7. Test Computations

Figure 7.41: (a) Position differences (b) velocity differences (c) acceleration differences (d) Fourier spectrum (dynamical restriction index window [1-5], reference motion: ellipse mode, $J_{\text{max}}=4$, $N_F=300$, white noise of 2 cm, observation type: Positions, method: flowchart Fig. 6.2, observation sampling rate: 30 sec., test sampling rate: 10 sec., without $\approx 5\%$ at boundaries $\equiv 9$ points or 80 sec.).

RMS

<table>
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<th>Vel.($m/s$)</th>
<th>Acc.($m/s^2$)</th>
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<td>59</td>
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<td>0.000721</td>
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</table>
Figure 7.42: (a) Position differences (b) velocity differences (c) acceleration differences (d) Fourier spectrum (dynamical restriction index window [1-59], reference motion: ellipse mode, $J_{max}=4$, $N_F=300$, white noise of 2 cm, observation type: Positions, method: flowchart Fig. 6.2, observation sampling rate: 30 sec., test sampling rate: 10 sec., without $\approx 5\%$ at boundaries $\equiv 9$ points or 80 sec.).

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<th>RMS (Acc. m/s²)</th>
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<td>59</td>
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<td>0.000001</td>
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Figure 7.43: (a) Position differences, (b) velocity differences, (c) acceleration differences, (d) Fourier spectrum (dynamical restriction index window [1-59], reference motion: ellipse mode, \( J_{\text{max}} = 4 \), \( N_F = 60 \), white noise of 2 cm, observation type: Positions, method: flowchart Fig. 6.2, observation sampling rate: 30 sec., test sampling rate: 10 sec., without \( \approx 5\% \) at boundaries \( \equiv 9 \) points or 80 sec.).

RMS

<table>
<thead>
<tr>
<th>( \sigma_d^2 )</th>
<th>Pos. (m)</th>
<th>Vel. (m/s)</th>
<th>Acc. (m/s^2)</th>
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</thead>
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<tr>
<td>( 10^{-1} )</td>
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<td>0.000777</td>
<td>0.000061</td>
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<td>( 10^{-3} )</td>
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<td>0.000987</td>
<td>0.000080</td>
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<td>( 10^{-5} )</td>
<td>0.024032</td>
<td>0.000681</td>
<td>0.000051</td>
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<td>( 10^{-9} )</td>
<td>0.052265</td>
<td>0.001150</td>
<td>0.000004</td>
</tr>
</tbody>
</table>
7.4. Reduced-Kinematical Precise Orbit Determination (RKPOD)

Figure 7.44: (a) Position differences (b) velocity differences (c) acceleration differences (d) Fourier spectrum (dynamical restriction index window [1-59], reference motion: ellipse mode, $J_{\text{max}}=4$, $N_F=60$, white noise of 5 cm, observation type: Positions, method: flowchart Fig. 6.2, observation sampling rate: 30 sec., test sampling rate: 10 sec., without $\approx 5\%$ at boundaries $\equiv 9$ points or 80 sec.).

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<th>Vel. (m/s)</th>
<th>Acc. (m/s$^2$)</th>
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<td>$10^{-9}$</td>
<td>0.070613</td>
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Figure 7.45: Difference between CHAMP dynamical orbit of different degree and order of the Earth gravity field and full CHAMP dynamical orbit with an Earth gravity field degree and order $N_F=300$ without $\approx 5\%$ at boundaries.
8. Discussion and Conclusions

In this dissertation, a new approach for the determination of satellite orbits based on densely and homogeneously distributed observations is presented. These observations are code pseudo-range and carrier phase measurements between a Low Earth Orbiter (LEO) and the satellites of any of the Global Navigation Satellite Systems (GNSS) such as GPS, GLONASS or in future GALILEO. The orbit determination is restricted to short arcs; the use of precise short arcs of low-flying satellites has been demonstrated recently to be the proper tool for various applications in Satellite Geodesy, especially for the determination of static and temporal gravity field models, based on the observations related to the new satellite gravity missions such as CHAMP and the twin satellite mission GRACE.

