 = 3 + \varepsilon, \quad (3.60)
\]

with \(\varepsilon \ll 1\) the vacuum energy is

\[
\frac{V_0}{\mu^4} \sim \varepsilon. \quad (3.61)
\]
This puts a constraint on \( c \) and \( \mu^2 \). The solution to first order in \( \varepsilon \) is

\[
c \approx \mu^2 \left( 2 - \sqrt{3} + \frac{\sqrt{3}}{6} \varepsilon \right), \tag{3.62}
\]

\[
\phi_0 \approx \sqrt{3} - 1 + \frac{\sqrt{3} - 3}{6} \varepsilon. \tag{3.63}
\]

The supersymmetric AdS minimum of the \( T \)-subsector is given by eq. (3.18). Combining the two sectors, their minima will shift according to eqs. (3.41) – (3.42). The shift of the individual minima can also be understood from their masses. The masses of the Polonyi field and the gravitino are of comparable size

\[
m_\phi \sim m_{3/2} = e^{G/2} \sim |W_{\text{MAT}}(\phi_0)| \sim \mu^2, \tag{3.64}
\]

with \( \mu^2 \) setting the scale, whereas the mass of the modulus is

\[
m_T \sim (a T_0) m_{3/2} \sim (a T_0) \mu^2 \tag{3.65}
\]

such that \( T \) is heavy. Fig. 3.1 confirms this result: the potential in the \( T \) direction is very steep around the minimum unlike the potential in the \( \phi \) direction. As a result \( T \) only slightly shifts from the original position and its contribution SUSY breakdown is suppressed. Finally, the resulting vacuum energy can be made arbitrarily small by adjusting \( \varepsilon \) and without affecting other aspects of the system. A concrete example based on the hidden sector gauge group SU(5) is shown in fig. 3.1 and tab. 3.1 summarizes the main parameters.

As a final remark we would like to stress that a deviation from the constraints eqs. (3.62) and (3.63) (which actually only cover the \( \phi \)-subsector) will cause the minimum of the potential to move along the \( T \) direction. For a delicate choice of parameters \( c \) and \( \mu^2 \) the modulus can be stabilized at \( \Re T_0 \sim 2 \).

### 3.4.4 Soft breaking terms

In the scheme of uplifting via matter fields we can identify three sources of SUSY breakdown coming from the \( T \) modulus, the matter field \( \phi \) and the SUGRA compensator \( C \), respectively. The auxiliary components are

\[
F_T \sim \frac{m_{3/2}}{a}, \tag{3.66}
\]

\[
F_\phi \sim m_{3/2}, \tag{3.67}
\]

\[
F_C \sim \frac{m_{3/2}}{C_0}. \tag{3.68}
\]

This is different from the KKLT scheme where the contribution from the SUSY breaking fields is sequestered from the \( T \) modulus and the visible matter fields.

In the \( F \)-uplifting scheme, the uplifting sector is (generically) not sequestered and affects the mediation of the SUSY breakdown as well as the resulting phenomenology. In section 3.4 we saw that the suppression of \( F_T \) makes the contribution from the conformal anomaly competitive to the tree-level modulus.
mediation. Furthermore, the soft breaking parameters experience loop contributions from Kähler anomalies and string threshold corrections which depend on $F^T$ and $F^0$. Since $F^T$ is suppressed we can neglect its involvement into such contributions. The unsuppressed $F^0$, however, triggers loop contributions that can become equally important to the contributions from modulus and anomaly mediations. There are two potential difficulties connected to these additional loop contributions. First, both introduce model-dependence, thereby lowering the predictivity of the scheme. Second, string threshold corrections depend on the detailed ultraviolet physics above the compactification/GUT scale. In principle this presents an uncontrollable contribution to the soft terms which could spoil the predictive power of the scheme. In other words, if such contributions become sizable no model-independent statement about the soft parameters can be made. Apart from that there might also be $\phi$-dependent string threshold corrections to the gauge kinetic functions [57, 60].

One possibility to avoid these troubles is to assume a discrete symmetry $\phi \rightarrow -\phi$ which is broken only by non-perturbative dynamics that is responsible for SUSY breaking. Such (approximate) symmetry can make the $\phi$-dependent contributions from Kähler anomalies and string threshold corrections negligible [92]. Another way out is to consider a class of models realizing $\phi_0 \ll 1$ [57, 60]. In this work we will focus on this kind of models. The construction of superpotentials realizing this situation is outlined in [57]. Assuming MSSM gauge fields originating from D7 branes, the (tree-level) gauge kinetic function and the Kähler potential are [47, 48]

$$f_a = T,$$

$$K = -3 \log \left( T + \bar{T} \right) + \phi \bar{\phi} + Q^i \bar{Q}^i Z_i,$$

(3.69)

(3.70)

with the Kähler metric for the visible fields

$$Z_i = (T + \bar{T})^{-n_i} [1 + \xi \phi \bar{\phi}],$$

(3.71)

where $Q^i$ are the visible fields with effective modular weights $n_i$ and $\xi_i$ describes the non-sequestered coupling between the visible fields, the $T$ modulus and the hidden matter field $\phi$. Using the formulae of appendix A.3, the soft breaking parameters at the GUT scale are given by

$$M_a = \frac{F^T}{T_0 + \bar{T}_0} + \frac{b_a g_{\text{GUT}}^2}{4 \pi^2 C_0} \frac{1}{4 \pi^2 C_0},$$

(3.72)

$$A_{ijk} = \left( -3 + n_i + n_j + n_k \right) \frac{F^T}{T_0 + \bar{T}_0} + \frac{\gamma_i + \gamma_j + \gamma_k}{4 \pi^2 C_0},$$

(3.73)

$$m_i^2 = (3 \xi_i - n_i) \frac{\left| F^T \right|^2}{\left( T_0 + \bar{T}_0 \right)^2} - \frac{\gamma_i}{4 \pi^2 C_0} + \frac{F^T \partial_{T} \gamma_i}{2} \frac{1}{4 \pi^2 C_0} + (1 - 3 \xi_i) m_{3/2}^2,$$

(3.74)
where \( b_a \) are the \( \beta \)-function coefficients and \( \gamma_i \) describes the RG running of the anomalous dimensions \( \gamma_i \).

Compared to KKLT scheme, \( F \)-uplifting provides the same pattern for the soft gaugino masses and the \( A \)-terms, i.e. modulus and anomaly mediation are of comparable size. We have to keep in mind that this result is only valid in models allowing \( \phi_0 \ll 1 \) such that additional contributions to eqs. (3.72) – (3.74) are suppressed. Then, as in KKLT, the non-universality in the gaugino masses at the GUT scale is given by the respective \( \beta \)-function coefficients. Since the RG running is described by the same \( \beta \)-functions the splitting disappears at \( M_{\text{MIR}} \) eq. (2.58) leading to the mirage unification of gaugino masses.

The structure of the soft scalar scared masses is richer as compared to KKLT. This is due to the non-sequestered form of the uplifting sector and endows the scheme with interesting features which we want to address.

**Tachyons ::** The second and the third term in eq. (3.74) represent pure anomaly [51, 52, 80] and mixed modulus-anomaly contributions [47], respectively and provide tachyonic sleptons and squarks. The presence of tachyonic fields indicates that at the GUT scale the boundary conditions might be ill-defined. Thus, the absence of tachyons at the GUT scale requires the modulus contribution to be slightly bigger than the anomaly contribution. The non-sequestered coupling between \( \phi \) and the visible fields, however, gives rise to a positive contribution \( \mathcal{O}(m_{3/2}^2) \) encoded in the last term of eq. (3.74). By introducing \( \xi_i \) we are entering the model-dependence sector which is the price we have to pay to have spontaneous SUSY breaking. But it has the advantage to provide a tool to remove the tachyons without affecting other soft terms. Note that in order to keep the last term in eq. (3.74) positive, \( 0 \leq \xi_i \leq 1/3 \) must be fulfilled.

**Flavor ::** In general, the couplings \( \xi_i \) are flavor dependent and thus present an additional source of flavor violation. In the KKLT scheme, sequestering ensures the absence of additional flavor violations. However, we can easily resolve this issue by choosing \( \xi_i \) generation independent, too. Furthermore, since the modular weights are in general flavor dependent too, we choose them to be universal.

**Mass pattern ::** While a mirage pattern occurs for the gaugino masses this is not necessarily true for the scalar masses. The reason is the additional contribution from the matter field \( \phi \) encoded in the last term in eq. (3.74). However, mirage unification is still realizable but only in models admitting \( \xi_i = \mathcal{O}(1/3) \). Thus, for \( \xi_i \ll 1/3 \) \( F \)-uplifting shows the relaxed mirage pattern

\[
M_d \ll m_i \sim m_{3/2}, \tag{3.75}
\]

whereas for \( \xi_i = \mathcal{O}(1/3) \) the pure mirage pattern

\[
M_d \sim m_i \ll m_{3/2} \tag{3.76}
\]

is recovered.
3.4 The pattern of SUSY breaking in F-uplifting

Cosmology :: As in the KKLT scheme, the cosmological moduli and gravitino problems can be alleviated. The masses of the moduli are enhanced by the little hierarchy. The masses of the sfermions, however, are in general $O(m_{3/2})$. Thus, making the gravitino heavy results in heavy sfermions. Furthermore, one is also faced with the so-called moduli induced gravitino problem [127, 128], which states that in models like KKLT the branching ratio for the $T$ decays into gravitini is $O(1)$ which leads to abundant gravitino production and severe cosmological problems. In F-uplifting such a problem is usually absent [60, 129] since the uplifting field $\phi$ typically has a mass $O(m_{3/2})$. This is because the uplifting potential is not very steep [130–132]. Thus $\phi$ dominates the energy density of the universe at late times, but its decay into gravitini is suppressed and the moduli induced gravitino problem is absent.

Finally, using our parameterization eqs. (2.51) and (2.52), we can write the soft parameters eqs. (3.72) and (3.74) in a compact form. Moreover, in order to compare the last term in eq. (3.74) with the remaining terms we introduce

$$\eta_i^2 = (1 - 3 \xi_i) (16\pi^2)^2.$$  \hfill (3.77)

Note that $\xi_i = 1/3$ corresponds to $\eta_i = 0$ which gives a pure mirage pattern. For $\eta_i = O(1)$ the deviation from the pure mirage picture is small and increases with an increasing $\eta_i$. The decoupling limit $\xi_i = 0$ corresponds to $\eta_i \approx 158$ resulting in a maximally relaxed mirage pattern. Thus we arrive at (cf. appendix A.3)

$$M_a = M_0 \left[ \varrho + b_4 \varrho^2 \right],$$  \hfill (3.78)

$$A_{ijk} = M_0 \left[ (-3 \varrho + n_i + n_j + n_k) + (\gamma_i + \gamma_j + \gamma_k) \right],$$  \hfill (3.79)

$$m_i^2 = M_0^2 \left[ (3 \xi_i - n_i) \varrho^2 - \dot{\gamma}_i + 2 \varrho \Psi_i^T + \eta_i^2 \right],$$  \hfill (3.80)

where $\Psi_i^T$ denotes the $T$ dependence of the anomalous dimension.
Chapter 4
Downlifting in heterotic string theory

In this chapter, we study the impact of the matter sector in the context of heterotic orbifold compactifications. After specifying the structure of the low energy SUGRA we first review the difficulties of moduli stabilization in this framework. Then we show that moduli stabilization can be achieved quite easily, if we accept the existence of the so-called downlifting sector (similar to the uplifting sector in the type IIB case) which is in any case necessary to adjust the vacuum energy to an acceptable value. Afterwards, we determine the soft breaking parameters. Finally we present a possible application of this procedure in the framework of heterotic orbifolds with fluxes.

4.1 Modular invariance

Orbifold\(^1\) compactifications of (fluxless) heterotic string theory enjoy the simplicity of torus compactifications \([44,133–135]\), but due to the action of the discrete orbifold group (e.g. \(\mathbb{Z}_3\)) they lead to \(\mathcal{N} = 1\) SUSY in 4D compared to \(\mathcal{N} = 4\) in case of the torus. In addition to the dilaton \(S\), the presence of toroidal geometry gives rise to moduli which dictate the sizes and relative orientations of the tori. These moduli will enjoy various symmetry transformations that leave the spectrum and the equations of motion for the low energy effective theory unchanged. One usually refers to these symmetries as modular symmetries \([136–138]\). The symmetry group of target space modular transformations depends on the particular orbifold background. The low energy effective SUGRA from orbifold models contains (at least) three Kähler moduli \(T_i\) which describe the size of the three complex planes and hence the volume of the compact space. Depending on the orbifold action imposed, there might be also further Kähler moduli as well as some number of CSM \(U_i\) related to the deformations of the complex structure. These moduli (together with some charged matter fields) originate from the untwisted sector of the orbifold \([139]\). In addition, depending on the particular orbifold setup, the effective low energy theory usually involves several fields coming from the twisted sector of the orbifold \([139]\).

\(^1\)An orbifold is defined as a manifold divided by a discrete symmetry. In string theory context, this manifold is usually assumed to be a flat torus. Orbifolds are everywhere Riemann-flat except at some finitely many points where the curvature becomes singular. In other words, orbifold is a singular limit of a CY manifold.
The most important property arising in (fluxless) orbifold compactifications is the appearance of an exact SL(2, \mathbb{Z}) global symmetry\footnote{At the quantum level such symmetries are typically anomalous [140, 141]. This anomaly is canceled in the effective theory by a sort of Green–Schwarz mechanisms and model-dependent string threshold corrections [142, 143].} acting on a generic single index modulus \( M_i \) as [144, 145]

\[
M_i \rightarrow \frac{a_i M_i - i b_i}{1 c_i M_i + d_i}, \quad a_i d_i - b_i c_i = 1, \quad a_i, b_i, c_i, d_i \in \mathbb{Z},
\]

and a charged matter field \( \phi_{\alpha} \) transforms as

\[
\phi_{\alpha} \rightarrow \phi_{\alpha} \prod_i (ic_i M_i + d_i)^{-n_{\alpha}^i},
\]

where \( n_{\alpha}^i \) denotes the modular weight of the matter fields.\footnote{Fields in the twisted sectors with the same modular weight can mix among themselves under SL(2, \mathbb{Z}) transformations [146].}

The dilaton \( S \) remains invariant under the modular transformations \( (4.1) \). This set of transformations can be generated from the two underlying transformations including duality \( M_i \rightarrow 1/M_i \) and imaginary shifts \( M_i \rightarrow M_i + i \). Observe that there exist two points which are left invariant under these modular transformations. These so-called self-dual points (SDP) are

\[
M_i^{(SDP)} = \left\{ 1, e^{i\pi/6} \right\}.
\]

In the present work we focus our attention on orbifold models that give rise to an effective low energy theory with untwisted Kähler moduli. The tree-level Kähler potential for models with a single untwisted Kähler modulus \( T \) is given by [70–72]

\[
K = -\log(S + \bar{S}) - 3 \log(T + \bar{T}) + \sum_{\alpha} \phi_{\alpha} \phi_{\alpha} (T + \bar{T})^{-n_{\alpha}^i},
\]

with the effective modular weights \( n_{\alpha} = \sum_i n_{\alpha}^i \) being \( O(1) \) integers. Eqs. (4.1) and (4.2) induce a transformation on the Kähler potential of the form \( K \rightarrow K + \mathcal{K} + \bar{\mathcal{K}} \), with \( \mathcal{K} = 3 \log(i c T + d) \).

The invariance of the effective SUGRA action under modular transformations implies the invariance of the SUGRA potential and, since the scalar potential is defined in terms of the Kähler function \( G \), also the modular invariance of \( G \). This can be achieved, provided that the superpotential transforms as

\[
W \rightarrow (i c T + d)^{-3} W.
\]

In the fluxless heterotic setup the \( T \) moduli cannot be stabilized perturbatively thus one is left to consider non-perturbative superpotentials consistent with modular invariance. In [136, 137] it was shown that the \( T \)-dependence of an effective non-perturbative superpotential satisfying eq. (4.5) must be of the form

\[
W(T) \sim \eta^{-6}(T),
\]
where $\eta(T)$ is the Dedekind $\eta$-function
\[
\eta(T) = e^{-\frac{2\pi i T}{\tau}} \prod_{n=1}^{\infty} \left(1 - e^{-2\pi n T}\right),
\]
which transforms under eq. (4.1) as
\[
\eta(T) \rightarrow (i c T + d)^{1/2} \eta(T).
\]
Moreover it is a periodic function of $\Im T$ and vanishes exponentially at $\Re T \rightarrow 0$ and $\Re T \rightarrow \infty$. This can be straightforwardly generalized to multi moduli case.

### 4.2 Dilaton and a modulus

Guided by modular invariance of the effective action in orbifold models, we want to consider a low energy effective 4D $N = 1$ SUGRA theory containing the dilaton $S$ and an overall Kähler modulus $T$. We will assume that there are no light (hidden sector) matter fields present (or assume that their VEVs are negligible). Then the effective theory is described by
\[
f_a = S,
\]
\[
K = -\log(S + \overline{S}) - 3 \log(T + \overline{T}).
\]
Since $S$ and $T$ do not receive a perturbative superpotential we consider a non-perturbative superpotential induced by gaugino condensation \([21–26, 76]\)
\[
W(S, T) = \frac{\Omega(S)}{\eta^6(T)},
\]
where we leave the form of the gaugino condensate $\Omega(S)$ undetermined at this stage. The $S$-dependence of the condensate follows from the gauge kinetic function $f_S = S$. The corresponding $F$ terms and the scalar potential are given by
\[
F^S = \frac{e^{G/2}K_{ss}^{-1}G_S}{\Omega_S(S + \overline{S})} \left(\Omega_S(S + \overline{S}) - \Omega\right),
\]
\[
F^T = \frac{e^{G/2}K_{tt}^{-1}G_T}{\Omega_T(T + \overline{T})} \left(\Omega_T(T + \overline{T}) - \Omega\right),
\]
\[
V = e^G \left[K_{ss}^{-1}G_S G_S + K_{tt}^{-1}G_T G_T - 3\right]
\]
\[
= \frac{\Omega_S(S + \overline{S}) - \Omega^2 + 3 |\Omega|^2 \left[(T + \overline{T})^2 |E(T, \overline{T})|^2 - 1\right]}{(S + \overline{S})(T + \overline{T})^3 |\eta(T)|^{12}},
\]
Fig. 4.1 :: Scalar potential eq. (4.14) in the $T$-direction. Panel (a) illustrates the case $F^S = 0$. In panel (b) the impact of a non-zero $F^S$ on the potential is shown in the $\Re e T$ direction with $\Im m T = 0$. The case $\alpha = 0$ corresponds to $F^S = 0$. For each value of $\alpha$ the scalar potential has been rescaled in order to place all plots on the same figure.

where we have introduced the modified Eisenstein function

$$\mathcal{E}(T, \overline{T}) = \frac{1}{T + \overline{T}} + \frac{2}{\eta(T)} \frac{d\eta}{dT},$$

(4.15)

which vanishes at the SDP and their modular transformed images.

Without any further analysis one can draw a general conclusion from the form of the scalar potential. In the decompactification limit $\Re e T \to \infty$ (and its dual $\Re e T \to 0$), the scalar potential diverges, $V \to \infty$, due to $(T + \overline{T})^3 |\eta(T)|^{12} \to 0$.

Another generic feature is that at the SDP (and their modular transformed equivalents) the $T$-modulus does not break supersymmetry as the $T$ auxiliary field, eq. (4.13), vanishes at these points. Recall from eq. (3.3) that supersymmetric configurations always correspond to local extrema of the scalar potential. Such a scenario is typically called dilaton domination [97, 98]. Also supersymmetric configurations in the $S$ direction extremize the scalar potential. They correspond to moduli domination scenarios [98]. Apart from that there exist generically other (non-supersymmetric) configurations.

In case $F^S = 0$, the scalar potential eq. (4.14) has extrema at the SDP which, as illustrated in fig. 4.1.a, correspond a saddle point in the case of $T_0 = 1$ and a local maximum for $T_0 = e^{1/\pi/6}$. Clearly the pure supersymmetric case $F^S = F^T = 0$ is unrealistic as it does not correspond to a stable solution. There are, however, generically nearby extrema in the $\Re e T$ direction and some of them correspond to local minima. The existence of these nearby minima can be inferred from the properties of the $\eta$-function. Consider for instance $T_0 = 1$, which is a maximum in $\Re e T$. Keeping $\Im m T = 0$ we obtain $V \to \infty$ for $0 \leftarrow \Re e T \to \infty$ as discussed above. Thus, for $1 < \Re e T < \infty$ there must exist a minimum and its dual at $0 < \Re e T < 1$. Since $T_0 = 1$ is a minimum in the $\Im m T$, the nearby minima are genuine minima of the scalar potential. This happens at $T_0 = 1.23$ and $T_0 = 1/1.23$. Note that at
these points $0 < \mathcal{E} < 1$ and thus SUSY is broken by $F_T$ and the vacuum energy is large and negative. From the modular invariance of the scalar potential, the latter is a periodic function of $\Im T$, such that there is a whole family of local extrema with the same $\Re T$ but shifted $\Im T = \Im T + n$ with $n \in \mathbb{Z}$ (see fig. 4.1.a).

Introducing a non-zero $F_S$ will switch on the first term in the numerator in eq. (4.14). In case of gaugino condensates, this term will be proportional to the condensate times a factor proportional to $(\beta$-function$)^{-1} \gg 1$. Let us parameterize the $F_S$ contribution to eq. (4.14) by $|\Omega_S(S + \bar{S})| = \alpha |\Omega|^2$ with $\alpha = 0$ corresponding to $F_S = 0$. As the size of the dilaton $F$-term increases, the shape of the scalar potential changes. Along the $\Re T$ direction it becomes increasingly shallow, as displayed in fig. 4.1.b. For some critical value $\alpha \sim 0.5$ the nearby extrema in the $\Im T$ direction empty into the SDP. Moreover, the SDP undergo a transition; $T_0 = 1$ becomes a minimum in the $\Re T$ direction and a maximum in $\Im T$, and $T_0 = \alpha^{2 \pi/6}$ is now a local minimum. Thus, a non-perturbative superpotential for the dilaton forces the $T$-moduli to be stabilized at one of their SDP. In case of gaugino condensation one typically has $\alpha \gg 1$, hence the first term in eq. (4.14) will be energetically favored and provides a large and positive vacuum energy.

### 4.3 Stabilization of the dilaton

Let us assume that the $T$-modulus is fixed at one of its SDP and focus on the stabilization of the dilaton $S$. In this case the scalar potential reduces to

$$V = e^G \left[ K_{S\bar{S}}^{-1} G_S G_{\bar{S}} - 3 \right]. \quad (4.16)$$

#### 4.3.1 No-go with a single condensate

To generate a superpotential for the dilaton we consider gaugino condensation in a hidden sector involving a pure SU($N$) gauge theory. For a single condensate the superpotential is given by

$$\Omega(S) = A e^{-a S}, \quad (4.17)$$

with $a = 8\pi^2/N \gg 1$ and $A = O(1)$. To analyze whether such a superpotential can lead to stable minima we first impose the stationary point condition $V_S = 0$. This relates the derivatives of $G$ as

$$0 = G_S \left[ K_{S\bar{S}}^{-1} |G_S|^2 + \left( K_{S\bar{S}}^{-1} \right)_S G_{\bar{S}} - 1 \right], \quad (4.18)$$

$$= \left( \frac{1}{S + \bar{S}} + a \right) \left[ 1 + 2 \left( 1 + a(S + \bar{S}) \right) - |1 + a(S + \bar{S})|^2 \right] \quad (4.19)$$

Obviously, two sorts of extrema come into consideration: supersymmetric with $G_S = 0$ and non-supersymmetric corresponding to $G_S \neq 0$. The condition for a
supersymmetric extremum leads to
\[ \Re S_0 = \frac{-1}{2a} < 0. \tag{4.20} \]
From the phenomenological point of view, eq. (4.20) does not correspond to a reasonable solution. This can be seen from the fact that the VEV of the dilaton determines the gauge coupling constant via \( g_a^{-2} = g \Re f_a = \Re S_0 \) and consequently eq. (4.20) implies imaginary gauge coupling.

