Essays on
International Policy Coordination
in Interdependent Economies

Inaugural-Dissertation
zur Erlangung des Grades eines Doktors
der Wirtschafts- und Gesellschaftswissenschaften
durch die
Rechts- und Staatswissenschaftliche Fakultät
der Rheinischen Friedrich-Wilhelms-Universität
Bonn

vorgelegt von
Michael Evers
aus Rheine.

Tag der Promotion: 11. Oktober 2007
Dekan: Prof. Dr. Gerhard Wagner
Erstreferent: Prof. Dr. Jürgen von Hagen
Zweitreferent: Prof. Dr. Ludger Linnemann

Tag der mündlichen Prüfung: 11. Oktober 2007

Diese Dissertation ist auf dem Hochschulschriftenserver der ULB Bonn http://hss.ulb.uni-bonn.de/diss_online elektronisch publiziert.
to Verena and my parents
Acknowledgements

While I was writing this thesis, I received help from many people and in various ways. I am grateful to my supervisor Jürgen von Hagen for his inexhaustible patience and his guidance. I benefitted a lot from many discussions with him and comments from him.

I also want to thank Ludger Linnemann for many helpful comments and for acting as a referee on my thesis committee.

I’m grateful to Michael Devereux, Martin Hellwig, Kenneth Kletzer, Gernot Müller, Stefan Niemann, Matthias Paustian, Marc Schiffbauer, and Martin Uribe for valuable comments and helpful discussions on the various chapters.

I want to thank the Bonn Graduate School of Economics and the German Science Foundation DFG (through the GRK ”Quantitative Economics” and SPP 1142) for financial support. I also want to thank Urs Schweizer, Georg Nölsche, and Jürgen von Hagen for managing the Bonn Graduate School of Economics and the Graduiertenkolleg.

I am glad about and thankful for the friendship with my fellow graduate students in Bonn that enriched my life tremendously.

Most of all I have to thank Verena and my family for their grandiose support and the endless patience with me.
Contents

I.1 A Brief Review of International Policy Coordination .................. 3
  I.1.1 International Monetary Policy Coordination .................. 3
  I.1.2 International Fiscal Policy Coordination .................. 5
I.2 Outline of the Chapters ............................................. 6

1 Optimal Monetary Policy in an Interdependent World ............ 9
  1.1 The Model ................................................. 13
     1.1.1 Firms and Technologies .......................... 13
     1.1.2 The Households .................................. 14
     1.1.3 Governments’ Budget Constraints ................. 16
  1.2 Equilibrium Allocation ........................................ 17
     1.2.1 Goods Prices and the Terms of Trade ............. 17
     1.2.2 Ex Post Equilibrium Allocation .................. 17
     1.2.3 Distribution of the Equilibrium Allocation .......... 20
  1.3 Short-Run Monetary Policy ................................... 21
     1.3.1 Inflation Target and the Money Supply as Two Distinct Policy Instruments ................................ 22
     1.3.2 Policymakers’ Objective ............................. 26
  1.4 Optimal Cooperative Monetary Policy ......................... 28
     1.4.1 The Optimal Nominal Interest Rate .................. 28
     1.4.2 The Optimal Money Supply ........................... 29
  1.5 Noncooperative Monetary Policy ............................... 32
     1.5.1 The Nominal Interest Rate ......................... 32
     1.5.2 The Money Supply .................................. 33
  1.6 A Numerical Example ......................................... 34
## 1.7 Conclusion of Chapter 1