The approach investigated in this research is characterized by the fact that the satellite arcs are represented by a semi-analytical series. This kind of orbit representation not only allows to determine arbitrary functionals of the satellite’s orbit, such as velocities and accelerations of the satellite, but it is also possible to use geometrical, kinematical, but also dynamical observations for the determination of the orbit parameters. The interpretation of the term “kinematical” is different from that what is usually understood by kinematical positioning in the context of GPS positioning methods. Here, the notion “kinematics” used as in physics which is defined as the theory of the motion of mass points and closely related to terms such as velocity and acceleration. While the definition of the term “geometrical” is reserved for the point-wise determination of positions by purely geometric observations. In that case, there is no connection between subsequent positions, and consequently, no information about the velocity or the acceleration of the satellite.

The orbit determination approach is based on the formulation of a boundary value problem to Newton-Euler’s equation of motion, either as absolute or relative orbit, in form of an integral equation of Fredholm type. The solution of this integral equation is formulated as a function which consists of three parts: a first one describing a linear combination of the boundary position vectors (either a straight line or an ellipse connecting the end points of the arc or a dynamical reference orbit), a second one which consists of polynomials of Euler and Bernoulli type of various degrees and a third one consisting of a series of sine functions, described by an - in principle - infinite number of terms. Because of the limited number of observations in an orbit determination process, the number of parameters has to be restricted adequately, ensuring the envisaged accuracy. Theoretically, the series in terms of Euler and Bernoulli polynomials could completely represent the orbit in addition to the linear combination of the boundary vectors. But it turned out that this is not possible with sufficient numerical accuracy. Also, the sine series as the solution of the Fredholm integral equation could be used to represent the orbit in addition to the linear combination of the boundary vectors. This is possible also numerically by extending the upper summation index of the sine series to a sufficient high number. But in that case, there are not enough observations to determine a sufficient number of series terms, on the one hand, or the Gibb’s effect is too large, if the summation index is selected corresponding to the number of equally distributed observations, on the other hand. A compromise is achieved by combining both representations. Because of the fact that the Euler-Bernoulli polynomial series are constructed such that its derivatives coincide up to a sufficient high degree with the number of derivatives of the satellite arc at the boundaries, the derivatives of the difference function becomes zero at the arc’s boundaries and a smooth difference function results. This function can be represented by a very fast converging sine series with small resulting Gibb’s effect at the arc boundaries, if periodically continued with the period of twice of the arc length.

Because of the close relation of the series coefficients with the force function model, the orbit determination can be designed as a pure dynamical but also as a pure kinematical orbit determination technique. If only weak dynamical restrictions are introduced in the adjustment process then a reduced kinematical orbit results. The series coefficients of this approximation function can be fixed by the adapted force function model and the boundary vectors can be determined as free parameters by fitting the observations in a best possible way representing a dynamical orbit. If all free parameters are determined completely by a least squares adjustment procedure then a kinematical orbit results. The observations are based on precise GNSS measurements of various types. GNSS provides accurate code pseudo-range and carrier phase observations that can be used to estimate the geometrical absolute positions of the LEOs.
In Chapter 1, the subject of orbit determination was introduced along with its applications especially in the field of Satellite Geodesy. Then the pre-requisites of the orbit determination were treated in Chapter 2, including some essentials of the GNSS, the reference systems and transformation issues as well as a discussion of various error sources of the GNSS point positioning.