On the other hand, non-supersymmetric extrema (vanishing of the square brackets in eq. (4.19)) appear at
\[ \Re S_0 = \pm \frac{1}{a \sqrt{2}}, \]  
\( (4.21) \)
of which only positive solutions are of interest to us. To analyze stability of the non-supersymmetric extremum we can use the general result in section 3.1. Plugging eq. (4.21) into eq. (3.5) yields
\[ V_{SS} = - \left( 2 + \sqrt{2} \right) a^3 A^2 e^{-\sqrt{2}}, \]  
\( (4.22) \)
resulting in at least one negative or zero eigenvalue of the Hessian. This shows that the non-supersymmetric solution does not correspond to a minimum. This is schematically shown in fig. 4.2.a. Apart from that, \( \Re S_0 = 1/a \sqrt{2} \ll 1 \) corresponds to strong (universal) gauge coupling, thus invalidating the perturbative SUGRA approach. Finally, as \( \Re S \to +\infty \) the dilaton potential eq. (4.16) exponentially approaches zero.

Therefore, given a single gaugino condensate eq. (4.17) and a tree-level Kähler potential eq. (4.10), the dilaton either enters in a strong coupling regime or runs away to infinity (decoupling limit) resulting in a free theory. In order to allow for a formation of stable minimum additional DOF are necessary.

### 4.3.2 Racetracks

In a more general situation a hidden sector gauge group is expected to be a product of simple groups
\[ \mathcal{G} = \prod_{a=1}^{n} \mathcal{G}_a \otimes U(1)^m. \]  
\( (4.23) \)
Some of the \( \mathcal{G}_a \) will be asymptotically free and can therefore form gaugino condensates. In absence of (light) charged matter it is natural to assume that the corresponding superpotential will be a sum of various non-perturbatively generated superpotentials. For the simplest case of two condensates in the hidden sector the superpotential takes the form
\[ \Omega(S) = A_1 e^{-a_1 k_1 S} + A_2 e^{-a_2 k_2 S}, \]  
\( (4.24) \)
4.3 Stabilization of the dilaton

Fig. 4.2: Dilaton scalar potential from gaugino condensation. One condensate leads to a run-away potential displayed in panel (a). Observe that the potential energy is positive and very large. Two (or more) condensates can lead to a local minimum shown in panel (b). Here we have assumed a real dilaton field and \( T \) to be fixed at one of the SDP.

with \( a_i = \frac{8\pi^2}{N_i} \), for SU(\( N_i \)) gauge groups, \( A_i \) are assumed real and \( k_i \) represent differing affine levels for the gauge groups. Imposing the stationary point condition \( V_S = 0 \), a solution with a positive definite Hessian is obtained for

\[
\text{Re} S_0 \simeq \frac{1}{a_2 k_2 - a_1 k_1} \log \left( \frac{A_2 k_2 a_2}{A_1 k_1 a_1} \right),
\]

where the VEV of \( \text{Im} S_0 \) is always such that the coefficients \( A_i \) at the minimum have opposite signs. Superpotentials of the form eq. (4.24) are called racetracks [148, 149], since one has to balance two exponential functions against each other in a delicate way in order to provide the formation of a minimum for the dilaton.

As an illustrative (toy) example consider a hidden sector comprising \( G_1 = \text{SU}(7) \) and \( G_2 = \text{SU}(8) \) with \( k_1 = k_2 = 1 \), \( A_1 = 1.03 \) and \( A_2 = -1 \). The corresponding scalar potential, fig. 4.2.b, develops non-trivial extrema. The asymptotic behavior for \( \text{Re} S \to \infty \) as well as the unbounded-from-below direction for \( \text{Re} S \to 0 \) are still present. The major achievement over the single condensate is the appearance of a minimum which is separated by a maximum from the run-away minimum at \(+\infty\).

Without further ingredients, however, the minima are phenomenologically unattractive. For reasonable choices of \( G_a \) and \( A_i \) the dilaton is stabilized at \( \text{Re} S \ll 1 \), suggesting a strong coupling regime. Thus, large hidden sector gauge groups (often beyond the limits of the weakly coupled heterotic strings) would be necessary to achieve \( \text{Re} S = O(1) \). Another severe problem is that the minima have a large negative vacuum energy.\(^4\) As stated by the no-go theorem of section 3.1

\(^4\) Even though racetrack models in presence of matter fields can stabilize the dilaton at acceptable values \( \text{Re} S = \frac{\gamma^2}{\text{SGUT}} \approx 2 \), the resulting minima are plagued by the negative vacuum energy [147].
this is a common difficulty in models with a tree-level Kähler potential. For this reason we will consider corrections to the Kähler potential in the next section.

### 4.3.3 Kähler stabilization

To lift the AdS vacua while retaining the classical Kähler potential eq. (4.10) one would have to include further modifications which could have back-reactions on the dynamics of moduli stabilization.

An alternative approach is to consider quantum corrections to the tree-level Kähler potential eq. (4.10). Indeed, this mechanism has been extensively studied in the literature [150–153] under the name of Kähler stabilization.

In principle the Kähler potential can receive perturbative as well as non-perturbative (S-dependent) corrections. Since in the known examples of orbifold compactifications perturbative corrections to the dilaton Kähler potential turn out to be very small [150], one is led to explore the non-perturbative ones. According to the investigation in [154], the dilaton Kähler potential can experience stringy non-perturbative effects which may be sizable, even in the weak coupling regime.

The form of such non-perturbative Kähler potentials has been argued to be [150, 154]

\[
K_{\text{TREE+NP}} = \log \left[ \frac{1}{S + \bar{S}} + d \left( \frac{S + \bar{S}}{2} \right)^{p/2} e^{-b \sqrt{\frac{S + \bar{S}}{2}}} \right].
\]

(4.26)

The first term represents the tree-level contribution and \(d, p, b > 0\) are real numbers parameterizing the non-perturbative correction. Eq. (4.26) does not change the transformation properties of the Kähler potential under \(\text{SL}(2, \mathbb{Z})\) due to the invariance of \(S\) under these transformations. Observe that in the decoupling limit \(\text{Ref} S \to \infty\) eq. (4.26) yields the tree-level form whereas for \(\text{Ref} S = O(1)\) the non-perturbative part can indeed be large in magnitude.

Consider now the run-away situation in the single condensate case of section 4.3.1. The presence of the additional term in the Kähler potential will enable the Kähler metric to have zeros for some values of the parameters \(d, p, b\) as illustrated in fig. 4.3.a. A vanishing Kähler metric results in the appearance of singularities in the scalar potential eq. (4.16) as it depends on the inverse Kähler metric. Requiring a positive kinetic term for the dilaton, \(K_{S} > 0\) at \(\text{Re} S \sim 2\), will restrict the values of the parameters \(d, p, b\). The analysis in [153] shows that physically meaningful choices correspond to \(p \sim b = O(1)\). With \(d > p, b\) one can arrange for a singularity in \(K_{S}^{-1}\) and reduce this singularity to a maximum by fine-tuning \(d\). Then, as evident from fig. 4.3.b, the potential will have a minimum very close to the barrier. However, for all reasonable values of the parameters \(d, p, b\) the vacuum energy turns out to be positive and large in magnitude [153].

\[^{3}\text{Yet another possibility is the inclusion of fluxes [45, 46].}\]
Fig. 4.3 **Panel (a)** shows the Kähler metric for different values of the parameter $d$. By fine-tuning $d$ one can adjust the minimum very close to zero. This results in a barrier in the scalar potential displayed in panel (b). The red (dashed) curve represents negative kinetic terms.

Although non-perturbative corrections to the Kähler potential are capable in stabilizing the dilaton at $\Re e S \sim 2$ in the presence of a single condensate, dS vacua with nearly vanishing CC cannot be achieved. It is interesting to note that in this scenario one can realize a phenomenologically acceptable gravitino mass $m_{3/2} = O(\text{TeV})$ while at the same time keeping the dilaton very heavy. This is because the mass of the dilaton $m_S = V_{SS} k^{-1}_{SS}$ is proportional to the inverse Kähler metric which induces an enhancement in the mass of several orders of magnitude. In the example shown in fig. 4.3.b we have $m_{3/2} \approx 2 \text{ TeV}$ and $m_S \approx 5 \times 10^7 \text{ TeV}$.

Stabilization of the dilaton in this framework crucially depends on the size of the non-perturbative correction. Fig. 4.3.b shows the sensitivity of the barrier on the parameter $d$. If it is too small the barrier will disappear resulting in a run-away potential for the dilaton. In the multi-condensate case the situation gets not improved. The characteristic racetrack AdS minimum would still be present at $\Re e S \ll 1$.

### 4.4 $F$-downlifting

In the previous sections we saw that it is quite difficult to simultaneously stabilize the dilaton at an acceptable VEV and to assure broken SUSY in a dS space with a nearly vanishing CC. On top of that one has to ensure a reasonable gravitino mass. In analogy to the type IIB case studied in section 3.3 we would like to investigate the impact of hidden sector matter fields in the presence of a single gaugino condensate and a tree-level Kähler potential.
4.4.1 Introducing matter fields

Consider a low energy effective 4D $\mathcal{N} = 1$ SUGRA theory originating from orbifold compactifications of weakly coupled (fluxless) heterotic string theory, involving the dilaton $S$ and a universal Kähler modulus $T$. The new ingredient we want to introduce is an additional hidden sector containing a (single) matter field $\phi$. The corresponding tree-level Kähler potential for the $ST\phi$ system is given by

$$K = -\log (S + \bar{S}) - 3\log (T + \bar{T}) + \phi \bar{\phi} (T + \bar{T})^{-n_{\phi}},$$

(4.27)

where $n_{\phi}$ denotes the modular weight of $\phi$. Demanding modular invariance of the scalar potential requires the superpotential to transform with modular weight $-3$ (cf. eq. (4.5)). Thus the field dependence in the superpotential must generically be of the form

$$W(S, T, \phi) = \frac{\Omega(S, \eta^{2n_{\phi}}(T) \phi)}{\eta^{3} (T)}.$$

(4.28)

Note that the $\eta$-function presents the only possible $T$-dependence consistent with modular invariance. In this work we will assume that $\phi$ has modular weight zero. This will forbid possible couplings between $T$ and $\phi$ in the Kähler potential providing a diagonal Kähler metric.

The scalar potential induced by eqs. (4.27) and (4.28) is

$$V = e^{\frac{K}{2}} \left[ K^{-1}_{S} G_{S} G_{S} + K^{-1}_{\bar{T}} G_{T} G_{\bar{T}} + G_{\phi} G_{\bar{\phi}} - 3 \right]$$

$$= \frac{e^{\phi \bar{\phi}}}{(S + \bar{S})(T + \bar{T})^{3} |\eta(T)|^{12}} \left[ \eta^{S} |^{2} + |\gamma^{\phi}|^{2} - 3 \delta |\Omega|^{2} \right],$$

(4.29)

where we have introduced

$$\gamma^{S} = \Omega_{S} (S + \bar{S}) - \Omega,$$

(4.30)

$$\gamma^{\phi} = \Omega_{\phi} - \phi \Omega,$$

(4.31)

$$\delta = 1 - (T + \bar{T})^{2} |\mathcal{E}(T, \bar{T})|^{2},$$

(4.32)

such that

$$F^{S} = e^{G/2} \frac{S + \bar{S}}{\Omega} \gamma^{S},$$

(4.33)

$$F^{\phi} = e^{G/2} \frac{1}{\Omega} \gamma^{\phi},$$

(4.34)

$$|F^{T}|^{2} = e^{G} (T + \bar{T})^{2} (1 - \delta),$$

(4.35)

$^{6}$In orbifold compactifications the CSM can be easily fixed through the symmetries of the orbifold.
with $\mathcal{E}(T, \overline{T})$ being the modified Eisenstein function defined in eq. (4.15). The quantities $\gamma^S$ and $\gamma^\phi$ measure the contribution from the dilaton and the matter field to SUSY breaking and $\delta$ does the same for the $T$ modulus. Observe that due to $n_\phi = 0$ the $F$-terms do not mix.

As in section 4.3.3 we can deduce two general features of the potential from its modular invariance. First, in the decompactification limit $\mathfrak{Im} T \rightarrow \infty$ (and its dual $\mathfrak{Im} T \rightarrow 0$) the product $(T + \overline{T})^3 |\eta(T)|^{12}$ vanishes exponentially, hence the scalar potential diverges at those limits. Second, since $G_T$ vanishes at the SDP, they always correspond to extrema of the potential where the $T$ modulus does not break SUSY, $F_T = 0$. From the phenomenological point of view this presents an interesting situation since the modulus mediated soft breaking terms might encompass (additional) flavor violation [86,87]. Thus it is encouraging to study this particular case and analyze whether and under what circumstances the SDP do correspond to local minima of the scalar potential.

### 4.4.2 A matter field and a condensate

As far as the $S$ and $\phi$ dependence of the superpotential is concerned we would like to consider the interaction of a single condensate with one matter field. We will assume an additive superpotential of the form

$$\Omega(S, \phi) = \omega(S) + \tau(\phi),$$

$$= A e^{-aS} + \tau(\phi),$$

where $a = 8\pi^2/N$ for a pure SU($N$) gauge group. We leave the form of the matter superpotential $\tau(\phi)$ generic.

The system under consideration consists of three subsectors. In the $S$-subsector we have an unstabilized dilaton with a runaway potential. In the $T$-subsector the SDP, although corresponding to local extrema, are not necessarily minima of the potential. Given this setup we would like to analyze the role of the $\phi$-subsector in the total $ST\phi$-system. In particular we would like to know whether the interplay between $S$, $T$ and $\phi$ can provide local minima of the potential at a reasonable value of $S$ and with SUSY broken in a nearly Minkowski space.

Let us begin by imposing the stationary point conditions

$$V_S = -\frac{V}{S + \overline{S}} + \frac{e_i K}{|\eta|^{12}} \left[ \overline{\Omega}_S \gamma^S + \Omega_{SS} \left( S + \overline{S} \right) \gamma^S \right. $$

$$\left. + \overline{\phi} \Omega_S \gamma^\phi - 3 \delta \Omega_S \overline{\Omega} \right] \doteq 0, \quad (4.38)$$

$$V_\phi = \overline{\phi} V + \frac{e_i K}{|\eta|^{12}} \left[ - \Omega_\phi \gamma^S + \overline{\Omega} \gamma^\phi \right. $$

$$\left. + \left( \Omega_{\phi S} + \overline{\phi} \Omega_\phi \right) \gamma^\phi - 3 \delta \Omega_\phi \overline{\Omega} \right] \doteq 0, \quad (4.39)$$
\[ V_T = -3E V + e^{\xi} \left[ 6(T + T) |E|^2 \\
+ 3(T + T)^2 E_T \bar{E} + 3(T + T)^2 E E_T \right] \equiv 0, \quad (4.40) \]

where as usual we use the subscripts on \( V \) and \( \Omega \) to denote differentiation with respect to the fields. In our analysis we are mainly interested in the local behavior of the scalar potential. Without loss of generality, assume that eqs. (4.38) – (4.40) are satisfied at
\[ S_0 = O(1), \quad \phi_0 \leq O(1), \quad T_0 = \{1, e^{3 \eta/6}\}. \quad (4.41) \]

To analyze stability of the stationary point we have to compute the eigenvalues of the Hessian. The corresponding second derivatives of the potential evaluated at eq. (4.41) are given by

\[ V_{SS} = \frac{\epsilon_{KK_0}}{|\eta_0|^{12}} \left[ (\Omega_{SS} (S_0 + \bar{S}_0) + \Omega_{SS}) \gamma^S \\
+ (2\Omega_2 (S_0 + \bar{S}_0) - 3\Omega_0) \Omega_{SS} + \bar{\phi}_0 \Omega_{SS} \gamma^\phi \right], \quad (4.42) \]

\[ V_{SS} = \frac{\epsilon_{KK_0}}{|\eta_0|^{12}} \left[ |\Omega_{SS} (S_0 + \bar{S}_0)|^2 + \left( |\phi_0|^2 - 2 |\Omega_5|^2 + \Omega_{SS} \gamma^S + \Omega_{SS} \gamma^\phi \right) \right], \quad (4.43) \]

\[ V_{\phi\phi} = -\bar{\phi}_0 V_0 + \frac{\epsilon_{KK_0}}{|\eta_0|^{12}} \left[ -\Omega_{\phi\phi} \gamma^\phi + 2 \Omega_2 \phi_0 \Omega_\phi \right. \\
\left. + \left( \Omega_{\phi\phi} + \phi_0 \Omega_{\phi\phi} \right) \gamma^\phi - \Omega_0 \Omega_{\phi\phi} \right], \quad (4.44) \]

\[ V_{\phi\phi} = \left( 1 - |\phi_0| \right)^2 V_0 + \frac{\epsilon_{KK_0}}{|\eta_0|^{12}} \left[ |\Omega_{\phi\phi} + \phi_0 \Omega_\phi|^2 + \Omega_0 \phi_0 \Omega_\phi \right. \\
\left. + \Omega_0 \phi_0 \Omega_\phi + |\Omega_0|^2 \right], \quad (4.45) \]

\[ V_{\phi\phi} = \frac{6 \lambda \Omega_0^2}{(T_0 + \bar{T}_0)^2}, \quad (4.46) \]
\( V_T = \frac{3 V_0}{(T_0 + T)^2} + \frac{3(1 + |A|^2)|\Omega_0|^2}{(T_0 + T)^2}, \)  

(4.49)

where the subscript \( 0 \) denotes the VEV of a quantity and

\[
\lambda = \frac{3}{2} \frac{2(T_0 + T)^2}{\eta_0} \frac{d^2 \eta}{dT^2}. 
\]

(4.50)

Note that \( V_T S = V_T \phi = V_T S = V_T \phi = 0 \) identically.

Depending on the relation between \( \gamma^S \) and \( \gamma^\phi \) one can have different scenarii of SUSY breaking. From the dilaton stationary point conditions eq. (4.38) we obtain

\[
\left(1 + e^{G_0}V_0\right)G_S + 2\left(S_0 + \bar{S}_0\right)|G_S|^2 + \left(S_0 + \bar{S}_0\right)^2 G_{SS}G_S = -G_{S\phi}G_{\phi}, \]

(4.51)

at the stationary point. If \( G_\phi = 0, G_S = 0 \) is required by stationarity. This corresponds to a purely supersymmetric configuration. Consider now the dilaton domination scenario where \( |G_\phi| \ll |G_S| \). In this limit eq. (4.51) requires an unnaturally large \( G_{S\phi} \). This requirement is difficult to achieve as one usually has \( |G_{S\phi}| = O(1) \). In the mixed dilaton-matter case with \( |G_S| \sim |G_\phi| \), eq. (4.51) again requires an unnaturally large \( G_{S\phi} \) as typically \( |G_{SS}| \gg 1 \) in models with gaugino condensation. On the other hand, in the matter domination case with \( |G_S| \ll |G_\phi| \) eq. (4.51) is easily satisfied due to large \( G_{SS} \). Thus, given our particular setup eqs. (4.27), (4.28) and (4.36), matter-dominated SUSY breaking scenario seems to be particularly suited for phenomenological considerations.

### 4.4.3 Adjusting the vacuum energy

In the light of a vanishing \( CC \), eq. (4.29) yields

\[
K_{SS}^{-1} |G_S|^2 + |G_\phi|^2 - 3 = 0. 
\]

(4.52)

in the stationary point eq. (4.41). For matter dominated SUSY breaking this implies \( |G_\phi| = O(1) \) since \( |G_S| \ll |G_\phi| \). Hence one can parameterize

\[
|G_\phi|^2 = 3 + \epsilon, 
\]

(4.53)

with \( \epsilon \ll 1 \) yielding \( e^{-G_0} V_0 \sim \epsilon \). By fine-tuning the parameters of the matter superpotential \( \tau(\phi) \) the vacuum energy can be adjusted arbitrarily small and positive without affecting other aspects of the system.

### 4.4.4 Matter dominated SUSY breaking

For matter-dominated SUSY breaking in a Minkowski vacuum, we can already deduce some features of the superpotentials realizing this situation. With the
requirements $|G_5| \ll 1$ and $G_\phi = O(1)$ or equivalently $|\gamma^5| \ll |\Omega_0|$ and $|\gamma^\phi| \sim |\Omega_0|$, eq. (4.30) immediately yields $|\Omega_5| = |\omega_5| \sim |\Omega_0|$ and, since $\omega$ contains a gaugino condensate,

$$|\Omega_5| = |\omega_5| \sim a |\Omega_0| \gg |\Omega_0|.$$  \hspace{1cm} (4.54)

From eq. (4.31) we obtain

$$|\Omega_\phi| = |\tau_\phi| \sim |\Omega_0|.$$  \hspace{1cm} (4.55)

Under these circumstances, $|V_5| \gg |V_{\phi\phi}|$, $|V_{\phi5}| > |V_{\phi\phi}|$. Higher derivatives of the superpotential with respect to $\phi$ remain undetermined at this stage. To obtain a particularly transparent expression for the Hessian let us choose (yet unconstrained) $\Omega_{\phi\phi\phi}$ such that the matrix element $V_{\phi\phi}$ is small. Then

$$\frac{\partial^2 V}{\partial \phi_i \partial \phi_j} \approx \frac{\phi^K_0}{|\eta_0|^{1/2}} \begin{pmatrix} |\Gamma|^2 & 0 & 0 & 0 & \Gamma \bar{\theta} & 0 \\ 0 & |\Gamma|^2 & 0 & 0 & 0 & \Gamma \theta \\ 0 & 0 & \Delta \Xi & \Delta \Upsilon & 0 & 0 \\ 0 & 0 & \Delta \bar{\Xi} & \Delta \bar{\Upsilon} & 0 & 0 \\ \Gamma \theta & 0 & 0 & 0 & \Delta & 0 \\ 0 & \Gamma \bar{\theta} & 0 & 0 & 0 & \Delta \end{pmatrix},$$  \hspace{1cm} (4.56)

with

$$\Gamma = \Omega_{55} \left( S_0 + \bar{S}_0 \right),$$ \hspace{1cm} (4.57)

$$\theta = -\Omega_\phi,$$ \hspace{1cm} (4.58)

$$\Delta = |\Omega_0|,$$ \hspace{1cm} (4.59)

$$\Upsilon = \frac{6}{(T_0 + \bar{T}_0)^2} \lambda,$$ \hspace{1cm} (4.60)

$$\Xi = \frac{3}{(T_0 + \bar{T}_0)^2} \left( 1 + |\lambda|^2 \right),$$ \hspace{1cm} (4.61)

where $|\Gamma| \gg |\theta|, \Delta$. For convenience, the indices of the Hessian are defined as $(x_1, x_2, x_3, x_4, x_5, x_6) = (S, \bar{S}, T, \bar{T}, \phi, \bar{\phi})$. The eigenvalues of eq. (4.56) are

$$\frac{1}{2} \left( \Delta + |\Gamma|^2 + \sqrt{|\Gamma|^4 + 4 |\Gamma|^2 |\theta|^2 - 2 \Delta |\Gamma|^2 + \Delta^2} \right) \approx \frac{\Delta}{2} + |\Gamma|^2,$$ \hspace{1cm} (4.62)

$$\frac{1}{2} \left( \Delta + |\Gamma|^2 - \sqrt{|\Gamma|^4 + 4 |\Gamma|^2 |\theta|^2 - 2 \Delta |\Gamma|^2 + \Delta^2} \right) \approx \frac{\Delta}{2},$$ \hspace{1cm} (4.63)

$$(\Xi - |\Upsilon|) \Delta,$$ \hspace{1cm} (4.64)

$$(\Xi + |\Upsilon|) \Delta.$$ \hspace{1cm} (4.65)

Note that the first two eigenvalues are degenerate. All eigenvalues are positive provided $\Xi > |\Upsilon|$. This poses a constraint on the $T$-subsector. In particular it
implies $V_{TT} > |V_{TT}|$ at the minimum. A numerical inspection reveals that at the
SDP (which we assume to be the minima)

$$
\Xi - |\Upsilon| \left|_{T_0=1} \approx 0.14, 
\Xi - |\Upsilon| \left|_{T_0=\Xi/\Omega_0} \approx 0.77.
$$

Thus indeed the stationary point eq. (4.41) is a minimum!