### 2 Optimum Policy Domains in an Interdependent World

2.1 The Model

2.1.1 Firms

2.1.2 Households

2.1.3 Governments’ Budget Constraints

2.2 Equilibrium in Closed Form

2.2.1 Equilibrium Prices and the Terms of Trade

2.2.2 Ex Post Equilibrium Allocation

2.2.3 Ex Post Public Policy Intervention to Consumption Spending: Money Supply and Consumption Taxes

2.2.4 Distribution of the Ex Post Equilibrium Allocation

2.2.5 Ex Ante Public Policy Intervention to Wage Setting: Nominal Interest Rate and Labor Income Tax

2.2.6 The Objectives in Closed Form

2.3 International Cooperation of Public Policies

2.3.1 Optimal Public Policy Coordination

2.3.2 Non-Cooperative Public Policy: Nash

2.4 Monetary Cooperation and Fiscal Independence

2.5 A Numerical Example

2.6 Conclusion of Chapter 2

### 3 Federal Fiscal Transfers in Monetary Unions: A NOEM Approach

3.1 The Model

3.1.1 Goods Production

3.1.2 Households

3.1.3 Local Governments and Federal Fiscal Arrangements

3.1.4 Market Clearing, Terms of Trade, and the Real Exchange Rate

3.2 Solution of the log-linearized Model

3.2.1 Long-run implications of productivity and demand shocks

3.2.2 Short-run implications of productivity and demand shocks

3.2.3 Welfare
3.3 Federal Fiscal Arrangements ....................................................... 81
  3.3.1 Stabilizing Demand Shocks ................................................... 81
  3.3.2 Stabilizing Productivity Shocks .............................................. 83
3.4 Conclusion of Chapter 3 ......................................................... 88
A Appendix to Chapter 1 ................................................................. 95
  A.1 Model Specification ................................................................. 95
  A.2 The Explicit Distribution of the Equilibrium Allocation ................. 99
  A.3 Optimal Monetary Policy Coordination ......................................... 104
  A.4 Non-cooperative Monetary Policy: Nash ....................................... 110
B Appendix to Chapter 2 ................................................................. 113
  B.1 Optimal Public Policy Coordination ........................................... 113
  B.2 Non-cooperative Public Policy: Nash .......................................... 114
  B.3 Monetary Cooperation and Fiscal Independence .............................. 115
C Appendix to Chapter 3 ................................................................. 121
  C.1 Summarizing the Equilibrium Conditions ..................................... 121
  C.2 The Log-linearized Equilibrium Conditions .................................. 123
  C.3 Aggregates and Country Differences ......................................... 125
  C.4 Comparing Steady States ......................................................... 128
  C.5 Short-run equilibrium responses ................................................ 130
  C.6 Policy Analysis: Calculating the Transfers .................................. 134
  C.7 Approximating Welfare ............................................................ 137

References ................................................................................. 138
### List of Tables

1.1 Ex Post period equilibrium allocation for given $Z$ and $ToT$  
1.2 Gains from International Monetary Policy Coordination  
2.1 The ex post period equilibrium allocation given monetary and fiscal policy  
2.2 Gains from monetary policy coordination when fiscal authorities operate independently as compared to the case when both national monetary and fiscal policy are conducted non-cooperatively.  
2.3 Gains from fiscal policy coordination when monetary authorities already cooperate as compared to the case when monetary policy coordinate but fiscal authorities operate independently.
Introduction

Throughout the last couple of decades the world has experienced a strong and steady increase in the economic interdependence among national economies. Accordingly, because national monetary and fiscal policies do have an important impact on national macroeconomic activities, they are also subject to a steady increase in mutual interdependence. As a consequence, because conflicting national policy interests might lead to international disagreements, the necessity of the international coordination of macroeconomic policies has become a central postulation within both the public as well as the academic debate: Countries should coordinate macroeconomic policies in order to incorporate externalities of national policies on other countries and they should do so as to overcome inefficiencies arising from strategic considerations to exploit the international transmission of national macroeconomic policies in one country’s own favor.

The idea that countries should consult about their economic policy conduct is by far not merely a thought experiment. There are in fact many important examples of international institutions whose sole purposes are to form the basis for policy coordination. The Bretton Woods System (1944-1973) that regulated international monetary and financial relations among the participating countries is the first example of an international institution designed to facilitate macroeconomic and in particular monetary policy coordination. The principal feature of the Bretton Woods system were the pegged exchange rates of its members and the control of temporary imbalances of payments. Influential institutions as the International Monetary Fund (IMF) or the International Bank for Reconstruction and Development (EBRD) find their roots in Bretton Woods. Another most important because present in Europeans’ everyday life example is the European Union (EU) and the formation of the Economic and Monetary Union (EMU) within the European Union.