In Chapter 3, some important aspects of the pre-processing of code pseudo-range and carrier phase GPS-SST observations were represented. Among various observation techniques to determine the LEO orbit, the GNSS observations especially the high-low GPS-SST observations play an important role. Among the GPS-SST observations, the code pseudo-range GPS-SST observations are frequently used to determine initial LEO orbits. Final improvements are performed based on high-low GPS-SST carrier phase observations. The code pseudo-range and carrier phase GPS-SST observations have to be cleaned from outliers and cycle slips, respectively. Therefore, the outliers in the code pseudo-range observations were screened using the majority-voting strategy, which was described in Sec. 3.1.2. With this method, the outliers in the code pseudo-range observations could be detected. It was found in this research that some of the low elevated GPS satellites show outliers, and it is recommended for the zero difference data processing mode if only GPS satellites with a sufficient high elevation are used. Therefore, a cut-off angle of 15° was applied to the GPS-SST processing procedure. The same screening algorithm can be applied for the subsequent time differenced carrier phase observations. In this case, the limiting criterion of the performance of the screening algorithm is the quality of the LEO absolute positions which are determined based on the code pseudo-range observations. Therefore, alternative observation combination techniques (described in Sec. 3.2.4) have to be used along with the screening algorithm to ensure the removal of the outliers (or cycle slips) in the carrier phase GPS-SST observations. In case of the GPS-SST data processing, proper observation weights have to be used in the adjustment. Different weighting methods were proposed in Sec. 3.3. The GPS-SST data weighting has to be based on geometrical or physical criteria, or both of them. One useful criterion is the weighting according to the zenith distance. Especially in case of the CHAMP and the GRACE satellites, SNR (Signal to Noise Ratio) values are given in RINEX format. Therefore, they can be used as a physical interpretation of the GPS signal strength of the GPS-SST observations. The geometrical and physical weighting methods show approximately the same results for the LEO precise orbit determination. Because of the dependency of the SNR values on the GPS receiver, their interpretation in case of CHAMP is difficult with respect to the geometrical criteria, e.g. based on the zenith distance. Therefore, in this research, only the zenith distance weighting method was used in the GPS-SST data processing.

In Chapter 4, the geometrical precise orbit determination strategy was discussed. In this method, only geometrical connections (in the sense of distances) between the GPS satellites and the LEOs are used to determine the position coordinates of the LEOs. With this approach, the absolute positions are determined point-wise based on the code pseudo-range and carrier phase observations. With the ionosphere-free code pseudo-range GPS-SST observations, the absolute positions are estimated with meter-accuracy. Improved LEO positions can be estimated based on the accurate carrier phase GPS-SST observations. If the correction models are applied to the observation equation then an accuracy of 2 – 3 cm can be expected for low-flying satellites like CHAMP. From the various important key factors, the accuracy of the LEO GPS-SST carrier phase observations, the geometrical strength of the GPS satellites and the number of GPS satellites at every epoch are the most important pre-requisites to estimate successfully the geometrical orbit with high precision. From the different GPS-SST data processing techniques, the zero difference mode proved to be an efficient choice for the LEO precise orbit determination using only undifferenced GPS-SST observations. To externally validate and compare the estimated geometrical results, the PSO CHAMP dynamical orbit of the GFZ Potsdam and Švehla’s CHAMP dynamical solution orbit were used. The geometrical results show systematic differences from both dynamical reference orbits. They can be interpreted as dynamical mis-modelings of the PSO orbits and Švehla’s solution in the years 2002 and 2003 as described in Chapter 7.

In Chapter 5, various important theoretical and numerical features related to the representation of the short arc of the proposed integrated orbit determination approach were presented. The geometrically estimated LEO orbit is given point-wise; therefore there is no connection between subsequent positions, and consequently, no information about the velocity or even the acceleration of the LEO is available. To describe the time dependency of the satellite motion, the semi-analytical orbit representation based on the boundary positions, the Euler-Bernoulli polynomials and the residual sine series was used. The semi-analytical representation function is based on the reference motion (a straight line, an ellipse or a dynamical reference orbit).
of the LEO. It was found in this research that the type of reference motion is important for the size of the sine amplitudes, and also for the size of the remainder function of the approximate orbit representation. The accuracy of the LEO is much better when using the dynamically determined arc as reference motion than a straight line or an ellipse reference motion. Because of the fact that the computation of the dynamical reference LEO orbit is much more costly than the use of an ellipse reference motion, the ellipse reference motion was used as an acceptable compromise.