Moreover, the spectrum consists of six states. Two of them are heavy with
masses of order $|\Gamma| ~ |\Omega_{SS}|$ and correspond to the dilaton. The remaining four
states are light and have masses of order $\sqrt{\AA} \sim |\Omega_0|$. They correspond to the $T$
modulus and the matter field $\phi$.

We note that the SDP are not the only minima in the $T$-direction. There also
exist nearby minima with $F^T \neq 0$.

In our analysis we have assumed an additive superpotential eq. (4.36). It is also
possible to construct models with more general superpotentials allowing for a
mixing between $S$ and $\phi$ along the lines of [57]. In a very recent publication [155]
the case with non-zero modular weights has been studied. It was found that
the requirement of $T$ being stabilized at one of the SDP significantly restricts the
possible values for the modular weights $n_\phi$.

## 4.5 The pattern of SUSY breaking in $F$-downlifting

Let us examine the structure of the SUSY breaking parameters appearing in the
matter domination case. Since we have stabilized $T$ at one of its SDP, the modulus
subsector is not involved in SUSY breaking, $F^T = 0$. To estimate the size of the
dilaton auxiliary field, we recall that for a Minkowski vacuum the stationary
point conditions eqs. (4.38) – (4.40) yields

$$
\Omega_{SS} (S + S) \gamma^S + \Omega_{SS} \gamma^S = 3 \Omega_0 \Omega_0 - \phi_0 \Omega_5 \gamma^\phi,
$$

where $\Omega_{SS} \sim a \Omega_0 \gg \Omega_0$ and the right-hand side is $\mathcal{O}(\Omega_0)$. Plugging this into
eq \text{eq. (4.33)} gives

$$
P^S \sim \frac{m_3/2 \Omega_{SS}}{\Omega_{SS}} \sim \frac{m_3/2}{a}.
$$

Therefore, the SUSY breaking contribution from the dilaton is suppressed. For
the matter field we have $|\gamma^\phi| \sim |\Omega_0|$, hence eq. (4.34) straightforwardly yields

$$
P^\phi \sim m_3/2,
$$

implying that $\phi$ is the dominant source of SUSY breaking, as expected for matter
domination.
Next we would like to estimate the masses of the fields involved in the effective SUGRA. First of all, the mass of the gravitino is given by

\[ m_{3/2} = e^{\kappa_0/2} = e^{\kappa_0} |W_0| \sim |\Omega_0| \sim a |\omega_0|, \]

where we have used \( e^{\kappa_0}|\eta_0|^{-6} = O(1) \). Hence it is related to gaugino condensation.

The mass of the dilaton is given by

\[ m_2^S = \frac{V_{SS}}{K_{SS}} \sim |\Omega_0|^2 \sim (a \Re S_0)^2 |\Omega_0|^2 \sim (a \Re S_0)^2 m_{3/2}^2, \]

indicating that the dilaton is heavy compared to \( m_{3/2} \): its mass is enhanced by the same factor which suppresses \( F^5 \). For the matter field \( \phi \) we obtain

\[ m_2^\phi = \frac{V_{\phi\phi}}{K_{\phi\phi}} \sim |\Omega_0|^2 \sim m_{3/2}^2, \]

verifying that the matter field is indeed a light DOF. This is also consistent with the fact that its \( F \)-term is not suppressed. Finally, for the \( T \) modulus we find

\[ m_T^2 = \frac{V_{TT}}{K_{TT}} \sim |\Omega_0|^2 \sim m_{3/2}^2, \]

thus \( T \) remains light.

### 4.5.1 The little hierarchy

If we compare the results obtained in the context of (fluxless) heterotic orbifold models to the type IIB case we, in fact, end up with very similar conclusions. There is, however one significant difference between these two frameworks. In the type IIB case the starting point was a supersymmetric theory in an AdS vacuum with all moduli fixed. The matter sector was then responsible for breaking SUSY and uplifting the vacuum energy to a desired value. In the (fluxless) heterotic setup we started with an unstabilized (run-away) dilaton. The superpotential interaction involving matter fields (together with the requirement of modular invariance) provides the stabilization of the dilaton and the Kähler modulus. We would like to emphasize that the stabilization of the dilaton at the phenomenologically favored value \( \Re S \sim 2 \) is possible with just one gaugino condensate and \( T \) can be stabilized at the SDP. Moreover, the matter sector also indicates the breakdown of SUSY.

Recall from section 4.3.1 that in the absence of the matter sector the vacuum energy is large and positive. The impact of the matter sector is such that it changes the shape of the scalar potential as to form local minima and “downlifts” the vacuum energy to a small positive or zero value. All this results from the large magnitude of the \( F \)-term of the hidden sector matter field. For this reason we will refer to this procedure as \( F \)-downlifting [58].
4.5 The pattern of SUSY breaking in F-downlifting

The pattern of SUSY breaking in F-downlifting

\[ F \downarrow \text{lifting} \]

<table>
<thead>
<tr>
<th>( c )</th>
<th>( \mu^2 )</th>
<th>( \phi_0 )</th>
<th>( F^S )</th>
<th>( F^\phi )</th>
<th>( m_{3/2} )</th>
<th>( m_S )</th>
<th>( m_\phi/m_T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5 \times 10^{-16} )</td>
<td>( 2 \times 10^{-15} )</td>
<td>( 0.73 )</td>
<td>( 2 \times 10^{-16} )</td>
<td>( 3 \times 10^{-15} )</td>
<td>( 5 \text{ TeV} )</td>
<td>( 365 \text{ TeV} )</td>
<td>( 8 \text{ TeV} )</td>
</tr>
</tbody>
</table>

**Tab. 4.1** :: Sample spectrum for a hidden SU(4) and \( A = 3 \).

The hierarchical structure among the masses and the F-terms has its origin in the appearance of the factor

\[ a \text{Re} S_0 \sim \log \left( \frac{A}{\Omega} \right) \sim \log \left( \frac{M_P}{m_{3/2}} \right), \]

known as the little hierarchy \([47,50]\). It suppresses the dilaton contribution to the soft breaking terms and enhances its mass

\[ m_S \sim (a \text{Re} T_0) m_{3/2} \sim (a \text{Re} T_0) m_T \sim (a \text{Re} T_0) m_\phi. \]

4.5.2 Another example

As a concrete realization of the downlifting procedure consider a Polonyi-type superpotential \([9,126]\)

\[ W = c + \mu^2 \phi - A e^{-a S}, \]

where \( c \) and \( \mu^2 \) are real constants and we consider a SU(4) hidden sector gauge group. For simplicity we treat \( S \) and \( \phi \) as real fields. Demanding the condition \( V = \partial_S V = \partial_\phi V = \partial_T V = 0 \) at \( S_0 = \text{Re} S_0 = 2, T_0 = 1 \) will provide a vacuum configuration with vanishing energy and all moduli fixed in a local minimum as illustrated in fig. 4.4. The values of parameters of the Polonyi subsector and the representative quantities in the vacuum are summarized in tab. 4.1. From the shape of the scalar potential fig. 4.4.a we see that it is much steeper in the \( S \)-direction than in the \( T \)-direction (or in the \( \phi \)-direction) indicating that the dilaton is heavier than the Kähler modulus.
4.5.3 Soft breaking terms

In the scheme of $F$-downlifting in the framework of heterotic string theory, the situation is (very) similar to the $F$-uplifting in the type IIB context. The sources of the soft breaking terms are given by

\[ F_S \sim \frac{m_{3/2}}{a}, \quad (4.78) \]
\[ F_T = 0, \quad (4.79) \]
\[ F^\phi \sim m_{3/2}, \quad (4.80) \]
\[ \frac{F_C}{C_0} \sim m_{3/2}, \quad (4.81) \]

in case if the Kähler modulus is stabilized at the SDP. We recall from the above discussion that the downlifting sector does not necessary take a sequestered form. Therefore, in addition to the conformal anomaly mediation, it can induce contributions to the soft parameters coming from Kähler anomalies and string threshold corrections. Such contributions affect the predictability of the scheme and model-independent statements will not be possible [92]. As discussed in section 3.4.4 one can suppress such dangerous contributions by assigning discrete symmetries to the downlifting sector. In models realizing $\phi_0 \ll 1$ such contributions are subleading, too.

Since the $T$ modulus does not communicate the breakdown of SUSY we can neglect it and formulate an “effective” theory described by

\[ f_a = S, \quad (4.82) \]
\[ K = -\log (S + \overline{S}) + \phi \overline{\phi} + Q^i Q^j Z_{ij}, \quad (4.83) \]

with the visible Kähler metric

\[ Z_i = 1 + \xi \phi \overline{\phi}, \quad (4.84) \]

where $Q^i$ are visible fields and $\xi_i$ measures the coupling between visible and hidden matter in the Kähler potential. As usual we assume MSSM matter content and require $g_{\text{GUT}}^{-2} \sim 2$ at the GUT scale. Using the formulae of appendix A.4 the soft terms just below the GUT scale are

\[ M_a = \frac{F_S}{S_0 + \overline{S}_0} + \frac{b_a}{4} g_{\text{GUT}}^2 \frac{1}{4\pi^2 C_0} F_C, \quad (4.85) \]
\[ A_{ijk} = -\frac{F_S}{S_0 + \overline{S}_0} + \frac{\gamma_i + \gamma_j + \gamma_k}{4} \frac{1}{4\pi^2 C_0} F_C, \quad (4.86) \]
\[ m_i^2 = \xi_i \left[ \frac{|F_S|^2}{(S_0 + \overline{S}_0)^2} - \frac{\gamma_i}{4} \frac{1}{4\pi^2 C_0} \right] + \frac{F_S \partial_S \gamma_i}{2} \frac{1}{4\pi^2 C_0} + (1 - 3 \xi_i) m_{3/2}^2, \quad (4.87) \]
where \( b_i \) are the \( \beta \)-function coefficients and \( \gamma_i \) describes the RG running of the anomalous dimensions \( \gamma_i \).

The soft terms look very similar to the ones obtained in the \( F \)-uplifting scenario eqs. (3.72) – (3.74). Since the dilaton contribution is suppressed by the little hierarchy, anomaly mediation becomes competitive, leading to a mixed dilaton-anomaly mediation. In case of the gaugino masses we have a universal contribution from the dilaton and a non-universal one from the conformal anomaly in terms of the MSSM 1-loop \( \beta \)-function coefficients. At 1-loop the gaugino masses evolve with the same \( \beta \)-functions, hence at an intermediate scale anomaly mediation cancels the RG evolution again leading to a mirage unification of the gaugino masses at \( \mathcal{M}_{\text{Mir}} \) given by eq. (2.58).

The form of the \( A \)-terms and the scalar squared masses is similar to eqs. (3.73) – (3.74), with the difference that the modular weights are now absent. There is, however, another subtle difference between the results in the type IIB and the heterotic case: in the heterotic case the anomaly mediated contributions to the \( A \)-terms and scalar squared masses appear enhanced compared to the type IIB situation, where the modulus mediated part contains a factor of 3 originating from \(-3 \log(T + T)\) as compared to \(-\log(S + S)\). As we shall see in section 5.5, this will result in a slightly different low energy phenomenology between \( F \)-uplifting and \( F \)-downlifting. We can summarize the pattern of the scalar squared masses as follows.

**Tachyons ::** Anomaly mediated contributions to the scalar squared masses encoded in the second and the third term in eq. (4.87) induce tachyonic squarks and sleptons \([47, 51, 52, 80]\). Since the tree-level dilaton contribution is suppressed by a factor of 3 as compared to eq. (3.74), we expect a larger tachyonic region here. Thus in the heterotic framework models with a too small \( F \) are disfavored. However, since the matter/downlifting contribution is not (fully) sequestered, the scalar squared masses are likely to receive contribution of \( \mathcal{O}(m_{3/2}^2) \) from the matter field. Hence, for \( \xi_i \ll 1/3 \), tachyons do not appear.

**Flavor ::** In this regard the situation is the same as in the type IIB case. Demanding flavor-independent couplings \( \xi_i = \xi \forall i \) can avoid additional flavor violation.

**Mass pattern ::** Since for \( \phi_0 \ll 1 \) we neither expect \( \phi \)-dependent corrections to the gauge kinetic function nor additional loop contributions beyond that of conformal anomaly, the soft gaugino masses show a mirage pattern as illustrated in fig. 2.3.b.

The fate of the scalar masses depends on the parameter \( \xi_i \). Models with \( \xi_i \ll 1/3 \) show a relaxed mirage pattern

\[
M_a \ll m_i \sim m_{3/2},
\]  

(4.88)
and those with $\xi_i = O(1)$ allow for a pure mirage pattern

$$M_a \sim m_i \ll m_{3/2}. \quad (4.89)$$

**Cosmology** :: One way to avoid cosmological gravitino and moduli problems [113] in string derived SUGRA theories, is to make these particles sufficiently heavy. In the framework of $F$-downlifting, the little hierarchy enhances the mass of the dilaton and suppresses the soft masses (at least those of the gauginos) and thus can serve to alleviate these problems. The $T$ modulus, however, has a mass comparable to the gravitino and might give rise to the moduli induced gravitino problem [127,128]. To mitigate this issue one has to rely on suitable extensions of the downlifting scheme which goes beyond the scope of this work.

Using our parameterization eq. (2.51), (2.52) and (3.77) we can recast the soft parameters eqs. (4.85)–(4.87) as (cf. appendix A.4)

$$M_a = M_0 \left[ \rho + b_\alpha \delta_{\alpha}^2 \right], \quad (4.90)$$

$$A_{ijk} = M_0 \left[ - \rho + \left( \gamma_i + \gamma_j + \gamma_k \right) \right], \quad (4.91)$$

$$m_i^2 = M_0^2 \left[ \xi_i \rho^2 - \gamma_i^2 + 2 \rho \Psi_i^S + \eta_i^2 \right], \quad (4.92)$$

with $\Psi_i^S$ denoting the $S$ dependence of the anomalous dimension.

### 4.6 Application

As one possible application of the $F$-downlifting mechanism we consider the issue of moduli stabilization in the context of heterotic orbifold compactifications with fluxes$^7$ [65, 156, 157].

The presence of fluxes can generate a perturbative superpotential for the Kähler moduli and the CSM, however not for the dilaton. Thus, a non-perturbatively generated superpotential for the dilaton is required, for which hidden sector gaugino condensation [21–26, 76] is well equipped. Moreover, it provides a dynamical mechanism to explain a hierarchically small scale of the gravitino mass

$$m_{3/2} \sim \frac{\Lambda^3}{M_P^2}, \quad (4.93)$$

and $F \sim \Lambda^3 / M_P^2$, where $\Lambda$ denotes the RG invariant scale (condensation scale) of a hidden sector gauge group. It requires $\Lambda$ to be at an intermediate scale if $m_{3/2}$ is at the (multi) TeV scale.

---

$^7$This setup goes beyond the “standard” CY compactifications. Modular symmetry eq. (4.1) might in general be absent in such constructions.
In the framework of heterotic orbifolds with fluxes, Derendinger, Kounnas, and Petropoulos (DKP) recently identified an even stronger (quadratic) suppression of the gravitino mass \( m_{3/2} \sim \frac{\Lambda^6}{M_p^5} \). (4.94)

In a rather natural way \( \Lambda \) could be identified with the GUT scale \( M_{\text{GUT}} \) or the compactification scale \( M_{\text{COMP}} \) (typically assumed to be near the GUT scale), thereby avoiding an intermediate scale. Thus a single \( \Lambda \) might represent \( M_{\text{GUT}} \sim M_{\text{COMP}} \) as well as the hierarchically small scale \( m_{3/2} \).

While fluxes combined with gaugino condensation are sufficient to stabilize all moduli in many cases, they fail to do so in the model of DKP. The obtained solution with the doubly suppressed gravitino mass contains two unstabilized moduli. They appear as a consequence of a restricted no-scale ansatz and it remains to be seen whether the doubly suppressed solution survives in the process of moduli stabilization.

In this section we would like to study an alternative approach realizing the doubly suppressed solution of DKP. The major difference to the DKP is that we first attempt to stabilize all moduli and then adjust the vacuum energy adopting the \( F \)-downlifting mechanism.

**4.6.1 The model of DKP**

The low energy effective SUGRA from \( \mathbb{Z}_2 \times \mathbb{Z}_2 \) orbifold compactifications [159–161] with fluxes [65, 156, 157] involves the dilaton \( S \), three Kähler moduli \( T_1, T_2, T_3 \) and three CSM which we will generically denote by \( U \). The tree-level Kähler potential is given by [158]

\[
K = -\log (S + \bar{S}) - \sum_{i=1}^{3} \log (T_i + \bar{T}_i) - 3 \log (U + \bar{U}).
\] (4.95)

The particular idiosyncrasy of the DKP model is founded in the special form of the superpotential which is assumed to be generated by fluxes and gaugino condensation,

\[
W = 3 \hat{A} U + \hat{D} U^3,
\] (4.96)

with

\[
\hat{A} = [\alpha + \alpha' w(\tilde{S})] \xi + D w(\tilde{S}),
\] (4.97)

\[
\hat{D} = [\delta + \delta' w(\tilde{S})] \xi + D w(\tilde{S}),
\] (4.98)

where \( \xi = T_1 - T_2, \ w(\tilde{S}) = e^{-\tilde{S}} \) describes the gaugino condensate and \( \tilde{S} = 8\pi^2 S/N \) for a hidden SU(N) gauge group. The parameters \( \alpha, \alpha', \delta, \delta', A, D \) are flux coefficients.
Aiming at a no-scale configuration, DKP make a specific ansatz demanding SUSY broken in Minkowski space, which implies

\[ \langle V \rangle = 0, \quad \langle W \rangle \neq 0, \] (4.99)

in the minimum. Then, the stationary point condition \( \partial_j V = 0 \ \forall j \) splits the moduli into two categories, with either \( \langle \partial_j W \rangle = 0 \) and \( \langle F^j \rangle \neq 0 \) or \( \langle F^j \rangle = 0 \). The index \( j \) runs over the moduli. The first category contains the three Kähler moduli responsible for SUSY breaking. The second category contains the dilaton and the CSM and is not involved in SUSY breaking. Moreover the stationary point conditions also requires \( \Re \xi = 0 \) [158].

From the stationarity conditions one obtains two conditions: \( \tilde{S} = \tilde{s} - \text{in}/2 \) and \( U = u \) real. Everything is consistent provided \( \alpha, \delta, A, D \) are real and \( \alpha', \delta' \) are imaginary [158]. The no-scale requirement allows one to express \( \xi \) and \( u \) as functions of \( \tilde{s} \) through

\[
\begin{align*}
  u(\tilde{s}) &= \sqrt{\frac{\hat{A}}{D}}, \\
  \xi(\tilde{s}) &= -\frac{1}{4} \frac{D \alpha + A \delta + (3 D \alpha + A \delta) w}{(\alpha + \alpha' w)(\delta + \delta' w)} w.
\end{align*}
\] (4.100)

As argued in [158] the dilaton can be stabilized at an acceptable VEV provided that the flux coefficients \( \alpha, \alpha', \delta, \delta', A, d \) are large, while their ratios are \( O(1) \). If this requirement is fulfilled, we can define a variable \( \rho \) as

\[ \rho = i \frac{D \alpha - A \delta}{D \alpha w}, \] (4.102)

which can be consistently taken to be \( O(1) \) since \( w \) is small and \( D \alpha / A \delta = O(1) \). This of course requires a certain amount of fine-tuning for \( D \alpha - A \delta \ll 1 \).

In the limit \( \alpha' = i \alpha, \delta' = -i \delta \), the mass of the gravitino is given by [158]

\[
\begin{align*}
e^{-K/2} m_{3/2} &= |W| \\
&\approx 2D e^{-2\tilde{s}} + \frac{A \delta - D \alpha}{\alpha} e^{-\tilde{s}} \left| \frac{3 \alpha}{\alpha'} \right|^{3/2} \\
&\approx 4D \left( -\frac{3 \alpha}{\alpha'} \right)^{3/2} \frac{\tilde{s}}{2\tilde{s} + 1} w^2,
\end{align*}
\] (4.103)

where the third line in eq. (4.103) is obtained under the above mentioned fine-tuning, stating that the gravitino mass scales as \( w^2 \). As studied in [59] the DKP fine-tuning is typically of order \( 10^{-3} \) and can be considered as rather mild.\(^8\)

Eq. (4.103) written in the form shows that the dilaton is stabilized through the presence of the condensate. Strictly speaking, this is not a racetrack mechanism.

\(^8\)It is worthwhile to recall that this fine-tuning is much less severe than that of the KKLT model which is of order \( 10^{-16} \) [49].
proper [148, 149], because we only have one condensate. However, the condensate enters into the superpotential eq. (4.96) in a rather complex way, and several terms are added together. It gives a result which effectively looks like a racetrack model.

Observe that the left-hand side of eq. (4.103) contains two unstabilized moduli, namely $T_1 + T_2$ and $T_3$. To stabilize these moduli one can assume $T$-dependent corrections to the Kähler potential [73, 162], demanding the flatness condition only locally. However, it is not easy to find a theoretical justification for this kind of corrections in string theory.

A common difficulty of the no-scale models are vanishing tree-level soft terms, since $F_S = 0$ in the minimum. Thus, loop-suppressed anomaly mediation will dominate the soft terms [51, 52, 80]. As already discussed above, pure anomaly mediation predicts tachyonic sleptons. To obtain a realistic model one can include radiative corrections to the gauge kinetic function [140, 163–165] and the Kähler potential [164, 166, 167]. Generically these corrections induce a mixing between $S$ and $T$ and can therefore lead to $F_S \neq 0$, thus reintroducing the tree-level contribution to the soft terms. The mixing between $S$ and $T$ also induces a shift in the location of the minimum. Such a shift will generically lead to a non-vanishing vacuum energy, spoiling the DKP no-scale configuration eq. (4.99). Thus the vacuum energy has to be tuned a second time. Moreover, the doubly suppressed solution might not survive this procedure. A more appealing approach would be to first stabilize all moduli and then take care of (the tuning of) the vacuum energy once and for all [59].

### 4.6.2 A benchmark model

In this section we would like to analyze whether the observed double suppression of the gaugino condensate can be realized in a more general setup, or whether it is tied to the specific ansatz adopted by DKP.

#### Basic ingredients

Let up recapitulate the basic ingredients needed for the double suppression. The obvious requirement is the absence of a perturbative superpotential for the dilaton. This is automatically fulfilled in the heterotic string theory as the dilaton appears only through the condensate. We also need some fine-tuning of parameters of the superpotential to suppress unwanted/disturbing contributions. One should also note that terms with $e^{-S}$ in the superpotential need to be multiplied by nontrivial functions of the $T$ moduli (a generic result obtained in heterotic string theory originating in world sheet instantons [168, 169]). Finally we need a superpotential with terms that allow large masses for the $T$ moduli, although the classical superpotential does not include quadratic terms in $T$ (but only constant and linear terms). This requirement has been studied in detail in [47] and strongly relies on the existence of the CSM.
Towards a resolution

Given the guidelines above we want to construct and analyze a simpler framework covering the main features of the DKP model. Thus we consider an effective low energy SUGRA approximation of a heterotic string theory setup with fluxes, containing the dilaton $S$, a universal Kähler modulus $T$ and a universal CSM $U$. Due to the presence of fluxes a nontrivial superpotential for the CSM is generated, resulting in the stabilization of $U$. Following the discussion in [47], integrating out the $U$ modulus will provide us with an effective superpotential which could include terms quadratic (and higher order) in $T$. We therefore assume the effective superpotential to be of the form \[ W = \mathcal{A}_0 e^{-a S} T + \mathcal{A}_1 T + \mathcal{A}_2 T^2 + \cdots + \mathcal{A}_n T^n, \] (4.104)

where $\mathcal{A}_0, \ldots, \mathcal{A}_n = O(1)$ and $a \gg 1$ are real constants.