Against this background, it appears natural that a large fraction of the international macroeconomics literature deals with the international transmission of national macroeconomic policies and the question of the desirability of institutional arrangements to coordinate national policy conduct. The academic literature evolved along two major strands following the natural separation of national macroeconomic policymaking into
monetary and fiscal policy. Reviewing the literature on international policy coordination, however, reveals two striking observations: First, the paradigm in the literature on monetary policy coordination is that gains from coordination are fairly small. Monetary policy is considered to be concerned with the stabilization of macroeconomic fluctuations only. Gains from policy coordination arise then from the inability of certain exchange rate systems to react appropriately to asymmetric shocks or from preventing countries from the use of stabilization policies to strategically manipulate the terms of trade. The second striking observation is that the analysis of international monetary policy coordination and the analysis of fiscal policy coordination in strategic setups were effectively uncoupled. Even though the profession understood well that both monetary and fiscal policy instruments have similar effects on the terms of trade\(^1\), a crucial question has not been addressed yet: How does the international coordination of only a part of national macroeconomic policies, say monetary policy within a monetary union, change the strategic behavior of the independently conducted remaining part of national policies, i.e. the still independently conducted fiscal policy?

The following three chapters of this thesis seek to contribute to these observations. Chapter 1 addresses the first observation and questions the professions’ concentration on stabilization issues only. It is demonstrated that the gains from stabilizing macroeconomic fluctuations are generically quite limited because they are of second-order. Instead, it is argued that gains from international monetary policy cooperation can be substantial when policymakers jointly prevent structural inefficiencies in the supply of labor and hence production directly rooting in strategic considerations rather than coordinate on the stabilization of exogenously driven fluctuations.

Chapter 2 deals with the second observation: It is argued that international policy coordination requires to include both monetary as well as fiscal policy because both sides dispose of effective policy instruments that enable the strategic manipulation of the country’s terms of trade. Hence, the coordination of one part of national macroeconomic policies through an international agreement still leaves room for national authorities to still unilaterally manipulate the terms of trade by means of different policy instruments. It is demonstrated that potential gains from international policy coordination are squandered if policymakers only cooperate, for instance, on monetary policy alone. Moreover, by letting the fiscal policy instruments be chosen non-cooperatively, monetary policy coordination might even create welfare losses as compared to no macroeconomic policy coordination at all.

The first two chapters have a clear proposition: Gains from policy coordination can be

---

\(^1\)The idea that a system of import tariffs and export subsidies form a substitute to a currency devaluation can already be traced back to Keynes (in Addendum I to the Macmillan Committee Report (Committee on Finance and Industry: Report, London, Government Printing Office, 1931; cited by Haberler (1969))) and to Hicks (1951).
I.1. A Brief Review of International Policy Coordination

In this section, I review the academic literature of international policy coordination in order to better illustrate how the present thesis relates to the literature. It will be brief, though, because every single chapter is written so as to be self-contained.