As already pointed out, the LEO arc can be represented by the reference motion and the Fourier series up to index \( n \) or by the reference motion and the Euler-Bernoulli polynomials up to an upper index \( J_{\text{max}} \), or by a combination of them. For the representation of the LEO arc by the boundary positions and the Fourier series, it was shown that this combination causes large Gibb’s effect at the arc boundaries in case of a low Fourier upper index. Due to the fact that there exists a functional dependency between the Fourier amplitudes and the Euler-Bernoulli coefficients, a series in terms of Euler-Bernoulli up to a sufficient high degree can be used to represent the LEO arc as well. It could be shown that the convergence of the Euler-Bernoulli polynomials requires a large upper index \( J_{\text{max}} \). In the practical application, the Euler-Bernoulli series has to be limited because of stability problems by a rather low index \( J_{\text{max}} \). But it was found that the orbit approximation of the series in terms of the Euler-Bernoulli polynomials is not better than a couple of centimeters in case of a maximum index \( J_{\text{max}} = 4 \). Both orbit representation can be used exclusively. Because of the disadvantages of both alternatives, a combined solution was proposed. A series in terms of Euler-Bernoulli polynomials has to be fitted to the geometrically determined arc with a reasonable upper degree \( J_{\text{max}} = 4 \), corresponding to sufficiently precise arc derivatives at the arc boundaries. In that case, the residual sine series shows a fast convergence and low residuals of the combined series when compared to the true ephemerides. Concluding it was proposed that the LEO short arc should be represented by the LEO arc boundary positions (connected by an ellipse), the Euler-Bernoulli polynomials up to degree \( J_{\text{max}} = 4 \) and a residual sine series up to a properly selected index \( \bar{n} \).

In Chapter 6, the integrated kinematic-dynamic orbit determination approach was formulated explicitly as well as different determination algorithms. In this chapter, it was found that the determination of the kinematical or dynamical orbits (or any reduced-kinematical orbit modification) can be performed based on the positions, derived in a first preparation step by a geometrical precise orbit determination procedure, or directly by the carrier phase GPS-SST observations, together with observation specific corrections. Both observation types have to deliver the same results if the full variance-covariance information (with the correlation information between the points in case of positions as pseudo-observation) are used in both algorithms. The estimated kinematical orbit is not only a continuous approximation of the LEO orbit in a kinematical sense, but it is also a solution of Newton-Euler’s equation of motion. If the LEO representation parameters are estimated by a least squares procedure only based on the carrier phase GPS-SST observations (or positions) without any dynamical information, then the LEO short arc is determined kinematically. By introducing dynamical information for the solution, a reduced-kinematical or a pure dynamical orbit determination can be realized. It was shown that a smooth transition from a kinematical orbit determination procedure to a dynamical orbit determination is possible by adding dynamical information (in the sense of Fourier amplitudes) together with its variance-covariance matrices to the observation equations. In other words, the estimated kinematical LEO short arc can be reduced with respect to the introduced dynamical information to the observation equation by loosing continuously the empirical orbit character.