As a next step let us fine-tune the coefficient $\mathcal{A}_1$ to be very small such that the term linear in $T$ becomes negligible. From the equation of motion for the $T$ modulus $F_T = 0$ we obtain

\[ 0 = W \partial_T K + \partial_T W, \]
\[ = \mathcal{A}_0 \left(-\frac{3}{2} + 1\right) e^{-a S} + \mathcal{A}_2 \left(-\frac{3}{2} + 2\right) T + \cdots + \mathcal{A}_n \left(-\frac{3}{2} + n\right) T^{n-1}, \] (4.106)

where for simplicity we have assumed real fields $S$ and $T$. For eq. (4.106) to be satisfied, the smallness of the condensate requires $T$ to be small. This implies that $T^3$ and higher powers of $T$ can be safely neglected in eq. (4.104). From the equation of motion eq. (4.106) one obtains

\[ T = \frac{\mathcal{A}_0}{\mathcal{A}_2} e^{-a S}. \] (4.107)

Consequently, we can “integrate out” the $T$ field and end up with the effective superpotential

\[ W_{\text{eff}} = 2 \frac{\mathcal{A}_0^2}{\mathcal{A}_2} e^{-2a S}. \] (4.108)

This is exactly the double suppression as obtained in the DKP model. The fact that the only $S$ dependence of the superpotential is encoded in the gaugino condensate can be identified as the crucial requirement for the double suppression.

The mild fine-tuning of DKP ($A \delta - D \alpha \ll 1$) has a counterpart in our benchmark model: the coefficient of a possible term linear in $T$ has to be small, otherwise, the double suppression would be spoiled.

At this stage, however, the dilaton is not yet stabilized since a single condensate leads to a run-away scalar potential. The remaining task to perform is to stabilize the dilaton and assure a reasonable vacuum energy. As we saw in section 4.4 these two operations can be done economically in one step by adopting the downlifting strategy.
4.6 Application

Downlifting the dilaton

Following the discussion in section 4.4, we consider the impact of hidden sector matter through the interaction with the effective theory obtained after integrating out \( U \) and \( T \) moduli. For concreteness and simplicity we will focus on a Polonyi-type superpotential \([9,126]\) so that the full superpotential is given by

\[
W_{\text{Polonyi}} = -A e^{-2a S} + c + \mu^2 \phi,
\]

where we have chosen \( 2A_{0}/A_{2} = -A \), \( c \) and \( \mu \) are real constants, and \( \phi \) represents a hidden sector matter field. The effective Kähler potential is

\[
K = -\log (S + \bar{S}) + \phi \bar{\phi}.
\]

As we saw above, systems of this type are capable of changing the shape of the runaway dilaton potential and lead to formation of stationary points. The stationary point in the configuration eqs. \((4.109)\) and \((4.110)\), turns out to be a local minimum. By appropriately choosing the parameters of the Polonyi sector the CC can be adjusted/fine-tuned to the desired value.

The consequence of the \( F \)-downlifting is the appearance of the little hierarchy \([58]\) originating from the factor

\[
2a \Re e S_{0} \sim \log \left( \frac{M_{P}}{m_{3/2}} \right),
\]

which is \( O(4\pi^{2}) \) for a (multi) TeV gravitino mass. In particular is leads to the suppression of the dilaton contribution to the soft terms

\[
F^{S} \sim \frac{m_{3/2}}{2a} = O\left( \frac{m_{3/2}}{4\pi^{2}} \right),
\]

such that SUSY breaking is dictated by the matter sector, \( F^{\phi} \sim m_{3/2} \). The scale of the soft terms is set by the gravitino mass

\[
m_{3/2} = e^{K^{/2}} |W| \sim \mu^{2},
\]

implying that \( \mu^{2} \) sets the scale of the gravitino (and also the mass of the Polonyi field). However, from eq. \((4.71)\) we immediately conclude that \( \mu^{2} \sim 2a e^{-2a S_{0}} \), consequently, the gravitino mass originates from gaugino condensation and is doubly suppressed. In tab.4.2 we present an explicit realization of the doubly suppressed gravitino mass based on the hidden sector gauge group SU(8), yielding a phenomenologically viable gravitino mass.

Small gravitino mass with a large \( \Lambda \)

In our benchmark model we have considered a hidden sector group SU(8) assuming a pure supersymmetric SU(8) gauge theory as well as the equality of
Downlifting in heterotic string theory

The gauge coupling constants of hidden and observable sector. This group could originate certainly from the SO(32) heterotic string theory but not so easily [170] from the $E_8 \times E_8$ theory favored by phenomenological arguments [61, 62]. String threshold corrections, however, might enlarge the hidden sector gauge coupling compared with the observable sector gauge groups and thus reopen many new ways for model building. In fact, in heterotic $M$ theory [171, 172], a larger coupling in the hidden sector might appear in a natural way [173-175]. Such models might then explain all scales directly from the string scale, without invoking the existence of an intermediate scale.

<table>
<thead>
<tr>
<th>$c$</th>
<th>$\mu^2$</th>
<th>$\phi_0$</th>
<th>$F^5$</th>
<th>$F^9$</th>
<th>$m_{3/2}$</th>
<th>$m_T$</th>
<th>$m_{\phi}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 \times 10^{-15}$</td>
<td>$6 \times 10^{-15}$</td>
<td>0.73</td>
<td>$3 \times 10^{-16}$</td>
<td>$7 \times 10^{-15}$</td>
<td>9 TeV</td>
<td>707 TeV</td>
<td>16 TeV</td>
</tr>
</tbody>
</table>

**Tab. 4.2**: Sample spectrum with a multi-TeV gravitino for SU(8).
Chapter 5

Phenomenology of uplifting/downlifting

We analyze in detail the phenomenological properties of the $F$-uplifting and the $F$-downlifting scheme. In particular, we discuss the behavior of the soft parameters at the GUT and the TeV scale. We consider a number of phenomenological constraints and find that there are considerable regions in the parameter space where the low energy spectra satisfy all of the constraints. Although the schemes of $F$-uplifting and $F$-downlifting have a very similar low energy phenomenology, they also exhibit some quantitative differences.

5.1 Preliminaries

In chapters 3 and 4 we have seen that under suitable theoretical assumptions the uplifting/downlifting procedure leads to a scenario where the soft SUSY breaking terms are induced by a hybrid mediation scheme and exhibit the so-called relaxed mirage pattern. Depending on a particular string theory setup the ratio between modulus (dilaton) and anomaly mediation, $\varrho$, will change, thereby affecting the pattern of the soft terms. The (geometrical) origin of matter fields influences the soft parameters via the modular weights $n_i$. In addition, the uplifting/downlifting sector provides a contribution to SUSY breaking encoded in the parameter $\xi_i$.

For the study of the low energy phenomenology we will not consider a specific string theory compactification. Instead, we follow a bottom-up approach. That is, we consider generic effective SUGRA models (which may originate from certain string theory setups) and treat $\varrho$, $m_{3/2}$, $n_i$ and $\xi_i$ as free parameters. To be concrete, we consider a class of models with negligible VEV of the uplifting/downlifting field $\phi_0 \ll 1$, as well as zero modular weights $n_i = 0$.

In our analysis henceforth, we will assume a GUT gauge group in the visible sector which is broken to the SM gauge group, with at least an MSSM chiral spectrum. For simplicity, we will present our results for SU(5) broken to the SM with just the MSSM chiral spectrum. All our results should hold for other GUT group breaking in the same way as well. Because of a GUT-like spectrum, the MSSM gauge couplings are unified at $2 \times 10^{16}$ GeV [27–30]. Since we are assuming an MSSM visible sector below the GUT scale, the gauge couplings are subject to
Phenomenology of uplifting/downlifting

F-uplifting (Type IIB framework)

\[ M_a = M_0 \left[ \varrho + b_a \frac{g_{\text{GUT}}^2}{16\pi^2} \right] \]

F-downlifting (Heterotic framework)

\[ M_a = M_0 \left[ \varrho + b_a \frac{g_{\text{GUT}}^2}{16\pi^2} \right] \]

\[ A_{ijk} = M_0 \left[ 3\xi_i \varrho^2 - \gamma_i + 2\varrho \Psi_i^T + \eta_i^2 \right] \]

\[ m_i^2 = M_0 \left[ 3\xi_i \varrho^2 - \gamma_i + 2\varrho \Psi_i^T + \eta_i^2 \right] \]

Table 5.1: MSSM soft breaking parameters at the GUT scale, where \( b_a \) are the \( \beta \)-functions, \( \dot{\gamma}_i \) denotes the running of the anomalous dimension \( \gamma_i \). The moduli and dilaton dependence of the anomalous dimension is encoded in \( \Psi_i^T \) and \( \Psi_i^S \), respectively. \( M_0 \equiv m_{3/2}/16\pi^2 \).

The parameter space

The soft breaking terms we are dealing with are non-universal at the GUT scale and are described by \( \varrho, M_0 \) and \( \eta_i \). The parameter \( \varrho \) (cf. eq. (2.51)) measures the ratio between modulus (dilaton) and anomaly mediation. In the limit \( \varrho \rightarrow 0 \) we recover pure anomaly mediation, while \( \varrho \gg 1 \) corresponds to pure modulus/dilaton mediation. The scale of the soft terms is set by the gravitino mass \( m_{3/2} \) through the parameter \( M_0 \) (cf. eq. (2.52)). The quantities \( \xi_i \) and \( \eta_i \) denote the contribution from the uplifting/downlifting sector. They are related by eq. (3.77), hence we use them as synonyms.

There are two more parameters. They are the Higgs mass parameters \( \mu \) and \( B_\mu \) responsible for electroweak symmetry breaking (EWSB). As required by experiment, the minimum of the Higgs scalar potential should break EW symmetry down to electromagnetism \( SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM} \). This breakdown is initiated through the VEV of the MSSM Higgs doublets \( H_u = (H^+_u, H^0_u) \) and \( H_d = (H^0_d, H^-_d) \). Moreover it is necessary that only electrically neutral Higgs field components acquire non-zero VEVs. Without loss of generality one can set \( \langle H^+_u \rangle = \langle H^-_d \rangle = 0 \) at the minimum using an SU(2)_L gauge transformation. Then,
5.2 Aspects of the soft terms at $M_{\text{GUT}}$

at tree-level, the neutral part of the Higgs scalar potential reads $[10, 11]$

$$V = \frac{1}{8} \left( g^2 + g'^2 \right) \left( |H^0_u| - |H^0_d| \right)^2 + \left( |\mu|^2 + m^2_{H_u} \right) |H^0_u|^2 + \left( |\mu|^2 + m^2_{H_d} \right) |H^0_d|^2 - \left( B\mu H^0_u H^0_d + \text{c.c.} \right),$$  \hspace{1cm} (5.1)

with $g$ and $g'$ being the SU(2)$_L$ and U(1)$_Y$ gauge couplings, respectively. From the minimization of eq. (5.1), the condition for a symmetry breaking stable vacuum yields two relations $[176]$

$$|\mu|^2 = \frac{m^2_{H_u} - m^2_{H_d} \tan^2 \beta}{\tan^2 \beta - 1},$$  \hspace{1cm} (5.2)

$$|B\mu| = \frac{\tan^2 \beta}{1 + \tan^2 \beta} \left( m^2_{H_u} + m^2_{H_d} + 2|\mu|^2 \right),$$  \hspace{1cm} (5.3)

where

$$\tan \beta = \frac{\langle H^0_u \rangle}{\langle H^0_d \rangle}.$$  \hspace{1cm} (5.4)

Since $\mu$ and $B\mu$ are responsible for EWSB, their magnitude is bounded by the scale of the soft masses, that is $O(1 \text{ TeV})$. However, models with $m_{3/2} \gg m_{\text{soft}}$ usually predict $B = O(m_{3/2})$ $[53, 105]$. Therefore a suitable mechanism is needed to obtain the desired values for $\mu$ and $B\mu$. Such mechanisms, however, are highly model dependent $[53, 105]$ and also require a certain degree of fine-tuning. Lacking a compelling model of generating $\mu$ and $B\mu$ we will treat them as adjustable parameters. The requirement of correct EWSB eqs. (5.2) and (5.3) determine the absolute value of $|\mu|$, whereas its sign remains a free parameter. The $B$-term can be traded for $\tan \beta$. Thus, the parameter space for phenomenological studies is spanned by

$$\{ g, m_{3/2}, \eta_i, \tan \beta, \text{sign} \mu \}.$$

5.2 Aspects of the soft terms at $M_{\text{GUT}}$

The class of (string inspired) SUGRA models that we have investigated in chapter 3 and 4 leads to a very distinct structure of the soft breaking parameters. In this section we study in detail the soft terms at the GUT scale.

Soft gaugino masses

Of all the soft terms gaugino masses have the simplest structure. Moreover they are exactly the same both in the type IIB and the heterotic framework. There is one contribution coming from pure modulus (dilaton) mediation and one
Phenomenology of uplifting/downlifting

from pure anomaly mediation. Just below the GUT scale the gaugino masses approximately read

\[
M_1 \approx (\varrho + 3.3) M_0, \quad M_2 \approx (\varrho + 0.5) M_0, \quad M_3 \approx (\varrho - 1.5) M_0.
\] (5.6)

The non-universality of the gaugino masses arises from the pure anomaly-mediated part which is proportional to the MSSM $\beta$-function coefficients $b_i$. At the GUT scale the gaugino masses are ordered as $M_1 > M_2 > M_3$ because $M_3$ is suppressed by the large negative $b_3$. Depending on the value of $\varrho$, this negative contribution might become more or less important. At $\varrho \approx 1.5$ it leads to a vanishing gluino mass. For increasing $\varrho$ the gaugino masses grow linearly whereas their ratios change such that for $\varrho \gg 1$ they unify.

**Soft $A$-terms**

The general structure of the $A$-terms at the GUT scale is the same in both the type IIB and the heterotic framework. The $A$-terms contain a universal contribution from modulus (dilaton) mediation and a non-universal one from anomaly mediation. The non-universality is given by the MSSM $\gamma$-functions (see appendix B). As we mentioned in chapter 4, there is one subtle difference between the soft terms arising in the type IIB framework and those in the heterotic framework. The dilaton-mediated contribution in the heterotic framework is smaller by a factor of 3 compared to the type IIB framework. This is due to the fact that there is only one dilaton but three (or more) Kähler moduli, resulting in different effective Kähler potentials eqs. (3.27) and (4.27). Thus the soft terms in the heterotic framework are reduced. One typically has $|A_{ijk}| \sim 3|M_a|$ in the type IIB framework and $|A_{ijk}| \lesssim |M_a|$ in the heterotic framework.

**Soft scalar squared masses**

Also the structure of the soft scalar masses is identical between the type IIB and the heterotic framework.

Unlike the gaugino masses and the $A$-terms, scalar squared masses receive four different contributions. The term quadratic in $\varrho$ is a sort of a mixed modulus (dilaton)-matter mediation. In general, this contribution is non-universal as $\xi_j$ may be flavor dependent. The term proportional to $\dot{\gamma}_i$ corresponds to pure anomaly mediation and gives rise to tachyonic sleptons. Furthermore, we have a term linear in $\varrho$ which is due to the mixing between modulus (dilaton) and anomaly mediation. It comes from the moduli (dilaton) dependence of the anomalous dimension $\gamma_i$ and provides tachyonic squarks. Last but not least, $\eta_i$ describes the contribution from the uplifting/downlifting sector. This contribution is very special as it arises due to the lack of sequestering of the uplifting/downlifting sector.

Generically, the uplifting/downlifting sector also affects the structure of the gaugino masses and the $A$-terms. These additional contributions will be suppressed provided the VEV of the uplifting/downlifting field is small, $\phi_0 \ll 1$. 
Our attention is devoted to precisely this class of models. Finally, the dilaton mediated contribution to scalar squared masses in the heterotic framework is smaller by a factor of 3 compared to the type IIB framework.

**SUSY CP Problem**

Although the sources of SUSY breaking $F^T(F^S)$, $F^\phi$ and $F^C$ are in general complex, all their phases are dynamically aligned to gaugino mass phases \([47, 48, 53, 55]\). These phases can be rotated away with suitable PQ and R rotations. Thus one can always choose a field basis in which the gaugino masses and the $A$-terms are real. It also ensures that the $\varrho$ parameter eq. (2.51) is real (and positive).

**SUSY flavor problem**

This problem appears when the modular weights $n_i$ and the couplings $\xi_i$ are flavor dependent. In our analysis we simply assume $n_i = 0$. The model is free from dangerous SUSY flavor violation if $\xi_i$ are chosen to be flavor independent. In what follows we use the following notation

\begin{align}
\xi_i^{(SFERMIONS)} &\equiv \xi_i \quad \text{or} \quad \eta_i^{(SFERMIONS)} \equiv \eta_i, \\
\xi_i^{(Higgs)} &\equiv \xi' \quad \text{or} \quad \eta_i^{(Higgs)} \equiv \eta'.
\end{align}

**Tachyons**

Due to the contributions from anomaly mediation, tachyons do also appear in the scheme of $F$-uplifting/$F$-downlifting but only in a limited region of the parameter space eq. (5.5). To get rid of the tachyons a positive contribution to scalar squared masses is required. Such a positive contribution is provided by the modulus mediated part and is proportional to $\varrho^2$. For $\eta = \eta' = 0$ (pure mirage mediation) the absence of tachyons impose a lower bound on the parameter $\varrho$. In the type IIB framework, fig. 5.1.a, tachyons are absent for $\varrho > 4$. Due to the reduced dilaton contribution in the heterotic framework, fig. 5.1.b, absence of tachyons requires here $\varrho > 12$. Such a large value of $\varrho$ exceeds the realm of mirage mediation as it corresponds to dilaton dominated mediation. Moreover, an increasing $\varrho$ also affects gaugino masses and $A$-terms.

Another possibility to get rid of the tachyons is to consider $\eta, \eta' > 0$ (or equivalently $\xi, \xi' < 1/3$). This presents an interesting possibility which is unique for the scheme of $F$-uplifting/$F$-downlifting. In particular we can perform a tachyon scan. That is, for every $\varrho$ and $\tan \beta$ we scan over $\eta$ and $\eta'$ and exclude those values for which tachyons appear. Our results are presented in figs. 5.1.c and 5.1.d. The absence of tachyons on the type IIB framework poses a rather weak constraint as it requires

\begin{align}
\eta_{\text{TYPE IIB}} > 1.7 \quad \text{and} \quad \eta'_{\text{TYPE IIB}} > 1.5,
\end{align}
whereas in the heterotic framework absence of tachyons becomes more stringent

$$\eta_{\text{HETEROTIC}} > 3.5 \quad \text{and} \quad \eta'_{\text{HETEROTIC}} > 1.7.$$  \hfill (5.10)

Since gaugino masses and $A$-terms are independent of $\eta_i$ they are not affected by this procedure.

### 5.3 Constraints on the soft terms at $M_{\text{TeV}}$

In the following two section our intention will be to study the phenomenological properties of the low energy spectra of models arising in the scheme of $F$-uplifting (type IIB framework) and $F$-downlifting (heterotic framework). To verify the viability of these models we will impose several phenomenological constraints of theoretical and experimental nature. This section serves to discuss the most relevant constraints.
5.3 Constraints on the soft terms at $M_{\text{TeV}}$

5.3.1 General remarks

As discussed in the previous section, the soft parameters at the GUT scale of the SUGRA models under consideration are described by six free parameters eq. (5.5). Once $\tan \beta$ and $\text{sign} \mu$ have been chosen we remain with four free parameters.

The primary parameters here are $\varrho$ and $m_{3/2}^2$ as they tell us in which regime the mediation of the SUSY breakdown occurs and what the characteristic scale of the soft breaking parameters is. If $\varrho$ is too large the soft terms at the GUT scale unify and the resulting phenomenology resembles the MSUGRA picture $[106–109]$. Since our main interest is to study the phenomenology of mixed modulus(dilaton)-anomaly mediation we will restrict ourselves to $0 \leq \varrho \leq 12$.

In order to have a sparticle spectrum in the TeV domain, the gravitino mass parameter $m_{3/2}^2$ eq. (2.52) should not be too large. Following this requirement we consider the interval $0 < m_{3/2}^2 \leq 60$ TeV.

As far as the parameters $\eta$ and $\eta'$ eq. (3.77) are concerned their range can be specified by consistency considerations. The requirement of a positive coupling between visible and hidden matter in the Kähler metric eqs. (3.71) and (4.84) gives the upper bound $\eta, \eta' \leq 16\pi^2 \approx 158$. On the other hand, demanding a tachyon-free setup at the GUT scale poses a lower bound eqs. (5.9) and (5.10) such that $\eta, \eta' \gtrsim O(1)$ is required. Still, this is a wide range. To further restrict the values of $\eta$ and $\eta'$ we need a suitable selection scheme to specify those values which are favored by phenomenological arguments. In section 5.3.6 we will use the so-called MSSM hierarchy problem as a guideline to constrain $\eta$ and $\eta'$.

5.3.2 Electroweak symmetry breaking

Minimization of the MSSM Higgs scalar potential eq. (5.1) leads to the (tree-level) relation eq. (5.2). For moderate values of $\tan \beta$ this relation can be well approximated by

$$|\mu|^2 \approx -m_{H_u}^2 \left( \frac{m_Z^2}{2} \right), \quad (5.11)$$

evaluated at the TeV scale. As the right-hand side is always positive the occurrence of EWSB requires a negative $m_{H_u}^2$ at/near the TeV scale. Since we demand a tachyon-free setup $m_{H_u}^2$ is positive at the GUT scale. The value of $m_{H_u}^2$ at the TeV scale is obtained via its RG evolution from the GUT scale. The relevant contribution at 1-loop level is

$$\frac{dm_{H_u}^2}{d \log \mu} \approx \frac{3}{16\pi^2} \left( m_{H_u}^2 + m_{Q_0}^2 + m_{\tilde{t}}^2 + A_t^2 \right), \quad (5.12)$$

with $\mu$ denoting the RG scale. Further details are given in appendix C. The RG evolution is most sensitive to the gluino mass $M_3$ which induces an increase of
the squark masses $m^2_{Q^3_L}$ and $m^2_{t_R}$. Solving the 1-loop RG equations (for $\tan \beta = 5$) one obtains

$$m^2_{H_u}(M_{\text{TeV}}) \approx -2.3M^2_3(M_{\text{GUT}}) + 0.6m^2_{H_u}(M_{\text{GUT}})$$

where we have omitted terms with smaller numerical coefficients. This states that gluino is the leading force in driving $m^2_{H_u}$ to negative values at the TeV scale. As evident from eq. (5.6), in the mirage mediation scenario a cancellation between modulus (dilaton) and anomaly mediation occurs for small values of $\varrho$ leading to an ultra-light gluino around $\varrho \approx 1.5$. There eq. (5.13) is not sufficient to make $m^2_{H_u}$ negative and EWSB will not be possible. Thus the requirement of correct EWSB sets a lower bound on $\varrho$. However, one can improve this situation by making the squarks sufficiently heavy. This can be achieved with a non-zero $\eta$ parameter.

5.3.3 Color and charge breaking minima

Generically, supersymmetric models exhibit many flat directions in the field space. Usually, the SUSY breaking terms lift these directions, but may also induce global or deep color and charge breaking (CCB) minima [177]. Thus it is important to verify that such minima do not occur. Some of the dangerous CCB minima appear when the soft $A$-terms are sufficiently large. The absence of the CCB minima requires

$$A_i^2 \lesssim 3 \left( m^2_{H_u} + m^2_{Q^3_L} + m^2_{t_R} \right),$$

at the GUT scale. The $A$-terms boundary conditions, tab. 5.1, imply that this constraint is usually satisfied in the schemes of $F$-uplifting and $F$-downlifting.

Another type of constraints comes from the unbounded-from-below (UFB) directions in the full scalar potential [177]. The most serious constraint involves the up-sector Higgs and slepton fields. The absence of the UFB direction requires

$$m^2_{H_u} + \sum_{i \in \text{sleptons}} m^2_i > 0,$$

at the TeV scale. In the framework of mixed modulus(dilaton)-anomaly mediation this constraint is respected in viable regions of the parameter space. This is mainly due to the reduced gluino mass which results from the negative anomaly-mediated contribution. Thus, we can summarize that the absence of CCB minima does not constrain the model significantly.