I.1.1 International Monetary Policy Coordination

Early contributions focus on the international transmission of monetary policy and ask with regard to the Bretton Woods system whether exchange rates should be flexible rather than pegged. Building on the seminal work by James Meade (1951), the criterion of the optimality of exchange rate regimes was determined by the concept of internal balance (i.e. full employment and price level stability) and external balance (balance of international payments). Milton Friedman argued that in the Keynesian environment with rigid wages and prices a system of flexible exchange rates would provide the terms of trade with the flexibility to fulfill the adjustment process to achieve external balance. This implicitly absorbs macroeconomic shocks coming from abroad and insulates the domestic economy from foreign disturbances which maintains internal balance. In contrast, a system of fixed exchange rates would lead to an inherently unstable international financial system with substantial as they are of first-order, but policy coordination must in principle include all instruments that can be used by national authorities to unilaterally manipulate the country’s terms of trade. A particular way to coordinate monetary policy is the formation of a monetary union where all member countries lose their independent monetary policy instruments completely. By giving up the exchange rate flexibility, the member countries sacrifice an effective policy instrument that can be used to switch expenditures between member countries in order to offset economic shocks that have adverse impacts on these countries. From the second chapter follows that there is a definite role for fiscal policy to overcome this loss because fiscal policy can similarly offset adverse shocks. In Chapter 3, this issue is further elaborated. In particular, a specific form of fiscal policy coordination is analyzed which was put forth by Peter Kenen (1969): Because monetary unions cannot use monetary policy instruments to effectively handle with adverse shocks to the member states, the monetary union should be furnished with a built-in fiscal transfer system that collects taxes from some member countries and pays transfers to other member countries in order to alleviate the economic consequences of adverse shocks. In Chapter 3, the properties of different federal fiscal transfer schemes are explored with regard to their capability to stabilize national consumption, production, and employment in detail. Two transfer schemes are considered: direct transfers among private sectors and indirect transfers among national fiscal authorities. It is shown that federal fiscal transfers schemes indeed provide perfect insurance against asymmetric shocks.
perseverative balance-of-payments crises (Friedman (1953)). In his "Theory of Optimum Currency Area", Robert Mundell acknowledged the line of economic reasoning behind Friedman's argument. He pointed out, however, that the argument supporting flexible exchange rates is not in general valid for nations and national currencies per se. Instead, he argued that the stabilization properties of a flexible exchange rate can only be fully seized in a world where production factors are perfectly mobile within a region but immobile across regions. Then each region should have its own currency that is flexible relative to all other currencies. His central proposition was that if national countries are heterogenous in structure and factors are immobile within countries, exchange rate flexibility is no means by which internal imbalances can be stabilized (Mundell (1961)). These arguments formed the basis for the discussion of international monetary policy coordination as evidenced, for instance, in the prearrangement to the European monetary unification (compare for example the McDougall Report (1977) and the Delors Report (1989)).

Accepting that national central banks do have (monopoly) power to alter the terms of trades - and even though the issue of strategic considerations of policymakers to manipulate the terms of trade has been raised much earlier by John Hicks (1951) for instance - it took until the 1970ies before economists began to formally theorize the strategic interaction of monetary policy conduct in open economies. Beginning with the seminal contribution by Hamada (1974), the first generation of game-theoretic models - represented most prominently by Hamada (1976), Oudiz and Sachs (1984), and Canzoneri and Gray (1985) - are based on Keynesian frameworks where policymakers are assumed to minimize an ad hoc motivated quadratic loss function that punishes deviations from given desired levels or blisspoints of the inflation rate, the increase in international reserves, or the output level. These models provide the theoretical rationale for international monetary policy coordination so as to overcome global inefficiencies induced by strategic considerations of independent monetary authorities. With the advent of the New Open Economy Macroeconomics (NOEM) that brought optimizing agents, monopolistic competition, and nominal rigidities into dynamic general equilibrium models, international economists were retooled with model frameworks that allowed first the derivation of their arguments from first principles and second a rigorous welfare foundation of the propositions they made. This New-Keynesian framework forms the basis of the second generation of monetary policy coordination models as in Obstfeld and Rogoff (2002a), Clarida et al. (2002), Devereux and Engel (2003), Benigno and Benigno (2006), and Pappa (2004). All these contributions found that there indeed exist gains to international monetary policy coordination as policymakers overcome inefficiencies arising from strategic considerations. In quantitative assessments, however, almost all authors find

\[2\text{Seminal contributions to this literature are Svensson and van Wijnbergen (1989) and Obstfeld and Rogoff (1995, 1996).}\]
I.1. A Brief Review of International Policy Coordination

that these gains are fairly small and rather negligible.