In this research, two LEO short arc determination strategies were proposed. In a first step, the Euler-Bernoulli polynomial coefficients up to degree \( J_{\text{max}} = 4 \) were derived based on the geometrically determined positions or directly by the carrier phase GPS-SST observations together with its variance-covariance matrices. Because of the reduced approximation accuracy of the orbit by a series in terms of the Euler-Bernoulli polynomials up to a maximum degree \( J_{\text{max}} = 4 \), it is sufficient to determine the coefficients of the Euler-Bernoulli polynomials without correcting the boundary positions. Another possibility is to perform a least squares fit of the Fourier series to the (pseudo) observations to determine the Euler-Bernoulli coefficients from the Fourier series amplitudes. The coefficients of the Euler-Bernoulli coefficients differ slightly depending on the estimation procedure, but the differences can be compensated by the residual sine series. In other words, the Euler-Bernoulli estimation procedure in the first case is performed in the space domain, and in the second estimation procedure, it is performed in the spectral domain.
Chapter 7 was dedicated to test computations, partly based on CHAMP orbit information of the SC7 simulation data set, partly based on real CHAMP data provided by GFZ Potsdam. In this chapter, geometrical, kinematical, reduced-kinematical and dynamical precise orbit determination modifications were tested. To unify the simulated cases with the real ones, the GPS-SST observations were provided in the RINEX format. A $15^\circ$ cut-off angle was used to avoid the processing of low elevated GPS satellites, and the zero difference GPS data processing mode was used for the orbit determination procedures.

It was found in the geometrical test computations of the real CHAMP data that the point-wise LEO orbit can be estimated based on the carrier phase GPS-SST observations with an accuracy of $2 - 3$ cm. It should be mentioned that the geometrical strength of the GPS satellites, the accuracy of the GPS-SST observations, and the number of GPS satellites plays an important role in the GPS data processing, especially for the geometrical orbit determination procedure. In all real geometrical orbit determination cases, the trend reassessed RMS values with respect to PSO and Švehla’s dynamical orbits are in the range of 1 to 2 cm, but the GPS-SST observation residuals show the smallest observation residuals. It should be mentioned that the geometrical orbit determination methods can not deliver directly the LEO kinematical parameters such as velocity and acceleration.

In Sec. 7.3.1, the kinematical orbit determination strategy was tested based on a 30 minute CHAMP short arc simulation scenario. Based on the results of the error-free simulation cases, it was concluded that the best case with minimal RMS values for ephemerides of positions, velocities and accelerations of 10 sec time difference based on observation series with a 30 sec sampling rate are achieved of using either GPS-SST observations directly or positions as pseudo observations indirectly by following the determination procedure as shown in the flowchart of Fig. 6.2. It is preferable to derive the Euler-Bernoulli series from the sine series coefficients, which are determined in a prior least squares adjustment step by simultaneous correcting the boundary values together with the sine coefficients.

If the observations (carrier phase GPS-SST observations or positions) are contaminated with white noise, two outcomes are of interest: first of all, the minimum RMS values for the positions, velocities and accelerations are not achieved for the so-called interpolation case (by extending the upper summation index of the sine series corresponding to the number of equi-distant observations) but for a much lower upper index, e.g. 30. Obviously, the restriction to a lower upper index acts as a filtering. Secondly, it was observed that the influence of white noise in the observations is reduced with the number of differentiations. While the RMS value for the positions is about four times larger for the noise case relative to the error-free case, for the accelerations this factor is reduced to only two, in case of an upper limit of the residual sine series of 30. Furthermore, it was noted that for an upper index of 59 (interpolation case), the RMS value represents a certain basic noise level which corresponds to the white noise level of the observations. These values can only be reduced by an additional filtering procedure, as it is realized in case of reducing the upper summation index to e.g. 30, but alternative filtering techniques might be even more successful.

The Gibb’s effect in the kinematical orbit determination procedure can be reduced down to a level of the maximum orbit representation accuracy of approximately 1 cm. It should be mentioned that in case of noisy observations, the Gibb’s effect is not visible anymore in the residuals; already in case of an upper sine series index of 30 compared to the general errors in the orbit determination results as a consequence of the observations errors. In the error-free case the Gibb’s effect is still clearly visible at the boundaries. In any case, it is useful to blank out the boundary regions in the size of 5% of the total arc length.