5.3.4 Neutralino dark matter

In the MSSM the neutral higgsinos $\tilde{H}^0_u, \tilde{H}^0_d$ mix with the neutral EW gauginos $\tilde{B}^0, \tilde{W}^0$ and form four eigenstates called neutralinos $\tilde{\chi}^0_i, i=1,2,3,4$. In the gauge eigenstate
basis \((\tilde{B}^0, \tilde{W}^0, \tilde{H}^0_u, \tilde{H}^0_d)\) the neutralino mass matrix is given by \([10, 11]\)

\[
M_N = \begin{pmatrix}
M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\
0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\
-m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\
m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0
\end{pmatrix}, \tag{5.16}
\]

with \(s_\beta = \sin \beta, c_\beta = \cos \beta, s_W = \sin \theta_W, c_W = \cos \theta_W\) as in eq. (5.4) and \(\theta_W\) is the weak mixing angle. Eq. (5.16) can be diagonalized by an orthogonal matrix \(Z\) such that the lightest neutralino is given by

\[
\tilde{\chi}_1^0 = Z_{11} \tilde{B}^0 + Z_{12} \tilde{W}^0 + Z_{13} \tilde{H}^0_u + Z_{14} \tilde{H}^0_d. \tag{5.17}
\]

Using this decomposition one defines

\[
\tilde{\chi}_1^0 = \begin{cases} 
\text{bino-like} & |Z_{11}|^2 + |Z_{12}|^2 > 0.9 \land |Z_{11}| > |Z_{12}|, \\
\text{wino-like} & |Z_{11}|^2 + |Z_{12}|^2 > 0.9 \land |Z_{11}| < |Z_{12}|, \\
\text{higgsino-like} & |Z_{11}|^2 + |Z_{12}|^2 < 0.1, \\
\text{mixed} & \text{otherwise}.
\end{cases} \tag{5.18}
\]

In models we are going to investigate the lightest neutralino happens to be the LSP in the most of the parameter space. Under the assumption of \(R\)-parity conservation it is stable \([10, 11]\). It can be considered as a good cold DM candidate since it is a weakly interacting particle. To get a consistent DM abundance one has to make sure that the neutralinos annihilate efficiently enough. Efficient annihilation mechanisms include light neutralinos and light sfermions, co-annihilations, resonance enhancement in the Higgs exchanges and annihilation into \(W\) boson pairs. In models under consideration the higgsino mass parameter \(\mu\) can be relatively small depending on the value of \(\rho\). This is mainly due to the suppressed gluino mass. Thus the lightest neutralino can contain a significant higgsino component which may enhance the annihilation cross section.

In the computation of the DM abundance we will assume that the LSP abundance is thermal. The results predicted in our models are compared with the value inferred from observations. In particular we use the 3\(\sigma\) limit from the Wilkinson Microwave Anisotropy Probe (WMAP) collaboration on the neutralino cold DM abundance \([111]\)

\[
0.087 \leq \Omega_{\chi^0} h^2 \leq 0.138. \tag{5.19}
\]

Regions of parameter space violating the upper WMAP bound are treated as forbidden, those within the bounds as favored and those below the lower bound as allowed. In the latter case the correct cosmological abundance of DM could be achieved with additional DM particles (beyond the MSSM) and/or a non-thermal origin.
5.3.5 Accelerator constraints

Direct collider searches set lower bounds on the sparticle spectrum and Higgs masses. We implement these bounds by applying the LEP2 constraints. The most important and restrictive bounds are due to the lightest Higgs boson mass $m_{h^0} > 114$ GeV, the lightest chargino mass $m_{\tilde{\chi}_1^+} > 103.5$ GeV and the lightest top squark mass $m_{\tilde{t}_1} > 95.7$ GeV [178–180]. Regions of parameter space deceeding one of these bounds are called “below LEP”.

Furthermore, the supersymmetric spectrum is constrained indirectly by the $b \to s\gamma$ decay. The most important supersymmetric contributions involve chargino–stop loops as well as charged Higgs–top loops. We impose the $3\sigma$ bound from the B-factories [181,182], $2.33 \times 10^{-4} \leq \text{BR}(b \to s\gamma) \leq 4.15 \times 10^{-4}$.

These constraints set a lower bound on the gravitino mass parameter $m_3/2$.

5.3.6 The MSSM hierarchy problem

The so-called MSSM hierarchy problem [183,184] is caused by Higgs sector of the MSSM. First of all, at tree-level the mass of the lightest Higgs boson is given by [185,186]

$$m_{h^0} < m_Z \left| \cos 2\beta \right|,$$

with $\beta$ as in eq. (5.4). Clearly, this badly violates the current experimental lower bound $m_{h^0} > 114$ GeV [187]. There are, however, sizeable radiative corrections to the tree-level value. The most sizable 1-loop contribution comes from top and stop loops. In case the gauge eigenstate stop masses $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ are much large than the top quark mass $m_t$ one finds [188]

$$\delta_{1\text{-loop}} m_{h^0}^2 \approx \frac{3y_t^2 m_t^2}{4\pi^2} \sin^2 \beta \log \left( \frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2} \right),$$

where $y_t$ being the top Yukawa coupling. To lift the Higgs mass above the experimental lower bound requires a rather larger stop mass $m_{\tilde{t}_1,2} \gtrsim 1$ TeV.

On the other hand, the 1-loop RG evolution of the up-sector Higgs boson mass from the GUT down to the TeV scale is given by

$$\delta_{\text{RG}} m_{H_u}^2 \approx \frac{3}{4\pi^2} \left( m_{H_u}^2 + m_{H_u}^2 \right) \log \left( \frac{\Lambda}{m_t} \right),$$

(5.22)

where $m_t = \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$ and $\Lambda$ is the scale at which the boundary conditions set in. For $\Lambda = M_{\text{GUT}} \approx 2 \times 10^{16}$ GeV one typically has $|\delta_{\text{RG}} m_{H_u}^2| = \mathcal{O}(m_t^2)$ at the TeV scale.

Finally, the requirement of correct EWSB at the TeV scale eq. (5.2) leads (at tree-level) to

$$\frac{m_Z^2}{2} \approx -|\mu|^2 - m_{H_u}^2,$$

(5.23)
indicating that large cancellations are necessary to obtain the experimental value of the $Z$ boson mass. In particular, with $-m_{H_u}^2 = O(1 \text{ TeV}^2)$, as required by the Higgs mass bound, the degree of fine-tuning in eq. (5.23) is $O(1\%)$ or more severe.

One can rewrite eq. (5.2) in terms of the boundary conditions at the GUT scale (at 1-loop level) as [189]

$$m_Z^2 \approx -1.8|\mu|^2 + 5.9M_3^2 - 0.4M_2^2 - 1.2m_{H_u}^2 + 0.9m_{Q(3)}^2 + 0.7m_{t_R}^2,$$

$$- 0.6A_1 M_3 + 0.4M_2 M_3$$

$$:= -1.8|\mu|^2 + \tilde{m}_Z^2,$$ \hspace{1cm} (5.24)

where we have used $\tan\beta = 5$ and neglected terms with smaller numerical coefficients. If all of the parameters on the right-hand side of eq.(5.24) are $O(100^2 \text{ GeV}^2)$ no significant fine-tuning is needed. The soft breaking parameters, however, are typically in the TeV range. Then the correct value of $m_Z$ can be obtained in two different ways.

The first possibility is to arrange for a cancellation between $\mu^2$ and $\tilde{m}_Z^2$ by adjusting $\mu$. But then the value of $\mu$ might have to be very large. If $\mu$ is too large, the Higgs fields are too massive to play a role in the EWSB. Thus, the second possibility is to arrange for a cancellation within $\tilde{m}_Z^2$ such that $\mu$ has a value of the order of the EW scale.

The largest contribution to $\tilde{m}_Z^2$ comes from the gluino. In order to keep $\tilde{m}_Z^2$ small one would have to keep $M_3$ under control \[104, 105\]. As evident from eq. (5.6) in mirage mediation the gluino mass is reduced for small $\varrho$ and vanishes at $\varrho \approx 1.5$. Being zero or very small at the GUT scale the gluino mass will be zero or very small at the TeV scale, too. This is ruled out, as the gluino would be the LSP. In addition, if the gluino is very light it cannot provide the necessary RG contribution to $m_{H_u}^2$, eq. (5.13), such that eq. (5.2) will no longer be satisfied and consequently EWSB will not be realized around $\varrho \approx 1.5$. Thus larger values of $\varrho$ are required.

Alternatively one may try to enhance the value of the $A$-terms such that $A_i \approx 10 M_3$. However, in mirage mediation we cannot vary the soft parameters independently at the GUT scale. An enhancement in $A_i$ is connected to larger values of $\varrho$ which would simultaneously enhance $M_3$.

In order to achieve a cancellation within $\tilde{m}_Z^2$ for moderate values of $\varrho$ one has to adjust the masses of the sfermions and Higgses. Here we can use the freedom of choosing $\eta$ and $\eta'$. As evident from eq. (5.24) the contribution from $m_{H_u}^2$ is negative and thus by increasing $m_{H_u}^2$ at the GUT scale one obtains a sizable term that could cancel the large contribution of $M_3$. The contribution from squarks is positive and one has to keep their masses low. However, we cannot choose $\eta$ too small; otherwise the squarks might become tachyonic at the GUT scale. The essential lesson we learn from these considerations is to keep $\eta$ as low as possible and then adjust $\eta'$. If we want to keep $\tilde{m}_Z^2 = O(10^5 \text{ GeV})$ a certain relation between $\varrho$, $m_{3/2}$, $\eta$ and $\eta'$ has to be fulfilled. In that sense the MSSM hierarchy problem can be ameliorated at the expense of a (fine-)tuning of $\eta'$. 

5.3 Constraints on the soft terms at $M_{\text{TeV}}$
5.4 Phenomenological aspects of $F$-uplifting

Having discussed the phenomenological properties of the soft breaking parameters at the GUT scale we would like to compute them at the TeV scale and apply phenomenological constraints discussed in the previous section.

As explained above we restrict ourselves to the range

\begin{align}
0 \leq \varrho &\leq 12, \\
0 < m_{3/2} &\leq 60 \text{ TeV}.
\end{align}

(5.25) (5.26)

As far as the parameters $\eta$ and $\eta'$ are concerned we use the MSSM hierarchy problem to pick up a suitable value. We will also consider the so called matter domination scenario; a scheme where the scalar masses are $O(m_{3/2})$. This corresponds to $\xi = O(1/\sqrt{s})$ or equivalently $\eta_i = O(16\pi^2)$. Furthermore we analyze regimes with low and high values of $\tan \beta$. For simplicity we fix the sign of the $\mu$ parameter to be positive. Throughout our analysis we take $m_t = 175$ GeV. For the calculation of low energy data we use the public codes SOFTSUSY [190] and micrOMEGAs [191].

5.4.1 Aspects of the soft terms at $M_{\text{TeV}}$

In this section we discuss the RG evolution of the non-universal boundary conditions tab. 5.1 arising in the scheme of $F$-uplifting (type IIB framework). For the qualitative discussion it is sufficient to use the 1-loop RG equations summarized in appendix C. We choose $\eta = 1.5$ and $\eta' = 1.7$ so as to ensure the absence of tachyons at the GUT scale.

Soft gaugino masses

The evolution of the gaugino masses $M_a$ is given by the evolution of the gauge couplings constants $g_a$. At 1-loop level the quantity $M_a/g_a^2$ does not run. As already explained in section 2.3.1 due to mixed modulus anomaly boundary conditions the gaugino masses unify at the mirage scale eq. (2.58). Fig. 2.3.b shows the mirage unification of the gaugino masses as well as their true unification above the GUT scale.

Observe that for $\varrho = 5$ the gaugino masses unify in the middle between $M_{\text{GUT}}$ and $M_{\text{TeV}}$. For this reason we will use $\varrho = 5$ as a benchmark point. At this point the contributions from modulus and anomaly mediation are of the same size.

For $\varrho = 5$ the gaugino masses are ordered as $M_1 \div M_2 \div M_3 \approx 1 \div 1.3 \div 2.5$ at the TeV scale, which differs significantly from the well known MSUGRA pattern ($M_1 \div M_2 \div M_3 \approx 1 \div 2 \div 6$) and the anomaly pattern ($M_1 \div M_2 \div M_3 \approx 3.3 \div 1 \div 9$). Since bino is the lightest gaugino at the TeV scale the neutralino LSP is likely to be bino-like. However, due to the reduction of the gluino mass in the scheme of mirage mediation, the hierarchy between the gaugino masses is weaker. For small values of $\varrho$ the $\mu$-term becomes around the masses of the bino and wino. Consequently
the lightest neutralino can contain a significant higgsino component. We can also have a higgsino-like neutralino at larger values of \( \varrho \) by appropriately adjusting the masses of the sfermions and the Higgses.

**Soft \( A \)-terms**

Within the MSSM framework we use the notation

\[
A_t \equiv A_{\tilde{Q}_3 H_d t_R}, \quad A_b \equiv A_{\tilde{Q}_1 H_d b_R}, \quad A_\tau \equiv A_{\tilde{L}_3 H_d \tau_R}.
\]

The RG evolution of the \( A \)-terms, eqs. (C.7) – (C.9), involves gauge and Yukawa couplings. The gauge terms push the already negative \( A \)-terms to more negative values, while the Yukawa terms do the opposite. Fig. 5.2.c illustrates the RG evolution of the \( A \)-terms in the benchmark point \( \varrho = 5 \) at \( \tan \beta = 5 \). Note that owing to Yukawa coupling effects the \( A \)-terms do not show mirage unification.

The flat running of \( A_t \) is due to the large top Yukawa coupling \( y_t \) which counterbalances the contribution from the gauge terms. The smallness of the bottom and \( \tau \) Yukawa couplings \( y_b \) and \( y_\tau \) cannot yield the same effect for \( A_b \) and \( A_\tau \) such that at the TeV one typically has \( |A_t| < |A_\tau| < |A_b| \).

At large \( \tan \beta \) also the bottom Yukawa coupling becomes sizable thereby affecting the RG evolution of \( A_b \) and \( A_\tau \). For \( A_t \) and also \( A_\tau \) gauge and Yukawa terms counterbalance each other providing a nearly flat running.

**Soft scalar squared masses**

In fig. 5.2.a we present the RG evolution of the scalars of the first two generations in the benchmark point \( \varrho = 5 \) at \( \tan \beta = 5 \). The GUT scale mass ordering

\[
m^2_{\tilde{e}_R} > m^2_{\tilde{\nu}_e} > m^2_{\tilde{d}_R} > m^2_{\tilde{u}_R} > m^2_{\tilde{Q}_3}
\]

becomes essentially inverted at the TeV scale in the process of RG evolution. This is mainly due to large RG effects for colored particles on account of supersymmetric quantum chromodynamics interactions. Nevertheless, the reduced gluino mass makes sure that RG effects do not significantly enhance the masses of colored particles over the uncolored particles. This clearly differs from the MSUGRA and anomaly mediation schemes where the mass gap between squarks and sleptons can be sizable.

The masses of the third generation scalars and the Higgs bosons feel the effect of the Yukawa couplings. Their RG equation approximately reads (cf. eq. (C.21))

\[
\frac{d m^2_i}{d \log \mu} \sim - \sum_a g^2_{ai} M^2_a C^a_i,
\]

where \( a = 1, 2, 3 \). Thus the masses of these sparticles behave in the similar way as the gaugino masses \( M_a \). In particular, they unify at (approximately) the same mirage scale eq. (2.58) and do not depend on \( \tan \beta \).

The masses of the third generation scalars and the Higgs bosons feel the effect of the Yukawa couplings. This results in a different RG behavior. As
shown in fig. 5.2.b the phenomenon of mirage unification is not shared by these particles. For low \( \tan \beta \) the mass ordering at the GUT scale is typically \( m_{\tilde{t}_R} > m_{\tilde{b}_R} > m_{\tilde{t}_L} > m_{\tilde{b}_L} > m_{\tilde{g}_R} \) with quite small splittings. This ordering becomes inverted at the TeV scale with relatively small mass splittings.

The RG evolution of the third generation scalars is governed by the \( A \)-terms which act to suppress their masses. This is most significant for \( m_{\tilde{t}_R} \), where the quantity \( X_{\tilde{t}_R} \), eq. (C.10), enters with a larger numerical coefficient in eqs. (C.16) and (C.17). This effect alongside with the large intragenerational mixing in the top squark sector leads to \( \tilde{t}_1 \) being the next to lightest supersymmetric particle (NLSP). As we shall see shortly, for low \( \tan \beta \) the lightest top squark can also be the LSP in some regions of the parameter space. Moreover, observe that for low \( \tan \beta \) the masses of \( \tilde{b}_R, \tilde{t}_R \) and \( \tilde{g}_R \) are only affected by the smaller Yukawa couplings \( y_b, y_t \ll y_t \), and the structure of their RG running is similar to eq. (5.28).

At large \( \tan \beta \) the bottom Yukawa coupling becomes relevant and modifies
the mass ordering at the GUT scale to \(m_{t_H} > m_{\tau_R} > m_{L_3} > m_{b_R} > m_{Q_3}\). In the large \(\tan\beta\) regime the suppression of \(m_{t_R}\) is more efficient than the suppression of \(m_{\tau_R}\) such that near the TeV scale the lightest \(\tau\) slepton often happens to be the NLSP and, in certain regions of the parameter space, also the LSP. In the large \(\tan\beta\) regime none of the third generation sparticles share the mirage unification feature due to the effect of Yukawa couplings.

Finally the RG evolution of \(m_{H_u}^2\) is controlled to a large extent by \(m_{Q_3}^2\) and \(A_t\). Since gluino drives the squark masses it also controls the mass of the up-type Higgs boson. Both \(M_3\) and \(A_t\) push \(m_{H_u}^2\) to large negative values near the TeV scale. This is the well known mechanism of radiative EWSB.

5.4.2 Dependence on \(\eta\) and \(\eta'\)

As we have stated in section 5.3.1 the main parameters in our scheme are \(\varrho\) and \(m_{3/2}\). Before we perform a parameter space analysis we would like to specify suitable values of \(\eta\) and \(\eta'\). As mentioned in section 5.3.6 we use the MSSM hierarchy problem as a guideline. We proceed as follows. In the benchmark point \(\varrho = 5\) we choose \(m_{3/2} = 30\) TeV and fix \(\tan\beta = 5\). We then scan over \(\eta\) and \(\eta'\) and apply the constraint for correct EWSB section 5.3.2. Our result is shown in fig. 5.2.d.

For \(\eta = \eta' = 0\) the \(\mu\) parameter is quite large such that the fine-tuning in eq. (5.24) will be large. If we increase \(\eta'\) the Higgs bosons become heavier. This in turn reduces \(\tilde{m}_Z^2\) in eq. (5.24) resulting in a reduction of \(\mu\). With increasing \(\eta'\) the large \(m_{H_u}^2\) acts to suppress the masses of the lightest top squark such that (for small \(\eta\)) it becomes the LSP. For \(\eta' > 8\) the mass of \(H_u\) exceeds a critical value and \(\tilde{m}_Z^2\) becomes negative implying that EWSB is not realized.

By increasing \(\eta\) we can counteract with heavier scalars, thereby circumventing the stop LSP. However, the spectrum of sleptons and squarks becomes heavier as we proceed to increase \(\eta\) and \(\eta'\). In the consideration of a \(\mu\)-term of order the EW scale we have to stay close to the “No EWSB”. As evident from fig. 5.2.d and also from eq. (5.24), \(\eta' > \eta\) can be considered as a rule of thumb.

5.4.3 Low energy spectroscopy

In this section we perform a scan over the parameters \(\varrho\) and \(m_{3/2}\) for different values of \(\eta\) and \(\eta'\) at different \(\tan\beta\).

Low \(\tan\beta\) regime

According to the discussion in the previous section we pick up \(\eta = 4\) and \(\eta' = 8.5\) in the allowed region in fig. 5.2.d. The corresponding parameter space is shown if fig. 5.3.a.

The most severe constraints here are due to the requirement of correct EWSB. Around \(\varrho \approx 1.5\) the gluino is very light and thus too weak to drive \(m_{H_u}^2\) negative
at the TeV scale. Also the RG contribution from the $A$-terms is not sufficient to improve the situation, such that for $\rho < 4.5$ the condition for EWSB eq. (5.2) can not be satisfied. Viable spectra are obtained for $\rho > 5.5$.

In the region $4.5 \lesssim \rho \lesssim 5.5$ the large $A_t$ suppresses via RG the mass of the lightest top squark $m_{\tilde{t}_1}$ and makes it the NLSP. The mass of the lightest chargino $m_{\tilde{\chi}^+}$ is almost degenerate with the mass of the lightest neutralino $m_{\tilde{\chi}^0_1}$. They both have a significant higgsino component, since $|\mu| \lesssim M_1 (M_{3\nu})$.

For $5 \lesssim \rho \lesssim 6$ the lightest top squark becomes the LSP. This can be understood as follows. Close to the “No EWSB” region we have $|\mu| < M_1 (M_{3\nu})$. Consequently the lightest neutralino $\tilde{\chi}^0_1$ is higgsino-like. On the other hand, the large $A_t$ suppresses via RG the mass of $\tilde{t}_1$ such that it is very close to $m_{\tilde{\chi}^0_1}$. Due to the higgsino component the mass of $\tilde{\chi}^0_1$ grows rather fast with $\rho$ (it tries to follow $\mu$). For $\rho > 5$, $m_{\tilde{\chi}^0_1}$ overcomes $m_{\tilde{t}_1}$ and the stop quark becomes the LSP. As $\rho$ further increases the $\mu$-term becomes larger and the neutralino is dominated by the bino
component. This slows down the increase in $m_{\tilde{\chi}}^{0}$ whereas the increase in $m_{t_{1}}$ remains unchanged (cf. fig. 5.3.c). Consequently, for $\varrho > 6$ the neutralino again becomes the LSP. Thus the intermediate region with a stop LSP is a result of a higgsino-like neutralino around $\varrho \sim 5$ and a suppressed stop mass.

The LEP2 mass bounds for $m_{\tilde{\chi}}^{0}, m_{\tilde{\chi}^{+}}^{0}$ and $m_{t_{1}}$ are relatively weak compared to the EWSB constraint. Of all the mass bounds $m_{\tilde{\chi}}^{0}$ is most restrictive. The constraint from BR($b \rightarrow s\gamma$) does not further restrict the range of parameters. Altogether these constraints require $m_{3/2} \geq 4$ TeV.

The brown/strip in fig. 5.3.a satisfies the WMAP bounds eq. (5.19). This region of the parameter space is favored by the neutralino DM abundance. To the left of the brown/strip the neutralino abundance is below the WMAP bounds but still allowed if the DM production is non-thermal. To the right of the brown/strip the neutralino abundance is too large and thus ruled out by WMAP. In the allowed region the neutralino and the chargino have an important higgsino component. Fig. 5.3.c explains the shape of the allowed WMAP region by tracking the sparticle masses as a function of $\varrho$. For $\varrho \sim 4.5$ the mass gap $|m_{\tilde{\chi}}^{0} - m_{\tilde{\chi}^{+}}^{0}|$ is very small and the production of the relic abundance proceeds efficiently through the $\tilde{\chi}_{1}^{0}\tilde{\chi}_{1}^{+}$ co-annihilation. As $\varrho$ increases $m_{t_{1}}$ comes closer to $m_{\tilde{\chi}}^{0}$ such that around $\varrho \sim 5.5$ stop co-annihilation is at work. The mass gap between the neutralino and the chargino grows fast with increasing $\varrho$ and the annihilation cross section decreases. At $\varrho \sim 6$ we enter the WMAP strip. Although $m_{t_{1}} \sim m_{\tilde{\chi}}^{0}$ around $\varrho \sim 6$ it cannot overcome the decrease of the annihilation cross section caused by the meanwhile bino-like neutralino. For $\varrho > 6$ no further co-annihilation channels are available and the neutralino abundance rapidly exceeds the upper WMAP bound.

