

# Contents

<b>1</b>	<b>Introduction</b>	<b>7</b>
<b>2</b>	<b>Regional Gravity Field Recovery Using New Satellite Missions</b>	<b>9</b>
2.1	Overview of the Satellite Missions CHAMP, GRACE, and GOCE . . . . .	9
2.1.1	CHAMP . . . . .	9
2.1.2	GRACE . . . . .	10
2.1.3	GOCE . . . . .	11
2.2	Regional Gravity Field Recovery . . . . .	12
2.2.1	New Analysis Techniques for the New Satellite Missions . . . . .	12
2.2.2	Motivation for Regional Analysis . . . . .	14
2.2.3	Existing Approaches to Regional Gravity Field Modeling . . . . .	15
<b>3</b>	<b>Modeling of the Regional Gravity Field</b>	<b>17</b>
3.1	Fundamentals of Potential Theory . . . . .	17
3.2	Reproducing Kernel Hilbert Space . . . . .	18
3.3	Spherical Harmonics . . . . .	21
3.3.1	Mathematical Description . . . . .	21
3.3.2	The Use of Spherical Harmonics in Geodesy . . . . .	23
3.3.3	Covariance Function on the Sphere . . . . .	24
3.3.4	RKHS on the Sphere . . . . .	26
3.3.5	Upward Continuation . . . . .	28
3.4	Space Localizing Basis Functions . . . . .	28
3.4.1	Radial Basis Functions . . . . .	29
3.4.2	Spherical Splines . . . . .	30
3.4.3	Bandlimited Spline Functions . . . . .	31
3.4.4	Arrangement of the Basis Functions on the Sphere . . . . .	32
3.4.5	Convergence Issues: Summary . . . . .	34
3.5	Point Distributions on the Sphere . . . . .	36
3.5.1	Grids . . . . .	36
3.5.2	Applicability as Nodal Points for Splines . . . . .	42
3.5.3	Resolution . . . . .	52

---

<b>4</b>	<b>Setting up the Observation Equations</b>	<b>54</b>
4.1	Least Squares Approximation . . . . .	54
4.2	The Gravity Field and its Functionals in Terms of Splines . . . . .	55
4.2.1	Gravitational Potential . . . . .	56
4.2.2	Gravity . . . . .	56
4.2.3	Gravity Gradient . . . . .	57
4.3	Functional Model . . . . .	59
4.3.1	Precise Orbit Determination . . . . .	60
4.3.2	Low-Low Satellite-to-Satellite Tracking . . . . .	63
4.3.3	Satellite Gravity Gradiometry (GOCE) . . . . .	64
<b>5</b>	<b>Solving the System of Observation Equations</b>	<b>65</b>
5.1	Inverse Problems, Ill-posed Problems . . . . .	65
5.1.1	Ill-posedness of the Downward Continuation Process . . . . .	66
5.1.2	Singular Value Decomposition of the Design Matrix . . . . .	66
5.2	Regularization . . . . .	68
5.2.1	Tikhonov Regularization . . . . .	68
5.2.2	Regularization in the Finite Dimensional Model . . . . .	69
5.2.3	Regularization and Splines . . . . .	70
5.2.4	Variance Component Estimation . . . . .	73
5.2.5	Regionally Adapted Regularization . . . . .	74
5.3	Relationship Between Spline Approximation and Collocation . . . . .	76
<b>6</b>	<b>From Regional to Global Gravity Fields</b>	<b>78</b>
6.1	Conversion from a Global Spline Representation to Spherical Harmonics . . . . .	78
6.2	Patching of Individual Regional Solutions . . . . .	79
6.2.1	Quadrature Methods . . . . .	79
<b>7</b>	<b>Calculations and Results</b>	<b>88</b>
7.1	(Real) Data Analysis with the Programming System GROOPS . . . . .	88
7.1.1	Background Models . . . . .	92
7.2	Simulation Study: Basis Functions . . . . .	93
7.3	Gravity Field Solutions . . . . .	95
7.3.1	CHAMP . . . . .	98
7.3.2	GRACE . . . . .	102
7.3.3	Combination of GRACE and GOCE . . . . .	108

<b>8 Summary and Outlook</b>	<b>113</b>
<b>A Mathematical Fundamentals</b>	<b>116</b>
A.1 Function Spaces . . . . .	116
A.2 Linear Functionals and Linear Operators . . . . .	118
<b>B Derivation of the Weights for the Gauss-Legendre Quadrature</b>	<b>122</b>
<b>Abbreviations</b>	<b>124</b>
<b>List of Figures</b>	<b>125</b>
<b>List of Tables</b>	<b>128</b>
<b>References</b>	<b>129</b>