In Sec. 7.3.2, four examples of kinematical orbit determinations are discussed. All cases document the efficiency of the kinematical orbit determination strategy for the LEO short arcs. The results are more or less identical for the different upper indices of the residual sine series because of the large systematic derivations. The differences (bias and/or trend) between the results of the kinematical orbit determination and the PSO in the positions and velocities show (most probably) systematic mis-modeling effects of the dynamical CHAMP PSO orbits. Despite the large systematic differences in the positions and velocities, the small carrier phase observation residuals demonstrate the quality of the kinematical orbit determination procedure, so that the systematic deviations are caused mainly by the dynamical model used for the PSO orbits.
To show the flexibility of the proposed orbit determination strategy to vary between a kinematical orbit determination and a pure dynamical orbit determination by covering all levels of reduced-kinematical orbit determination cases, different simulation scenarios were tested in the error-free and error cases based on a 30 minute CHAMP short arc in Sec. 7.4. In the error-free case, no improvement of the orbit accuracy could be observed. In the error cases, an Earth gravity field up to degree and order 300 was used for the reduced-kinematical orbit determination. It represents a full and accurate dynamical information of the Earth gravity field at the altitude of the low-flying satellite.

The geometrical, kinematical, reduced-kinematical and dynamical precise orbit determination of low-flying satellites based on only high-low GPS-SST observations can be performed with sufficient accuracy. Further test with real data, based on the relative motion of the GRACE twin satellites, on Satellite Laser Ranging (SLR) data and altimetry cross-over tests are necessary. The proposed orbit determination strategy opens also new applications in the case of kinematical and dynamical observables in connection with satellite formation flight configurations.
List of Abbreviations

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
<th>Location</th>
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<td>AC</td>
<td>Analysis Center</td>
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<td>ANTEX</td>
<td>ANTenna EXchange format</td>
<td>see Sec. 2.2</td>
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<td>CCD</td>
<td>Charge Coupled Device</td>
<td>see Sec. 2.2</td>
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<td>CHAMP</td>
<td>CHAllenging Minisatellite Payload</td>
<td>see Sec. 2.2</td>
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<td>CNES</td>
<td>Centre National d’Études Spatiales</td>
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<td>DD</td>
<td>Double Difference</td>
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<td>DOD</td>
<td>Department Of Defense</td>
<td>see Sec. 2.2</td>
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<td>DOP</td>
<td>Dilution Of Precision</td>
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<td>DORIS</td>
<td>Doppler Orbitography and Radiopositioning Integrated by Satellite</td>
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<td>DPOD</td>
<td>Dynamical Precise Orbit Determination</td>
<td>see Sec. 2.2</td>
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<td>EGM</td>
<td>Earth Gravitational Model</td>
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<td>EIGEN</td>
<td>European Improved Gravity model of the Earth by New techniques</td>
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<td>ERP</td>
<td>Earth Rotation Parameter</td>
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<td>GeoForschungZentrum</td>
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<td>GLONASS</td>
<td>Global Navigation Satellite System</td>
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<td>GNSS</td>
<td>Global Navigation Satellite System</td>
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<td>GOCE</td>
<td>Gravity field and steady-state Ocean Circulation Explorer</td>
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<td>Gauss-Markov Model</td>
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<td>GPS</td>
<td>Global Positioning System</td>
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<td>GRACE</td>
<td>Gravity Recovery And Climate Experiment</td>
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<td>ICRF</td>
<td>International Celestial Reference Frame</td>
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<td>IGG</td>
<td>Institute for Geodesy and Geoinformation at university of Bonn</td>
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<td>International GNSS Service</td>
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<td>ITRF</td>
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<td>Jet Propulsion Laboratory</td>
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<td>Precise Point Positioning</td>
<td>see Sec. 2.2</td>
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<td>Precise Range And Range rate Equipment</td>
<td>see Sec. 2.2</td>
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<td>PSO</td>
<td>Post-processed Science Orbit</td>
<td>see Sec. 2.2</td>
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<td>RKPOD</td>
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