An interesting situation occurs when the soft scalar masses become very heavy. For concreteness let us consider $\eta = \eta' = 112$ ($\xi = \xi' = 1/6$). Fig. 5.4.a shows the corresponding parameter space. We are dealing here with relaxed mirage mediation. That is, all scalar masses are $O(m_{3/2})$ and the large contributions from $\eta$ and $\eta'$ make them unify at the GUT scale, while gaugino masses and $A$-terms are much lighter. The gaugino masses continue to unify at the mirage scale $M_{\text{mir}}$ eq. (2.58). The shape of the “No EWSB” area changes. This constraint is now less restrictive for small $m_{3/2}$. Again this can be understood from eq. (5.24). In the case of heavy scalars, the dominant contribution is given by $-1.2m_{H_{u}}^{2} + 0.9m_{Q^{(3)}}^{2} + 0.7m_{f_{k}^{+}}^{2}$, which must be positive. For small values of $\varrho$ this opens up a new region in the parameter space. In particular $\varrho \leq 3$ becomes accessible. For such small values of $\varrho$ the mirage unification of the gaugino masses occurs near the TeV scale $M_{\text{mir}} \approx M_{\text{av}}$. Therefore for $\varrho < 3$ and $m_{3/2} \sim 6$ TeV we have $M_{1}(M_{\text{av}}) \approx M_{2}(M_{\text{av}}) \approx M_{3}(M_{\text{av}})$. As usual we have an ultra-light gluino around $\varrho \approx 1.5$. Due to the enhancement of scalar masses accelerator constraints require now $m_{3/2} \geq 3$ TeV. The brown/strip in fig. 5.4.a represents the part of the parameter space favored by the WMAP constraint on the neutralino relic abundance. In the WMAP allowed region we have $M_{1} \approx M_{2}$ and $m_{\tilde{\chi}}^{0} \approx m_{\tilde{\chi}}^{+}$. 5.4 Phenomenological aspects of $F$-uplifting
Thus, strong co-annihilation of bino-like neutralinos with wino-like charginos gives the right amount of the relic LSP abundance.

Finally, a further increase of sfermion masses will dramatically influence the low energy phenomenology. In fig. 5.4.b we present the case with $\eta = 158$ and $\eta' = 112$ ($\xi = 0$ and $\xi' = 1/6$). This eliminates the “No EWSB” region since due to the heavy scalars the quantity $\tilde{m}_Z^2$ in eq. (5.24) is now positive. Thus also here low values of $\varrho$ are allowed. The appearance of the gluino LSP, however, can not be avoided as it is the consequence of the mixed modulus-anomaly boundary conditions for the gaugino masses. In the region allowed by the WMAP constraint we recover a mixed wino-bino neutralino, however, without any higgsino component (since $|\mu| \gg M_1$). Also here the co-annihilation with charginos is at work. Moreover, for small $\varrho$ we obtain a region where the lightest chargino is the LSP. Due to the very heavy sfermion spectrum the mass bounds from LEP2 become less restrictive such that $m_{3/2} \geq 1.5\,\text{TeV}$ is allowed.
Large $\tan \beta$ regime

If we increase $\tan \beta$ the constraint due to the correct EWSB becomes more severe. This is because the coefficient of the squark masses in eq. (5.24) decreases with $\tan \beta$ due to sbottom loops. On the other hand, the LEP2 mass bound for $m_{1\tilde{\ell}}$ becomes less restrictive. This comes from the fact that the radiative correction to the mass of the lightest Higgs boson eq. (5.21) favors large $\tan \beta$. Consequently the Higgs mass bound can be exceeded in larger portions of the parameter space.

Let us choose $\tan \beta = 30$ and analyze the parameter space fig. 5.3.b with $\eta = 4$ and $\eta' = 8.5$. Since the scalars are (relatively) light the “No EWSB” region experiences only a slight expansion compared to $\tan \beta = 5$. The LEP2 constraints allow now $m_{3/2} > 2.5$ TeV. In the large $\tan \beta$ regime the suppression of the top squark mass is reduced such that a stop NLSP usually does not occur. Only for smaller values of $m_{3/2}$ and moderate $\rho$ a region with a stop LSP, adjacent to the “Below LEP” region, emerges. Typically, we find that for moderate values of $\rho$ the NLSP is the lightest chargino, whereas for larger values of $\rho$ the lightest tau slepton $\tilde{\tau}_1$ becomes the NLSP. As before there is a region where the lightest chargino happens to be the LSP.\(^1\)

In the brown region of fig. 5.3.b, favored by the WMAP constraint, both the neutralino and the chargino have a significant higgsino component. The correct relic density is achieved due to neutralino annihilation as well as chargino co-annihilation processes. In addition, for small $m_{3/2}$ co-annihilation with the lightest top squark widens the allowed range of the parameter space.

For larger values of $\eta$ and $\eta'$ the phenomenology is essentially the same as in the low $\tan \beta$ regime. The WMAP allowed range lies on the edge of the “No EWSB” area. Co-annihilation processes with charginos give acceptable LSP abundance.

5.5 Phenomenological aspects of $F$-downlifting

In the previous section we saw that the low energy phenomenology of $F$-uplifting is quite distinct. Since the boundary conditions in the $F$-downlifting scheme are similar to those of $F$-uplifting we also expect a similar picture to emerge here as well. But we will also identify some differences. In analogy to the previous investigation we focus our attention on the range $0 \leq \rho \leq 12$, $0 < m_{3/2} \leq 60$ TeV and study cases with different $\tan \beta$ and $\eta$, $\eta'$. As before we keep $\mu > 0$ and $m_t = 175$ GeV.

---

\(^1\)In the region where we find a chargino LSP the gaugino masses and the $A$-terms feel the effect of anomaly mediation. A typical feature of anomaly mediation is that the mass gap between the lightest chargino and the lightest neutralino is $\mathcal{O}(200 \text{ MeV})$ [10]. This mass degeneracy can lead in some cases to long lived charginos, as long as anomaly mediation provides a significant contribution.
5.5.1 Dependence on $\eta$ and $\eta'$

The scheme of $F$-downlifting differs from that up $F$-uplifting in a reduced dilaton (gravity) mediated contribution to the soft $A$-terms and scalar squared masses. Therefore tachyonic fields from anomaly mediation cover a larger portion of the parameter space. At first sight this might imply that the boundary conditions are ill-defined. However, there is a (large) positive contribution coming from the downlifting sector encoded in the parameters $\eta$ and $\eta'$. The main difference to the uplifting case is that in the mirage mediation regime, $\varrho = O(5)$, non-zero values of $\eta$ and $\eta'$ are mandatory.

Using the MSSM hierarchy problem as outlined in section 5.3.6 we scan over $\eta$ and $\eta'$ in the benchmark point $\varrho = 5$ at $\tan \beta = 5$. Our result is displayed in fig. 5.5.d. Like in the case of $F$-uplifting a small magnitude of the $\mu$-term (less fine-tuning) requires $\eta' > \eta$.

As an illustrative example let us consider $\eta = 4$ and $\eta' = 6$.

5.5.2 Aspects of the soft terms at $M_{\text{TeV}}$

In models where the contributions from gravity and anomaly mediation are comparable we experience the phenomenon of mirage unification. The same happens, of course, in the present case of mixed dilaton-anomaly mediation.

**Soft gaugino masses**

The GUT scale boundary conditions tab. 5.1 for the gauginos in both frameworks are exactly the same. Therefore mirage unification of the gaugino masses occurs at the mirage scale $M_{\text{Mir}}$ eq. (2.58).

**Soft $A$-terms**

Due to the reduced contribution from dilaton mediation the magnitude of the $A$-terms is smaller compared to the $F$-uplifting scheme. At the GUT scale the $A$-terms are negative and typically $|A_{ijk}| \lesssim |M_3|$. The reduced magnitude of the $A$-terms influences their RG evolution. Now the contribution from the gauge terms in eqs. (C.7) – (C.9) cannot be counterbalanced by the Yukawa terms such that none of the $A$-terms has a flat running. As a result the $A$-terms are pushed towards more negative values during their RG evolution. Although their magnitude increases at the TeV scale, they are still a factor of $2 \ldots 3$ smaller compared to the $F$-uplifting scheme. Fig. 5.5.c the RG evolution of the $A$-terms in the benchmark point $\varrho = 5$ at $\tan \beta = 5$. Larger values of $\tan \beta$ do not significantly influence the evolution of the $A$-terms.

**Soft scalar squared masses**

The masses of the scalars of the first two generations, fig. 5.5.a, behave in the same way as in the scheme of $F$-uplifting. However, due to the reduced $A$-terms
5.5 Phenomenological aspects of $F$-downlifting

the RG evolution of the masses of the third generation scalars is slightly different. This mainly affects the mass of $\tilde{t}_R$. The reduced $A$-terms no longer provide the strong suppression of $m_{\tilde{t}_R}$ such that at the TeV scale $m_{\tilde{t}_R}$ and $m_{\tilde{\tau}_R}$ are almost degenerate. We illustrate this in fig. 5.5.b for the benchmark point $\rho = 5$ at $\tan \beta = 5$. Moreover, the reduced $A$-terms lead to a smaller intragenerational mixing. In the large $\tan \beta$ regime the picture is basically the same as in the $F$-uplifting scheme with the lightest $\tau$ slepton being the lightest sfermion. We would like to emphasize that due to the reduced $A$-terms the stop (and in some regions of the parameter space also the stau) is always heavier than the lightest chargino such that the stop (and in some cases the stau) cannot be the NLSP (or LSP).

5.5.3 Low energy spectroscopy

Let us study the parameter space spanned by $\rho$ and $m_{3/2}$ for different values of $\eta$, $\eta'$ and $\tan \beta$. 

![Fig. 5.5: Evolution of scalar masses of the first two generations (panel (a)) and the third generation (panel (b)). The evolution of the $A$-terms is shown in panel (c). Panel (d) shows the $\mu$-term as a function of $\eta$ and $\eta'$.](image)
Low $\tan \beta$ regime

We start with the case $\eta = 4$ and $\eta' = 6$. The corresponding parameter space is shown in fig. 5.6.a. Correct EWSB and current LEP2 bounds put severe constraints on $\rho$ and $m_{3/2}$. Particularly we find that for $\tan \beta = 5$ only $\rho \geq 5$ and $m_{3/2} \geq 8$ TeV are allowed.

The presence of the “No EWSB” region appears because at $\rho \approx 1.5$ the RG contribution from the gluino is too small to make $m_{H_u}^2$ negative at the TeV scale. In contrast to the type IIB framework, in the heterotic framework we do not have a region with stop LSP. However, a chargino LSP appears here as well.

For $\rho$ values close to the “No EWSB” region we have $|\mu| < M_1$ and the neutralino LSP is higgsino-like. Going to larger $\rho$ values the LSP becomes a mixed higgsino-bino state. From $\rho \sim 7$ we have a mostly bino-like LSP and also $|\mu| > M_1$.

The brown/strip in fig. 5.6.a shows the region of the parameter space which is favored by the WMAP results eq. (5.19). The region to the left/below the strip...
is allowed (lower abundance) and that to the right/above the strip is forbidden (too large relic abundance). The evolution of the relic density differs from the $F$-uplifting case (type IIB framework). For example, at $m_{3/2} = 40$ TeV and $\varrho \sim 5$ (see fig. 5.6.c) we are close to the “$\tilde{\chi}^+$/LSP” region and therefore we have $\tilde{\chi}_1^0 \tilde{\chi}_1^+ \tilde{\chi}_1^0$ co-annihilation, which enhances the annihilation cross section and lowers the relic abundance. The neutralino in this region is mostly higgsino-like. As $\varrho$ increases, the neutralino becomes mixed higgsino-bino and the $\mu$-term increases. The co-annihilation with the chargino gets reduced and the annihilation cross section decreases leading to a higher relic abundance, so that around $\varrho \sim 6$ we reach the brown/strip. When we proceed to increase $\varrho$ the neutralino becomes bino-like. However, we then reach $m_A/2 \sim m_{\tilde{\chi}_1^0}$. This happens because the reduced contribution from dilaton mediation results in a smaller mass for the pseudo-scalar Higgs $A$. Here the annihilation proceeds efficiently through the pseudo-scalar Higgs exchange $\tilde{\chi}_1^0 \tilde{\chi}_1^0 \rightarrow A \rightarrow f \bar{f}$ ($A$-funnel). This enhances the annihilation cross section and reduces the relic abundance well below the WMAP bounds. For $\varrho > 7$ the mass gap $|m_A/2 - m_{\tilde{\chi}_1^0}|$ grows and the efficiency of the $A$-funnel reduces rapidly. As there are no other co-annihilation channels available the cross section decreases and the relic abundance becomes too large.

For larger values of $\eta$ and $\eta'$ the low energy phenomenology is essentially the same as in the case of $F$-uplifting in the type IIB framework with the same conclusions.

**Large tan $\beta$ regime**

Consider again $\eta = 4$ and $\eta' = 6$ at $\tan \beta = 30$ (fig. 5.6.b). For large values of $\tan \beta$, the LEP2 mass constraints become less restrictive. However, the “No EWSB” region gets slightly bigger and the “$\tilde{\chi}^+$ LSP” region covers a larger part of the parameter space (compared to the case of small $\tan \beta$). The composition of the neutralino LSP is similar to the $\tan \beta = 5$ situation. For low $\varrho$ values (close to the “No EWSB” region) the neutralino is higgsino-like. Then, for larger $\varrho$ it becomes more and more bino-like.

The brown/strip in fig. 5.6.b, satisfying the WMAP limits, differs significantly from that at $\tan \beta = 5$. Now, a larger part of the parameter space is consistent with the correct amount of neutralino DM. This is because at large $\tan \beta$ the bottom Yukawa coupling $y_b$ becomes non-negligible and acts via RG effects to further suppress the mass of the pseudo-scalar Higgs. In addition, at large $\tan \beta$ the $A$-funnel provides a sizable contribution to the annihilation cross section. For $m_{3/2} = 30$ TeV and $\varrho \sim 5.5$ (cf. fig. 5.6.d) we have $m_{\tilde{\chi}_1^+} \sim m_{\tilde{\chi}_1^0} \sim \mu$ and chargino co-annihilation enhances the annihilation cross section and lowers the relic abundance. When $\varrho$ increases, $\mu$ gets larger and the neutralino becomes bino-like. For $\varrho > 7$ the mass gap $|m_{\tilde{\chi}_1^+} - m_{\tilde{\chi}_1^0}|$ grows; thus the $\tilde{\chi}_1^+ \tilde{\chi}_1^0$ co-annihilation channel no longer provides a sizable effect. As a result, the relic abundance grows above the upper WMAP bound. At the same time the mass of $\tilde{\chi}_1^0$ approaches the value $m_A/2$ and thus the pseudo-scalar Higgs exchange
begins to contribute. The cross section \( \sigma(\tilde{\chi}^0 \tilde{\chi}^0 \rightarrow b\overline{b}) \) grows with \( \tan^2 \beta \) and so the \( A \)-funnel overcomes the decrease of the annihilation cross section caused by the bino component of the neutralino. This keeps the relic abundance up to \( \varrho \sim 10 \) in the allowed range. For still larger \( \varrho \) finally the efficiency of the \( A \)-funnel decreases and the relic abundance becomes to large.

Again, for larger values of \( \eta \) and \( \eta' \) we obtain the same picture as in the uplifting case.

### 5.6 Numerical results

In tab. 5.2 we present low energy spectra for selected points from the allowed region of the parameter space in the scheme of \( F \)-uplifting and \( F \)-downlifting, respectively. Compared to e.g. the MSUGRA scheme \([106–109]\) we can clearly see that the spectra in the mirage mediation regime (small \( \eta \) and \( \eta' \)) and in relaxed mirage mediation (large \( \eta \) and \( \eta' \)) are rather “compressed” i.e. the mass splittings between the sleptons and squarks as well as between charginos and neutralinos are relatively small. Points \( D \) and \( E \) highlight the difference between the spectra obtained in the case of uplifting and downlifting for the same set of parameters.

### 5.7 Summary

The low energy phenomenology of \( F \)-uplifting and \( F \)-downlifting exhibits a rich structure. Apart form the non-universality of the boundary conditions at the GUT scale we can have a situation where the contribution from gravity mediation (modulus/dilaton) is comparable to that of anomaly mediation. In this case gaugino masses and masses of the scalars of the first two generations unify at an intermediate scale. The values of the gaugino masses and the \( A \)-terms cannot be varied independently (at the GUT scale). However, due to the unsequestered form of the \( F \)-uplifting/\( F \)-downlifting sector, the magnitude of sfermion and Higgs masses can be very different from that of the gaugino masses and the \( A \)-terms. In particular, scalars can be as heavy as the gravitino. Using the parameters of the \( F \)-uplifting/\( F \)-downlifting sector we can alleviate the MSSM fine-tuning problem to a certain degree.

The difference in the phenomenology between \( F \)-uplifting (type IIB framework) and \( F \)-downlifting (heterotic framework) is mainly due to the reduced contribution from gravity mediation, which results from different effective Kähler potentials in the respective effective theories. In particular the reduced \( A \)-terms in the scheme of \( F \)-downlifting prevent a stop (N)LSP.

The non-universality in the gaugino masses allows for \( M_1(M_{\text{EW}}) \approx M_2(M_{\text{EW}}) \) and leads to efficient chargino-neutralino co-annihilation. We do not observe any (significant) contribution from the \( A \)-funnel to the annihilation cross section in the scheme of \( F \)-uplifting. This is because the mass of the pseudo-scalar...
Higgs $A$ is typically too large. Instead, co-annihilation with stops lowers the neutralino relic abundance. On the other hand, in the $F$-downlifting scheme the reduced gravity-mediated contribution results in a slightly different sparticle spectrum and in particular in heavier stops but a lighter pseudo-scalar Higgs boson. This activates the $A$-funnel contribution to the annihilation cross section and especially in the large $\tan\beta$ regime a large portion of the parameter space can fulfill a series of phenomenological requirements. Interestingly, the KKLT prediction $\varrho \sim 5$ is always on the safe side.
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Conclusions

The key to understanding the low energy manifestation of string physics is understanding the dynamics that stabilize the moduli in a nearly 4D Minkowski vacuum. Although it is possible to stabilize all moduli by incorporating non-perturbative effects and background fluxes, this usually leads to vacuum solutions with unrealistic energy densities. One then needs an additional sector that adjusts the vacuum energy to a small positive value. In this work we have considered the possibility that such a sector is provided by hidden sector matter fields and have investigated its behavior in the framework of type IIB and heterotic string theory.

In the framework of type IIB string theory, moduli stabilization typically yields deep supersymmetric AdS vacua. We have shown that the matter sector can successfully “uplift” the vacuum energy to a realistic value without destabilizing the moduli vacuum configuration. In this context we refer to the matter sector as the uplifting sector. In the framework of heterotic string theory the situation is even more dramatic since moduli stabilization in conventional schemes has proven to be difficult and often yields a too large positive vacuum energy. Using a single gaugino condensate we have demonstrated that the matter sector can stabilize all (relevant) moduli. In addition, the matter sector provides a “downlift” of the vacuum energy to a smaller value, which is why in this context we refer to it as the downlifting sector. We have presented explicit examples of Minkowski vacua with spontaneously broken supersymmetry and a hierarchically small gravitino mass.

Furthermore, we have shown that the uplifting/downlifting sector is the dominant source of supersymmetry breaking. Under rather general circumstances this results in a mediation scheme where the tree-level moduli (gravity) mediation compete with loop effects from the uplifting/downlifting sector, leading to the phenomenon of mirage mediation. The moduli and the gravitino become rather heavy, whereas the soft gaugino masses are suppressed by a factor of the order \( \log(M_P/m_{3/2}) \sim 4\pi^2 \). Unlike the gaugino masses, the masses of squarks and sleptons exhibit a stronger model dependence and can be as large as the gravitino mass. The MSSM soft masses often show a symptomatic pattern, known as the mirage pattern, which is especially robust for the gaugino masses.
Another emphasis of this work has been a study of the low energy spectra arising in the schemes of \(F\)-uplifting and \(F\)-downlifting. We have argued that the soft terms are basically determined by just two continuous parameters and an additional parameter from the matter sector. This so-called \(\text{relaxed} \) mirage mediation scheme differs from the \(\text{MSUGRA}\), the anomaly and the \(\text{“pure} \) mirage mediation scenario in several phenomenological aspects. First of all, the scalars can be significantly heavier than the gauginos. This reduces the fine-tuning needed to suppress excessive CP and flavor violation on one hand, and can serve to alleviate the \(\text{MSSM} \) fine-tuning problem on the other hand. Our numerical analysis shows that there are regions in the parameter space where the neutralino LSP has a sizable higgsino component which is favored by dark matter considerations. We also find that charged or colored tachyons are usually absent. In addition, this scenario yields rather compressed low energy spectra and can avoid the cosmological moduli-induced gravitino problem.

The scheme of relaxed mirage mediation exhibits very distinct phenomenological properties and has a well motivated origin. In this scenario the soft gaugino masses appear to be least model-dependent and this allows rather robust statements about their masses. Bolstered by the fact that gaugino masses are (often) closely related to the gauge coupling constants, we hope that gaugino mass relations might play a crucial role in the upcoming searches for supersymmetry at the LHC. If gauginos (and eventually other particles) are found at the LHC, this could be the first hint towards the underlying structure of supersymmetry breaking and might even shed some light on string theory’s involvement in a unified description of nature.
Appendix A

Soft breaking terms in mixed modulus-anomaly mediation

We consider models where SUSY is broken spontaneously by fields which are assumed to be singlets under the SM gauge group, living in the so-called “hidden sector”. On the other hand, MSSM fields live in the observable sector, where the breakdown of SUSY should appear explicit but soft. Such hidden sector models are defined by the fact that the only couplings between the SUSY breaking (hidden) sector and the MSSM observable sector are gravitational and suppressed by inverse powers of $M_P$. Moreover, promoting global SUSY to local symmetry naturally leads to SUGRA, an effective (supersymmetric) theory of gravitation. Then the underlying scheme of mediating the breakdown via gravity is basically MSSM coupled to SUGRA. In string theory inspired environment, the fields that break SUSY spontaneously (through the VEV of an auxiliary field) are represented by moduli. The effective SUGRA is described by the Kähler potential

$$K_{\text{eff}} = K(X_I, \bar{X}_I) + Q^i \bar{Q}^j Z_i,$$

with $Z_i$ being the Kähler metric of the visible fields $Q_i$, the superpotential

$$W_{\text{eff}} = W(X_I) + \frac{1}{6} \lambda_{ijk} Q^i Q^j Q^k,$$

and the holomorphic gauge kinetic function $f_a$. Here, $X_I$ collectively denotes hidden sector and the constants $\lambda_{ijk}$ may in general depend on these fields.

After integrating out hidden sector fields one obtains of the soft breaking terms, which in the Einstein frame are given by

$$L_{\text{soft}} = -m_i^2 Q^i \bar{Q}^j - \left[ \frac{1}{2} M_a \lambda^a \lambda_a + \frac{1}{6} A_{ijk} y_{ijk} Q^i Q^j Q^k + \text{h. c.} \right],$$

where $\lambda^a$ are gauginos, $Q_i$ are sfermions and

$$y_{ijk} = \frac{\lambda_{ijk}}{\sqrt{Y_i Y_j Y_k}}$$

denote the canonically normalized Yukawa couplings. The quantities

$$Y_i = e^{-K/3} Z_i$$

are the so-called superspace wave function coefficients (wave function renormalization).
A.1 Soft terms at tree-level

In the following we derive the tree-level mediated soft breaking terms.

Gaugino masses

They are encoded in
\[
\mathcal{L} = \frac{1}{4} \int d^2 \theta f_a \Xi^a \Xi_a + \text{h.c.}, \tag{A.6}
\]
where $\Xi^a$ is the spinorial gauge field strength and $f_a$ denotes the gauge kinetic function for the $a$th gauge group. Inserting $X_I = \langle X_I \rangle + \theta^2 F^I$ one finds
\[
\mathcal{L} = \frac{1}{4} \int d^2 \theta \left[ f_a + \theta^2 F^I \partial_I f_a \right] \Xi^a \Xi_a + \text{h.c.} \tag{A.7}
\]
The first term yields the kinetic terms
\[
\mathcal{L}_{\text{kin}} = \frac{1}{4} \Im f_a F_a^{\mu \nu} F_a^\mu^\nu + i \Re f_a \overline{\chi}^a \gamma^\mu \partial_\mu \chi^a, \tag{A.8}
\]
such that $g_a^{-2} = \Re f_a$. Since $\Xi_a = -i \chi_a^a + \ldots$ \[192\] the second term leads to the gaugino mass term
\[
\mathcal{L} = -\frac{1}{4} \chi^a \chi^a F^I \partial_I f_a, \tag{A.9}
\]
and consequently the soft gaugino masses are
\[
M_{\text{moduli}}^a = F^I \partial_I \log (\Re f_a)
= \frac{1}{2 \Re f_a} F^I \partial_I f_a. \tag{A.10}
\]

A-terms

The trilinear couplings are encoded in
\[
\mathcal{L} = \int d^2 \theta \lambda_{ijk} Q^i Q^j Q^k. \tag{A.11}
\]
To arrive at canonical kinetic terms we perform a chiral rescaling of the visible fields $Q_i$
\[
Q_i \rightarrow Y_i^{-1/2} Q_i \left[ 1 - \theta^2 \frac{1}{Y_i} F^I \partial_I Y_i + \overline{\theta}^2 \frac{1}{Y_i} \overline{F}^I \partial_I Y_i \right], \tag{A.12}
\]
where we have taken $X_I = \langle X_I \rangle + \theta^2 F^I$. Using the canonically normalized Yukawa couplings eq. (A.4) we obtain

$$\mathcal{L} = \int d^2 \theta \frac{\lambda_{ijk}}{\sqrt{Y_i Y_j Y_k}} Q^i Q^j Q^k \left( 1 - \theta^2 F^I \partial_I \log Y_i \right) \times \left( 1 - \theta^2 F^I \partial_I \log Y_j \right) \left( 1 - \theta^2 F^I \partial_I \log Y_k \right) + \text{h.c.} \quad (A.13)$$

From this one finds

$$\mathcal{L} = \mathcal{L}_{\text{YUKAWA}} + \frac{\lambda_{ijk}}{\sqrt{Y_i Y_j Y_k}} \left[ \frac{\partial_I \lambda_{ijk}}{\lambda_{ijk}} \partial_I \log Y_i - \partial_I \log Y_j - \partial_I \log Y_k \right] + \text{h.c.} \quad (A.14)$$

This leads us to the tree-level soft trilinear couplings

$$A_{\text{MODULI}}^{ijk} = F^I \partial_I \log \left( \frac{\lambda_{ijk}}{Y_i Y_j Y_k} \right) = F^I \partial_I K - F^I \partial_I \log \left( \frac{\lambda_{ijk}}{Z_i Z_j Z_k} \right), \quad (A.15)$$

where the second equality follows from eq. (A.5).

**Scalar squared masses**

The masses of the scalar fields are encoded in

$$\mathcal{L} = \int d^4 \theta Y_i Q^i Q^i. \quad (A.16)$$

Substituting $X_I = \langle X_I \rangle + \theta^2 F^I$ yields

$$\mathcal{L} = \int d^4 \theta \left[ Y_i + \theta^2 F^I \partial_I Y_i + \theta^2 \bar{F}^I \partial_I \bar{Y}_i + \theta^2 \bar{F}^I \partial_I \partial_I Y_i \right] Q^i \bar{Q}^i. \quad (A.17)$$

Using chiral rescaling eq. (A.12) we obtain canonically normalized kinetic terms

$$\mathcal{L} = \int d^4 \theta \left[ 1 + \theta^2 \bar{F}^I \partial_I \partial_I Y_i - \theta^2 \bar{F}^I \partial_I \partial_I \bar{Y}_i \right] Q^i \bar{Q}^i. \quad (A.18)$$

This leads to the soft scalar squared masses

$$\left( m_{i \text{MODULI}} \right)^2 = -F^I \bar{F}^I \frac{\partial_I \partial_I Y_i}{Y_i} + F^I \bar{F}^I \frac{\partial_I \bar{Y}_i \partial_I Y_i}{Y_i^2}$$

$$= -F^I \bar{F}^I \partial_I \partial_I \log Y_i$$

$$= \frac{1}{3} F^I \bar{F}^I \partial_I \partial_I K - F^I \bar{F}^I \partial_I \partial_I \log Z_i, \quad (A.19)$$
where we have used eq. (A.5) to arrive at the last equality. It is important to stress that there is also a contribution from the vacuum energy to the scalar squared masses such that in total one has \[ (m_{\text{MODULI}}^i)^2 = \frac{2}{3} V_0 + \frac{1}{3} F^I \bar{F}^I \partial_i \partial_j K - F^I \bar{F}^I \partial_i \partial_j \log Z_i, \] (A.20)

with \( V_0 \) denoting the vacuum energy. Any additional source of vacuum energy density generically affects the soft scalar masses and should be taken into account.

A particularly interesting situation arises when the total vacuum energy is vanishing (Minkowski vacuum). The SUGRA potential eq. (2.2) can be rewritten as

\[ V = K_I F^I \bar{F}^I - 3 e^G, \]

where \( F^I = e^{G/2} K_I^{-1} G_I \) are the SUSY breaking F-terms. In a Minkowski minimum we have

\[ V_0 = K_I F^I \bar{F}^I - 3 m_{3/2}^2 \equiv 0, \] (A.21)

where \( m_{3/2} = \langle e^{G/2} \rangle \) is the gravitino mass. Merging eqs. (A.20) and (A.21) we obtain

\[ (m_{\text{MODULI}}^i)^2 = m_{3/2}^2 - F^I \bar{F}^I \partial_i \partial_j \log Z_i. \] (A.22)

Moreover, in Minkowski space the gravitino mass serves as a measure of SUSY breakdown. In particular, if SUSY is broken (at least one) \( G_I \), and with it \( F^I \), will be non-zero at the minimum. Then, eq. (A.21) implies that the gravitino mass must be non-zero.

### A.2 Soft terms at loop-level

Hidden sector models, in which SUSY breaking is communicated via (super)gravity, also possess the superconformal Weyl symmetry \([164, 165, 193–195]\) at tree-level. At quantum level this symmetry becomes anomalous. The superconformal Weyl anomaly always introduces a coupling of the SUGRA multiplet and the soft breaking terms. These couplings are determined through the supersymmetric RG functions.

To describe the effects of the superconformal anomaly it is convenient to use the off-shell formulation of SUGRA [8]. In this formalism, the auxiliary field of the SUGRA multiplet is placed inside a non-dynamical chiral superfield \( C \) known as the conformal compensator

\[ C = C_0 + \theta^2 F C. \] (A.23)
The auxiliary field $F^C$ of the SUGRA multiplet acquires a non-zero VEV through its coupling to the SUSY breaking sector, and couples to the MSSM fields through the superconformal anomaly. Therefore, this mechanism of mediating the breakdown of SUSY is called anomaly mediation [51, 52]. Since it occurs at loop-level, anomaly mediated soft terms will be suppressed by loop factors with respect to the tree-level gravity (or moduli) mediated soft terms. Thus, if the moduli auxiliary fields are not suppressed, $F^l \simeq F^C$, the contribution from anomaly mediation is negligible.

In chapters 2, 3 and 4, however, we have witnessed models that exhibit the so-called little hierarchy which suppresses the auxiliary fields of the moduli appearing in the gauge kinetic function $f_a$. Therefore, these models can lead to soft breaking terms which are induced by a mixed modulus-anomaly mediation, also known as mirage mediation [50].

The derivation of the anomaly mediated soft terms is carried out in the off-shell SUGRA format, following the prescriptions in [51]. To proceed we make the replacement

\begin{equation}
 f_a(X_I) \rightarrow f_a(X_I, C),
 \end{equation}

\begin{equation}
 Y_i(X_I, \overline{X}_I) \rightarrow Y_i(X_I, \overline{X}_I, C, \overline{C}).
\end{equation}

**Gaugino masses**

Anomaly mediated gaugino masses are obtained from eq. (A.10) by replacing $X_I$ with $C$

\begin{equation}
 M_a^{\text{ANOMALY}} = \frac{1}{2 \kappa f_a} \frac{\partial f_a}{\partial \log C} F^C,
\end{equation}

where we have used

\begin{equation}
 \frac{\partial f_a}{\partial C} = \frac{1}{C_0} \frac{\partial f_a}{\partial \log C}.
\end{equation}

As argued in [51], the $C$ dependence in the gauge kinetic function must be of the form

\begin{equation}
 f_a = f_a(X_I, \log \frac{\Lambda_{\text{UV}} C}{\mu}),
\end{equation}

with $\Lambda_{\text{UV}}$ denoting the ultraviolet cut-off scale and $\mu$ is the renormalization scale. Given its special form, $f_a$ has the property

\begin{equation}
 \frac{\partial f_a}{\partial \log C} = -\frac{\partial f_a}{\partial \log \mu}.
\end{equation}

From $f_a = g_a^2 + i \text{Im} f_a$ we obtain

\begin{equation}
 \frac{\partial f_a}{\partial \log C} = -\frac{\partial f_a}{\partial \log \mu} = \frac{2}{g_{\ast}^2} \frac{\partial g_{\ast}}{\partial \log \mu} = \frac{b_a}{16\pi^2},
\end{equation}

where $b_a$ is a constant determined by the gauge coupling $g_{\ast}$. This leads to

\begin{equation}
 M_a^{\text{ANOMALY}} = \frac{1}{2 \kappa f_a} \frac{\partial g_{\ast}}{\partial \log \mu} F^C.
\end{equation}
where the last equality follows from the definition of the $\beta$-function eq. (C.1). Thus we arrive at
\[
M_{\text{ANOMALY}}^2 = \frac{1}{2\pi f_a} \frac{b_a}{16\pi^2} F_C \frac{F^C}{C_0} = \frac{b_a g_a^2 F^C}{16\pi^2} \frac{F^C}{C_0}.
\]

$A$-terms

To derive the anomaly mediated $A$-terms we again take the tree-level result eq. (A.15) and replace $X_I$ by $C$,
\[
A_{ijk}^\text{ANOMALY} = F_C \frac{\partial}{\partial C} \log \left( \frac{\lambda_{ijk}}{Y_i Y_j Y_k} \right),
\]
(A.32)

As argued in [51] the $C$ dependence of $Y_i$ and $\lambda_{ijk}$ must be of the form
\[
Y_i = C \bar{C} Y_i(X_I, \bar{X}_I, \log \frac{\Lambda_{\text{UV}} \bar{C} C}{\mu^2}),
\]
(A.33)
\[
\lambda_{ijk} = C^3 \lambda(X_i).
\]
(A.34)

The pre-factor $C \bar{C}$ does not contribute to the soft breaking terms as it can be rotated away by a chiral rotation $C Q_i \rightarrow Q_i$. Since $\lambda_{ijk}$ comes from the superpotential it is protected by the non-renormalization theorem [5,15] and hence it has no RG dependence. Due to its special form $Y_i$ has the property
\[
\frac{\partial \log Y_i}{\partial C} = \frac{1}{C_0} \frac{\partial \log Y_i}{\partial \log C} = -\frac{1}{C_0} \frac{\partial \log Y_i}{\partial \log \mu^2} = -\frac{1}{C_0} \frac{1}{16\pi^2} \gamma_i,
\]
(A.35)

where the last equality follows from the definition of the anomalous dimension eq. (B.5). This leads to
\[
A_{ijk}^\text{ANOMALY} = \frac{F_C}{C_0} \frac{\partial}{\partial \log \mu^2} \log \left( Y_i Y_j Y_k \right) = \frac{\gamma_i + \gamma_j + \gamma_k}{16\pi^2} F_C \frac{F^C}{C_0}.
\]
(A.36)
(A.37)

Scalar squared masses

In contrast to the gaugino masses and trilinear couplings, scalar squared masses arise at 2-loop level. Rewriting the tree-level result eq. (A.19) we obtain
\[
(m_{i}^\text{ANOMALY})^2 + (m_{i}^\text{MIXED})^2 = -F_C \frac{F^C}{C_0} \frac{\partial^2 \log Y_i}{\partial C \partial \bar{C}}
- F_C \bar{F}^I \frac{\partial^2 \log Y_i}{\partial \bar{C} \partial X_I} - \bar{F}^I F^C \frac{\partial^2 \log Y_i}{\partial \bar{C} \partial X_I},
\]
(A.38)
where the first term corresponds to pure anomaly mediation and the last two terms arise due to the mixing between the moduli fields $X_I$ and the conformal compensator $C$. According to [51] the $C$ dependence in $Y_i$ must be of the form eq. (A.25). Again, the pre-factor $C \overline{C}$ can be rotated away and does not affect the soft terms. The $C$ and $\overline{C}$ derivatives in eq. (A.38) can be replaced following eq. (A.35). We then arrive at

$$\left( m_{\text{ANOMALY}}^i \right)^2 + \left( m_{\text{MIXED}}^i \right)^2 = -\frac{1}{16\pi^2} \frac{\partial \gamma_i}{\partial \log \mu^2} \left| \frac{FC}{C_0} \right|^2$$
$$+ F_I \frac{\partial \gamma_i}{\partial X_I} \frac{1}{16\pi^2} \frac{FC}{C_0} + \overline{F_I} \frac{\partial \gamma_i}{\partial \overline{X}_I} \frac{1}{16\pi^2} \frac{FC}{\overline{C}_0}$$

$$= -\frac{\dot{\gamma}_i}{16\pi^2} \frac{1}{FC} \left| \frac{FC}{C_0} \right|^2$$
$$+ \frac{F_I}{X_I + \overline{X}_I} \frac{\Psi^I_i}{16\pi^2} \frac{FC}{C_0} + \frac{\overline{F_I}}{X_I + \overline{X}_I} \frac{\overline{\Psi}^I_i}{16\pi^2} \frac{FC}{\overline{C}_0},$$  
(A.39)

where we have introduced the quantities

$$\frac{1}{16\pi^2} \dot{\gamma}_i = \frac{\partial \gamma_i}{\partial \log \mu^2},$$  
(A.40)

$$\Psi^i_i = (X + \overline{X}_I) \frac{\partial \gamma_i}{\partial X_I}.$$  
(A.41)

Eq. (A.40) describes the RG evolution of the anomalous dimension whereas eq. (A.41) results from the moduli dependence of the gauge couplings (i.e. gauge kinetic function).

Finally, we summarize the soft breaking parameters induced by the mixed modulus anomaly mediation:

$$M_a = \frac{1}{2\kappa} F_I \partial f_a + \frac{b_a g_s^2}{16\pi^2} \frac{FC}{C_0},$$  
(A.42)

$$A_{ijk} = F_I \partial K - F_I \partial I \log \left( \frac{\lambda_{ijk}}{Z_i Z_j Z_k} \right) + \frac{\gamma_i + \gamma_j + \gamma_k}{16\pi^2} \frac{FC}{C_0},$$  
(A.43)

$$m_i^2 = \left( m_3^2 / 2 + V_0 \right) - F_I \overline{F_I} \partial I \partial J \log Z_i - \frac{\dot{\gamma}_i}{(16\pi^2)^2} \frac{1}{C_0} \left| \frac{FC}{C_0} \right|^2$$
$$+ \frac{F_I}{X_I + \overline{X}_I} \frac{\Psi^I_i}{16\pi^2} \frac{FC}{C_0} + \frac{\overline{F_I}}{X_I + \overline{X}_I} \frac{\overline{\Psi}^I_i}{16\pi^2} \frac{FC}{\overline{C}_0},$$  
(A.44)

The explicit form of the parameters $\lambda_{ijk}$ depends on the theory of flavor and can only be addressed after realistic Yukawa flavor structures have been obtained. For simplicity, in this work we assume that $\lambda_{ijk}$ are moduli independent.
A.3 Soft terms in $F$-uplifting

In the scheme of $F$-uplifting the effective SUGRA theory (originating from type IIB string theory) is described by

$$f_a = T,$$  \hspace{1cm} (A.45)

$$K = -3 \log (T + \overline{T}) + \phi \overline{\phi},$$  \hspace{1cm} (A.46)

$$Z_i = (T + \overline{T})^{-n_i} \left[ 1 + \xi_i \phi \overline{\phi} \right],$$  \hspace{1cm} (A.47)

where we have assumed a universal (real) Kähler modulus $T$ and a single hidden sector (real) matter field $\phi$ with modular weight zero. The parameter $\xi_i$ describe the coupling between hidden and observable matter fields and $n_i$ denote effective modular weights. SUSY is broken by $F_T$ and $F_\phi$, with the latter providing the dominant contribution. Plugging eqs. (A.45) – (A.47) into eqs. (A.42) – (A.44) we obtain

$$M_a = \frac{F_T}{T_0 + \overline{T}_0} + \frac{b_a g_a^2 F_C}{16 \pi^2 C_0},$$  \hspace{1cm} (A.48)

$$A_{ijk} = \frac{F_T}{T_0 + \overline{T}_0} \left[ -3 + n_i + n_j + n_k \right] + \frac{\gamma_i + \gamma_j + \gamma_k}{16 \pi^2} \frac{F_C}{C_0},$$  \hspace{1cm} (A.49)

$$m_i^2 = \left( m_{3/2}^2 + V_0 \right) - n_i \left( \frac{|F_T|^2}{(T_0 + \overline{T}_0)^2} - \xi_i |F_\phi|^2 \right) - \gamma_i \frac{1}{(16 \pi^2)^2} \frac{F_C^2}{C_0} + 2 \frac{F_T}{T_0 + \overline{T}_0} \frac{\Psi_i^T F_C}{16 \pi^2 C_0},$$  \hspace{1cm} (A.50)

where we have assumed that the uplifting field $\phi$ is stabilized at $\phi_0 \ll 1$.

The scalar potential,

$$V = K_T \left| F_T \right|^2 + |F_\phi|^2 - 3 \phi \overline{\phi}^G,$$  \hspace{1cm} (A.51)

poses a relation among the SUSY breaking fields. The condition for having a Minkowski minimum gives

$$3 m_{3/2}^2 = 3 \left( \frac{|F_T|^2}{(T_0 + \overline{T}_0)^2} + |F_\phi|^2 \right).$$  \hspace{1cm} (A.52)

Thus, in a Minkowski vacuum we have

$$m_i^2 = (1 - 3 \xi_i) m_{3/2}^2 + (3 \xi_i - n_i) \left( \frac{|F_T|^2}{(T_0 + \overline{T}_0)^2} \right) - \gamma_i \frac{1}{(16 \pi^2)^2} \left( \frac{F_C^2}{C_0} \right) + 2 \frac{F_T}{T_0 + \overline{T}_0} \frac{\Psi_i^T F_C}{16 \pi^2 C_0}.$$  \hspace{1cm} (A.53)
Note that $\xi_i = 1/3$ corresponds to the case of KKLT models where the uplifting sector is assumed to be sequestered and hence in these models there is no contribution from the uplifting fields to the soft breaking terms.

Finally, using the parameterization

$$\varrho \equiv \frac{F_T}{T_0 + \bar{T}_0}, \quad (A.54)$$

$$M_0 \equiv \frac{m_{3/2}}{16\pi^2}, \quad (A.55)$$

$$\eta_i^2 \equiv (1 - 3\xi_i)(16\pi^2)^2, \quad (A.56)$$

$$\frac{F_C}{C_0} \equiv m_{3/2} + O\left(\frac{m_{3/2}}{16\pi^2}\right) \approx m_{3/2}, \quad (A.57)$$

we can write the soft terms in a compact form as

$$M_a = M_0\left[\varrho + b_a g_s^2S_{CUT}\right], \quad (A.58)$$

$$A_{ijk} = M_0\left[(-3\varrho + n_i + n_j + n_k) + (\gamma_i + \gamma_j + \gamma_k)\right], \quad (A.59)$$

$$m_i^2 = M_0^2\left[3\xi_i - n_i\right]g_s^2 - \gamma_i + 2\varrho \Psi_i^T + \eta_i^2, \quad (A.60)$$

### A.4 Soft terms in F-downlifting

In the scheme of F-downlifting the effective SUGRA theory (originating from heterotic string theory) is described by

$$f_a = S, \quad (A.61)$$

$$K = -\log\left(\frac{S + \bar{S}}{S}\right) + \phi\bar{\phi}, \quad (A.62)$$

$$Z_i = 1 + \xi_i\phi\bar{\phi}, \quad (A.63)$$

where we have assumed a real dilaton field $S$ and a single hidden sector (real) matter field $\phi$ with modular weight zero. Furthermore, it assumed that the Kähler moduli $T_p$ with $p = 1, 2, 3$ are stabilized at $F_{T_p} = 0$. Then the breakdown of SUSY is initiated through $F_S$ and $F_\phi$, with the latter providing the dominant contribution. Plugging eqs. (A.61) – (A.63) into eqs. (A.42) – (A.44) we obtain

$$M_a = \frac{F_S}{S_0 + \bar{S}_0} + \frac{b_a g_s^2}{16\pi^2 C_0} F_C, \quad (A.64)$$

$$A_{ijk} = \frac{F_S}{S_0 + \bar{S}_0}\left[-3 + n_i + n_j + n_k\right] + \frac{\gamma_i + \gamma_j + \gamma_k}{16\pi^2} \frac{F_C}{C_0}, \quad (A.65)$$

$$m_i^2 = \left(m_{3/2}^2 + V_0\right) - \xi_i \left|F_\phi\right|^2 - \gamma_i - \frac{1}{(16\pi^2)^2} \left|\frac{F_C}{C_0}\right|^2 + 2 \frac{F_S}{S_0 + \bar{S}_0} \frac{\Psi_i^S}{16\pi^2 C_0} F_C, \quad (A.66)$$
where we have assumed that the downlifting field $\phi$ is stabilized at $\phi_0 \ll 1$.

This time, the condition for having a Minkowski vacuum gives the relation

$$3m_{3/2}^2 = \frac{|F^S|^2}{(S_0 + \bar{S}_0)^2} + |F^\phi|^2,$$

which differs by a factor of 3 from eq. (A.52). Thus, in a Minkowski vacuum we have

$$m_i^2 = (1 - 3\xi_i) m_{3/2}^2 + \xi_i \frac{|F^S|^2}{(S_0 + \bar{S}_0)^2}$$

$$- \gamma_i^2 \frac{1}{(16\pi^2)^2} |F^C|^2 + 2 \frac{F^S}{S_0 + \bar{S}_0} \frac{\Psi^S_i \Psi^C}{16\pi^2 \bar{C}_0}.$$  \hspace{1cm} (A.68)

Using eqs. (A.55) and (A.57) together with

$$\rho M_0 \equiv \frac{F^S}{S_0 + \bar{S}_0},$$

we arrive at

$$M_a = M_0 \left[ \rho + b_a \frac{g^2}{\text{GUT}} \right],$$

$$A_{ijk} = M_0 \left[ - \rho + \left( \gamma_i + \gamma_j + \gamma_k \right) \right],$$

$$m_i^2 = M_0^2 \left[ \xi_i \rho^2 - \gamma_i^2 + 2 \rho \Psi^S_i + \eta_i^2 \right].$$  \hspace{1cm} (A.72)
Appendix B

MSSM parameters

Here we list various parameters which appear in the soft breaking terms eqs. (A.48), (A.49) and (A.53) as well as in eqs. (A.64), (A.65) and (A.68).

**β-function coefficients**

The 1-loop β-function coefficients are defined by

\[ b_a = -3C_a + \sum_i C^i_a, \]  

(B.1)

for \( a = 1, 2, 3 \). The quantity \( C_a \) is the quadratic Casimir invariant of the group being 0 for U(1) and \( N \) for SU(\( N \)). The \( C^i_a \) are the quadratic group theory invariants for the \( i \)th superfield defined in terms of the Lie algebra generators \( T^a \)

\[ (T^a T^a)^I = C^i_a \delta^I_i, \]  

(B.2)

with the gauge coupling \( g_a \). In order to agree with the canonical covariant derivative for grand unified unification of the SM gauge group SU(3)\(_C\) \( \times \) SU(2)\(_L\) \( \times \) U(1)\(_Y\) into SU(5) or SO(10) we choose the normalization (see e.g. [31])

\[ g_3 = g_\epsilon, \quad g_2 = g, \quad g_1 = \sqrt{\frac{5}{3}} g', \]  

(B.3)

where \( g' \) and \( g \) are the EW couplings with \( e = g \sin \theta_W = g' \cos \theta_W \).

For the MSSM matter content one has

<table>
<thead>
<tr>
<th>Superfields</th>
<th>( C^3 )</th>
<th>( C^2 )</th>
<th>( C^1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_p )</td>
<td>0 4/3</td>
<td>0 3/4</td>
<td>0 1/60</td>
</tr>
<tr>
<td>( \bar{u}_p )</td>
<td>0 4/3</td>
<td>0 3/4</td>
<td>0 1/15</td>
</tr>
<tr>
<td>( \bar{d}_p )</td>
<td>0 4/3</td>
<td>0 3/4</td>
<td>0 1/15</td>
</tr>
<tr>
<td>( L_p )</td>
<td>0 0 3/4</td>
<td>0 3/4</td>
<td>0 3/20</td>
</tr>
<tr>
<td>( \bar{e}_p )</td>
<td>0 0 3/4</td>
<td>0 3/4</td>
<td>0 3/5</td>
</tr>
<tr>
<td>( H_u )</td>
<td>0 0 3/4</td>
<td>0 3/4</td>
<td>0 3/20</td>
</tr>
<tr>
<td>( H_d )</td>
<td>0 0 3/4</td>
<td>0 3/4</td>
<td>0 3/20</td>
</tr>
</tbody>
</table>

Tab. B.1 :: Quadratic Casimirs for the MSSM fields.
where $Q_p$ denote the quark doublets, $\bar{u}_p$ are right-handed up-type quarks, $\bar{d}_p$ are the right-handed down-type quarks, $L_p$ denote the lepton doublets, $\bar{e}$ are the right-handed leptons, $H_u$ and $H_d$ are the two Higgs doublets and $p$ labels the generation. For this matter content one easily finds

$$b_3 = -3, \quad b_2 = 1, \quad b_1 = \frac{33}{5}.$$  \hfill (B.4)

### Anomalous dimension

The anomalous dimension describes the scale dependence of the wave function renormalization eq. (A.5)

$$\frac{1}{16\pi^2} \gamma_i = \frac{d \log Y_i}{d \log \mu^2},$$  \hfill (B.5)

where $i$ labels the MSSM superfields and $\mu$ denotes the renormalization scale. In particular, at 1-loop level one has [10],

$$\gamma_i = 2 \sum_a g_a^2 C_i^a - \sum_{jk} y_{ijk}^2 \frac{2}{2}.$$  \hfill (B.6)

The first sum runs over gauge group factors and the second sum runs over all Yukawa couplings that contain the $i$th fields with appropriate color factors included. For the MSSM one obtains

<table>
<thead>
<tr>
<th>Superfields</th>
<th>$\gamma_i$</th>
<th>$\gamma_i(M_{GUT})_{\tan \beta=5}$</th>
<th>$\gamma_i(M_{GUT})_{\tan \beta=30}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_3$</td>
<td>$\frac{8}{3} S_2^2 + \frac{3}{2} S_2^2 + \frac{1}{30} S_1^2 - (y_1^2 + y_5^2)$</td>
<td>1.84</td>
<td>1.83</td>
</tr>
<tr>
<td>$Q_{1,2}$</td>
<td>$\frac{8}{3} S_2^2 + \frac{3}{2} S_2^2 + \frac{1}{30} S_1^2$</td>
<td>2.14</td>
<td>2.14</td>
</tr>
<tr>
<td>$\bar{u}_3$</td>
<td>$\frac{8}{3} S_2^2 + \frac{4}{3} S_1^2 - 2y_1^2$</td>
<td>1.01</td>
<td>1.05</td>
</tr>
<tr>
<td>$\bar{u}_{1,2}$</td>
<td>$\frac{8}{3} S_2^2 + \frac{4}{3} S_1^2$</td>
<td>1.62</td>
<td>1.62</td>
</tr>
<tr>
<td>$\bar{d}_3$</td>
<td>$\frac{8}{3} S_2^2 + \frac{2}{3} S_1^2 - 2y_2^2$</td>
<td>1.36</td>
<td>1.01</td>
</tr>
<tr>
<td>$\bar{d}_{1,2}$</td>
<td>$\frac{8}{3} S_2^2 + \frac{2}{3} S_1^2$</td>
<td>1.42</td>
<td>1.42</td>
</tr>
<tr>
<td>$L_3$</td>
<td>$\frac{3}{2} S_2^2 + \frac{1}{30} S_1^2 - y_1^2$</td>
<td>0.93</td>
<td>0.87</td>
</tr>
<tr>
<td>$L_{1,2}$</td>
<td>$\frac{3}{2} S_2^2 + \frac{1}{30} S_1^2$</td>
<td>0.93</td>
<td>0.93</td>
</tr>
<tr>
<td>$\bar{e}_3$</td>
<td>$\frac{6}{5} S_1^2 - 2y_1^2$</td>
<td>0.61</td>
<td>0.49</td>
</tr>
<tr>
<td>$\bar{e}_{1,2}$</td>
<td>$\frac{6}{5} S_1^2$</td>
<td>0.61</td>
<td>0.61</td>
</tr>
<tr>
<td>$H_u$</td>
<td>$\frac{3}{2} S_2^2 + \frac{1}{30} S_1^2 - 3y_1^2$</td>
<td>0.02</td>
<td>0.07</td>
</tr>
<tr>
<td>$H_d$</td>
<td>$\frac{3}{2} S_2^2 + \frac{3}{10} S_1^2 - 3y_2^2 - y_1^2$</td>
<td>0.92</td>
<td>0.80</td>
</tr>
</tbody>
</table>

Tab. B.2 :: Anomalous dimension of the MSSM fields.
Running of the anomalous dimension

This is simply given by

\[ \frac{1}{16\pi^2} \dot{\gamma}_i = \frac{d\gamma_i}{d \log \mu^2}. \] (B.7)

In the MSSM one has (at 1-loop)

\[ \dot{\gamma}_i = 2 \sum_a \delta^a_i \, b_a \, C^a_i - \sum_{jk} \frac{y^2_{ij}}{2} \, b_{y_{jk}}, \] (B.8)

where \( b_{y_{ij}} \) describes the running of the Yukawa couplings (cf. eqs. (C.3) – (C.5)). Explicitly one finds

<table>
<thead>
<tr>
<th>Superfields</th>
<th>( \dot{\gamma}_i )</th>
<th>( \dot{\gamma}(M_{\text{GUT}}) )</th>
<th>( \dot{\gamma}(M_{\text{GUT}}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_3 )</td>
<td>(-8g_3^4 + \frac{3}{2}g_2^4 + \frac{11}{50}g_1^4 - (y_{\tau}^2 \beta_{\tau} + y_{t}^2 \beta_{t}))</td>
<td>-0.72</td>
<td>-0.65</td>
</tr>
<tr>
<td>( Q_{1,2} )</td>
<td>(-8g_3^4 + \frac{3}{2}g_2^4 + \frac{11}{50}g_1^4)</td>
<td>-1.59</td>
<td>-1.60</td>
</tr>
<tr>
<td>( \bar{u}_3 )</td>
<td>(-8g_3^4 + \frac{88}{25}g_1^4)</td>
<td>0.59</td>
<td>0.53</td>
</tr>
<tr>
<td>( \bar{u}_{1,2} )</td>
<td>(-8g_3^4 + \frac{88}{25}g_1^4)</td>
<td>-1.15</td>
<td>-1.16</td>
</tr>
<tr>
<td>( \bar{d}_3 )</td>
<td>(-8g_3^4 + \frac{22}{25}g_1^4 - 2y_{t}^2 \beta_{t})</td>
<td>-1.82</td>
<td>-1.63</td>
</tr>
<tr>
<td>( \bar{d}_{1,2} )</td>
<td>(-8g_3^4 + \frac{22}{25}g_1^4)</td>
<td>-1.82</td>
<td>-1.83</td>
</tr>
<tr>
<td>( L_3 )</td>
<td>(\frac{3}{2}g_2^4 + \frac{22}{25}g_1^4 - y_{t}^2 \beta_{t})</td>
<td>0.91</td>
<td>1.03</td>
</tr>
<tr>
<td>( L_{1,2} )</td>
<td>(\frac{3}{2}g_2^4 + \frac{22}{25}g_1^4)</td>
<td>0.91</td>
<td>0.91</td>
</tr>
<tr>
<td>( \tilde{e}_3 )</td>
<td>(\frac{198}{25}g_1^4 - 2y_{t}^2 \beta_{t})</td>
<td>2.04</td>
<td>2.26</td>
</tr>
<tr>
<td>( \tilde{e}_{1,2} )</td>
<td>(\frac{198}{25}g_1^4)</td>
<td>2.03</td>
<td>2.02</td>
</tr>
<tr>
<td>( H_u )</td>
<td>(\frac{3}{2}g_2^4 + \frac{22}{25}g_1^4 - 3y_{t}^2 \beta_{t})</td>
<td>3.52</td>
<td>3.43</td>
</tr>
<tr>
<td>( H_d )</td>
<td>(\frac{3}{2}g_2^4 + \frac{22}{25}g_1^4 - 3y_{t}^2 \beta_{t} - y_{t}^2)</td>
<td>0.92</td>
<td>1.33</td>
</tr>
</tbody>
</table>

Tab. B.3 :: Running of the anomalous dimension in the MSSM.

Moduli dependence of the anomalous dimension

In string inspired models the (unified) gauge coupling is given in terms of the gauge kinetic function which is moduli dependent. Thus also the anomalous
MSSM parameters

dimension eq. (B.6) is moduli dependent. Generically one finds [47,48]

\[
\frac{\partial \gamma_i}{\partial X_l} = - \sum_{jk} \frac{y_{ijk}^2}{2} \frac{\partial}{\partial X_l} \log \left( \frac{\lambda_{ijk}}{Y_i Y_j Y_k} \right) - 2 \sum_a g_a^2 C^a_i \frac{\partial}{\partial X_l} \log (\Re f_a) \tag{B.9}
\]

\[
= - \sum_{jk} \frac{y_{ijk}^2}{2} \frac{\partial}{\partial X_l} \log (\Re f_a), \tag{B.10}
\]

where the second equality is due to the assumption of \( \lambda_{ijk} \) being moduli independent. For the soft breaking terms it is convenient to consider the quantity eq. (A.41).

In the scheme of \( F \)-uplifting (type IIB strings) the effective SUGRA is described by eqs. (A.45) – (A.47). This yields

\[
\Psi^T_i = \sum_{jk} \frac{y_{ijk}^2}{2} (3 - n_i - n_j - n_k) - 2 \sum_a g_a^2 C^a_i. \tag{B.11}
\]

In case of zero modular weights one obtains

<table>
<thead>
<tr>
<th>Superfields</th>
<th>( \Psi^T_i )</th>
<th>( \Psi^T_i (M_{3/2})_{\text{tan} \beta = 5} )</th>
<th>( \Psi^T_i (M_{3/2})_{\text{tan} \beta = 30} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_3 )</td>
<td>(-\frac{3}{2} s^2 - \frac{3}{2} s_1 )</td>
<td>-0.92</td>
<td>-0.76</td>
</tr>
<tr>
<td>( L_{1,2} )</td>
<td>(-\frac{3}{2} s^2 - \frac{3}{10} s_1 )</td>
<td>-0.93</td>
<td>-0.93</td>
</tr>
<tr>
<td>( \tilde{e}_3 )</td>
<td>(-\frac{3}{2} s^2 + 6 y^2_0 )</td>
<td>-0.60</td>
<td>-0.27</td>
</tr>
<tr>
<td>( \tilde{e}_{1,2} )</td>
<td>(-\frac{3}{2} s^2 )</td>
<td>-0.61</td>
<td>-0.61</td>
</tr>
<tr>
<td>( H_u )</td>
<td>(-\frac{3}{2} s^2 - \frac{3}{10} s^2_1 + 9 y^2_0 )</td>
<td>1.80</td>
<td>1.63</td>
</tr>
<tr>
<td>( H_d )</td>
<td>(-\frac{3}{2} s^2 - \frac{3}{10} s^2_1 + 9 y^2_0 )</td>
<td>-0.92</td>
<td>-0.53</td>
</tr>
</tbody>
</table>

Tab. B.4 \( \equiv \) Moduli dependence of the anomalous dimension framework of type IIB string theory.

In the scheme of \( F \)-downlifting (heterotic strings) the effective SUGRA is described by eqs. (A.61) – (A.63). This yields

\[
\Psi^S_i = \sum_{jk} \frac{y_{ijk}^2}{2} - 2 \sum_a g_a^2 C^a_i. \tag{B.12}
\]
Explicit values for the MSSM superfields are given in tab. B.5. Note that the Yukawa terms in $\Psi_i^S$ are smaller by a factor of 3 compared to $\Psi_i^T$ due to the different (effective) Kähler potential.

<table>
<thead>
<tr>
<th>Superfields</th>
<th>$\Psi_i^S$</th>
<th>$\Psi_i^S(M_{GUT})_{\tan\beta=5}$</th>
<th>$\Psi_i^S(M_{GUT})_{\tan\beta=30}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q_3$</td>
<td>$-\frac{8}{3} S_3 - \frac{3}{2} S_2^2 - \frac{1}{30} S_1^2 + 3(y_t^2 + y_b^2)$</td>
<td>-1.84</td>
<td>-1.83</td>
</tr>
<tr>
<td>$Q_{1,2}$</td>
<td>$-\frac{8}{3} S_3 - \frac{3}{10} S_1^2$</td>
<td>-2.14</td>
<td>-2.14</td>
</tr>
<tr>
<td>$\bar{u}_3$</td>
<td>$-\frac{8}{3} S_3 - \frac{8}{15} S_1^2 + 6y_t^2$</td>
<td>-1.01</td>
<td>-1.05</td>
</tr>
<tr>
<td>$\bar{u}_{1,2}$</td>
<td>$-\frac{8}{3} S_3 - \frac{8}{15} S_1^2$</td>
<td>-1.62</td>
<td>-1.62</td>
</tr>
<tr>
<td>$\bar{d}_3$</td>
<td>$-\frac{8}{3} S_3 - \frac{8}{15} S_1^2 + 6y_b^2$</td>
<td>-1.42</td>
<td>-1.37</td>
</tr>
<tr>
<td>$\bar{d}_{1,2}$</td>
<td>$-\frac{8}{3} S_3 - \frac{8}{15} S_1^2$</td>
<td>-1.42</td>
<td>-1.42</td>
</tr>
<tr>
<td>$L_3$</td>
<td>$-\frac{2}{3} S_3^2 - \frac{3}{10} S_1^2 + 3y_t^2$</td>
<td>-0.93</td>
<td>-0.87</td>
</tr>
<tr>
<td>$L_{1,2}$</td>
<td>$-\frac{3}{2} S_3^2 - \frac{3}{10} S_1^2$</td>
<td>-0.93</td>
<td>-0.93</td>
</tr>
<tr>
<td>$\bar{e}_3$</td>
<td>$-\frac{6}{5} S_1^2 + 6y_t^2$</td>
<td>-0.61</td>
<td>-0.49</td>
</tr>
<tr>
<td>$\bar{e}_{1,2}$</td>
<td>$-\frac{6}{5} S_1^2$</td>
<td>-0.61</td>
<td>-0.61</td>
</tr>
<tr>
<td>$H_u$</td>
<td>$-\frac{2}{3} S_3^2 - \frac{3}{10} S_1^2 + 9y_t^2$</td>
<td>-0.02</td>
<td>-0.07</td>
</tr>
<tr>
<td>$H_d$</td>
<td>$-\frac{3}{2} S_2^2 - \frac{3}{10} S_1^2 + 9y_b^2 + 3y_t^2$</td>
<td>-0.92</td>
<td>-0.80</td>
</tr>
</tbody>
</table>

Tab. B.5: Dilaton dependence of the anomalous dimension framework of heterotic string theory.
Appendix C
Renormalization group

The soft terms obtained in appendices A.3 and A.4 are given just below the ultraviolet cut-off scale $\Lambda_{\text{UV}}$ where the effective theory sets in. In this work we adopt the MSSM matter content which yields an almost perfect unification of the gauge couplings of the SM gauge group around $M_{\text{GUT}} \approx 2 \times 10^{16}$ GeV. Thus we use $\Lambda_{\text{UV}} = M_{\text{GUT}}$. The values of the soft parameters below $M_{\text{GUT}}$ (i.e. at $M_{\text{UV}}$) are obtained via their RG evolution.

We stress that the framework of the MSSM requires SUSY respecting regularization and renormalization schemes. The most appropriate regularization scheme is the so-called regularization by dimensional reduction (DRED) with modified minimal subtraction ($\overline{\text{MS}}$) [196]. Moreover, the RG equations are governed by the non-renormalization theorem [5, 15]. In particular, it states that the logarithmically divergent contributions can always be absorbed into the wave function renormalization.

For a qualitative discussion of the low energy spectra it is sufficient to consider RG equations at 1-loop order (see e.g. [10]). In this work we use the approximation that only the third generation Yukawa couplings take on non-negligible values and assume that the soft trilinear couplings are proportional to the Yukawa matrices. We denote the renormalization scale by $\mu$.

Gauge couplings

$$\frac{dg_a}{d\log \mu} = \frac{1}{16\pi^2} b_a g_a^3,$$  \hspace{1cm} (C.1)

where $a = 1, 2, 3$ and $b_a$ are the 1-loop $\beta$-function coefficients eq. (B.4).

Gaugino masses

$$\frac{dM_a}{d\log \mu} = \frac{1}{8\pi^2} b_a g_a^2 M_a.$$  \hspace{1cm} (C.2)

The quantities $M_a/g_a^2$ are each constant and consequently do not run at 1-loop.
Yukawa couplings

\[ \frac{dy_t}{d \log \mu} = \frac{y_t}{16\pi^2} \left[ 6y_t^2 + y_b^2 - \frac{16}{3} \delta_3^2 - 3\delta_2^2 - \frac{13}{15} \delta_1^2 \right] \]  
\[ \frac{dy_b}{d \log \mu} = \frac{y_b}{16\pi^2} \left[ 6y_b^2 + y_t^2 + y_t^2 - \frac{16}{3} \delta_3^2 - 3\delta_2^2 - \frac{7}{15} \delta_1^2 \right] \]  
\[ \frac{dy_\tau}{d \log \mu} = \frac{y_\tau}{16\pi^2} \left[ 4y_\tau^2 + 3y_b^2 - 3\delta_2^2 - \frac{9}{5} \delta_1^2 \right] \]

\mu-term

\[ \frac{d\mu}{d \log \mu} = \frac{\mu}{16\pi^2} \left[ 3y_t^2 + 3y_b^2 + y_\tau^2 - 3\delta_2^2 - \frac{3}{5} \delta_1^2 \right] \]

Note that the \( \mu \) in the denominator denotes the renormalization scale.

A-terms

We use the abbreviations \( A_t \equiv A_{Q_{t_R}H_u t_R}, A_b \equiv A_{Q_{b_R}H_u b_R} \) and \( A_\tau \equiv A_{L_{\tau_R}H_d \tau_R} \).

\[ \frac{dA_t}{d \log \mu} = \frac{1}{16\pi^2} \left[ 12y_t^2 A_t + 2y_b^2 A_b + \frac{32}{3} \delta_3^2 M_3 + 6\delta_2^2 M_2 + \frac{26}{15} \delta_1^2 M_1 \right] \]
\[ \frac{dA_b}{d \log \mu} = \frac{1}{16\pi^2} \left[ 12y_b^2 A_b + 2y_t^2 A_t + 2y_\tau^2 A_\tau 
+ \frac{32}{3} \delta_3^2 M_3 + 6\delta_2^2 M_2 + \frac{14}{15} \delta_1^2 M_1 \right] \]
\[ \frac{dA_\tau}{d \log \mu} = \frac{1}{16\pi^2} \left[ 8y_\tau^2 A_\tau + 6y_b^2 A_b + 6\delta_2^2 M_2 + \frac{18}{15} \delta_1^2 M_1 \right] \]

The RG equations for the scalar squared masses can be written in a more suggestive form by using

\[ X_t = m_{Q_{t_R}}^2 + m_{H_u}^2 + m_{t_R}^2 + A_t^2, \]
\[ X_b = m_{Q_{b_R}}^2 + m_{H_u}^2 + m_{b_R}^2 + A_b^2, \]
\[ X_\tau = m_{L_{\tau_R}}^2 + m_{H_d}^2 + m_{\tau_R}^2 + A_\tau^2, \]
\[ S = \frac{1}{2} \sum_i Y_i m_i^2, \]

where the sum in eq. (C.13) runs over all MSSM scalar fields with hypercharge \( Y_i \). In most realistic models the contributions proportional to \( S \) are known to be relatively small.
Higgs mass squares

\[
\frac{d m_{H_u}^2}{d \log \mu} = \frac{1}{16 \pi^2} \left[ 6 y_t^2 X_t - 6 g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 + \frac{3}{5} g_1^2 S \right] \quad (C.14)
\]

\[
\frac{d m_{H_d}^2}{d \log \mu} = \frac{1}{16 \pi^2} \left[ 6 y_b^2 X_b + 2 y_t^2 X_t - 6 g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 - \frac{3}{5} g_1^2 S \right] \quad (C.15)
\]

Third generation squark mass squares

\[
\frac{d m_{\tilde{Q}_3}^2}{d \log \mu} = \frac{1}{16 \pi^2} \left[ 2 y_t^2 X_t + 2 y_b^2 X_b \right.
\]

\[
\left. - \frac{32}{3} g_3^2 M_3^2 - 6 g_2^2 M_2^2 - \frac{2}{15} g_1^2 M_1^2 + \frac{1}{5} g_1^2 S \right] \quad (C.16)
\]

\[
\frac{d m_{\tilde{t}_R}^2}{d \log \mu} = \frac{1}{16 \pi^2} \left[ 4 y_t^2 X_t - \frac{32}{3} g_3^2 M_3^2 - \frac{32}{15} g_1^2 M_1^2 - \frac{4}{5} g_1^2 S \right] \quad (C.17)
\]

\[
\frac{d m_{\tilde{b}_R}^2}{d \log \mu} = \frac{1}{16 \pi^2} \left[ 4 y_b^2 X_b - \frac{32}{3} g_3^2 M_3^2 - \frac{8}{15} g_1^2 M_1^2 + \frac{2}{5} g_1^2 S \right] \quad (C.18)
\]

Third generation slepton mass squares

\[
\frac{d m_{\tilde{L}^{(3)}}^2}{d \log \mu} = \frac{1}{16 \pi^2} \left[ 2 y_t^2 X_t - 6 g_2^2 M_2^2 - \frac{6}{5} g_1^2 M_1^2 - \frac{3}{5} g_1^2 S \right] \quad (C.19)
\]

\[
\frac{d m_{\tilde{\tau}_R}^2}{d \log \mu} = \frac{1}{16 \pi^2} \left[ 4 y_t^2 X_t - \frac{24}{5} g_1^2 M_1^2 + \frac{6}{5} g_1^2 S \right] \quad (C.20)
\]

First and second generation mass squares

\[
\frac{d m_i^2}{d \log \mu} = -\frac{1}{16 \pi^2} \left[ \sum_a 8 C_a^i g_a^2 M_a^2 + \frac{6}{5} Y_i g_1^2 S \right], \quad (C.21)
\]

where \(i\) runs over all first and second generation scalars, \(Y_i\) denotes the corresponding hypercharge and \(C_a^i\) are the quadratic Casimir for the \(a^{th}\) gauge group corresponding to the representation to which the \(i^{th}\) superfield belongs.
Bibliography


[63] K.-S. Choi and J. E. Kim, “Quarks and leptons from orbifolded superstring.”.