I.1.2 International Fiscal Policy Coordination

In the 1950ies and 1960ies, the focus of international fiscal policy coordination was on trade theory and the analysis of optimal tariff policy and protectionism. Nevertheless, the role of trade policy instruments such as import tariffs and export subsidies for correcting internal distortions was well taken as the highly influential contributions by Haberler (1950), Hagen (1958), and Bhagwati and Ramaswami (1963) document. In international macroeconomics, the potential role of national fiscal policies as stabilization instruments to external and internal imbalances in the sense of Meade was highlighted by Robert Mundell (1962) and Marcus Fleming (1962) and taken up among others by Anne Krueger (1965) and Richard Cooper (1969). The main conclusion that Mundell drew was that monetary policy should be used for attaining external balance whereas fiscal policy is the more appropriate instrument for achieving internal balance and hence full employment. Consequently, by the implicit isolation of the domestic economic activity from foreign fluctuations through monetary policy, the necessity for international fiscal policy coordination was not explicitly given. This is no longer true, however, for a currency union that does not suffice Mundell’s optimum criteria. As Kenen (1969) pointed out, because monetary policy does not dispose of the sufficient instruments to achieve both internal as well as external stability, there is a definite role for fiscal policy coordination among the member countries of a currency union. In particular, Kenen argued that a fiscal systems that spans all member regions or countries can be used to scrutinize fiscal budgetary policy instruments, i.e. collecting payments from some regions and make compensating payments to other regions, in order to offset economic shocks that affect the member countries adversely.

Elaborated game-theoretic arguments as in case of international monetary policy coordination were not provided until the 1980ies as for example by Hamada (1986), Kehoe (1987), and Chari and Kehoe (1990). The main theme again is that international coordination of policy conduct is desirable as it helps to overcome global inefficiencies induced by strategic considerations of independent fiscal policies. In the aftermath of the Maastricht Treaty and the Stability and Growth Pact (1997), however, the attention of the profession was drawn to the analysis of deficit rules and appropriate institutional arrangements of fiscal policy coordination within monetary unions.3 Surprisingly, in contrast to the wave of contributions to optimal international monetary policy conduct initiated by the developments of the NOEM framework, a new wave of a more general analysis of optimal international fiscal policy has not yet occurred. Instead, the profession has almost entirely concentrated on fiscal policy coordination within monetary unions.

3See, for example, Beetsma and Uhlig (1999), Dixit and Lambertini (2001, 2003), and Chari and Kehoe (1998).
I.2 Outline of the Chapters

In the first chapter, I develop a simple dynamic stochastic two-country model with sticky wages and a cash-in-advance restriction which is in the spirit of the New Open Economy Macroeconomics framework and similar in structure to Obstfeld and Rogoff (2002a) and Devereux and Engel (2003). In this environment, monetary authorities can manipulate the terms of trade by conducting a general short-run monetary policy using both the inflation target and the actual money supply. The money supply affects the terms of trade by altering the nominal exchange rate ex-post and it is used in the traditional way so as to stabilize macroeconomic fluctuations. The inflation target affects the terms of trade by changing expected inflation ex-ante. Within this framework, two important results emerge: First, the equilibrium and hence welfare effects of inflation targeting policy are of first order whereas those of the money supply management and hence stabilization policies are of second order. Second, regarding the inflation targeting policy, self-interested national policymakers indeed have an incentive to deviate from the globally optimal inflation. From a global perspective, it is optimal to follow the Friedman principle since it minimizes the wedge between the marginal rate of substitution and the marginal rate of transformation in all instances when wages are flexible and at least on average when wages are sticky. Independent and self-oriented national policymakers, however, strive for an appreciation of the terms of trade in order to improve the domestic labor-leisure trade-off in all instances when wages are flexible and at least on average when wages are sticky. A simple numerical example demonstrates that international coordination of inflation targeting policy can amount to welfare gains four orders of magnitude larger than gains from coordinating stabilization policies through money supplies.

In Chapter 2, I augment the model presented in Chapter 1 by two national fiscal authorities. Both dispose of two different policy instruments: distortionary taxes on labor income and distortionary taxes on consumption expenditures. The key property of the different monetary and fiscal policy instruments is that the labor income tax and the inflation target on the one hand and the consumption tax and money supply on the other hand are perfectly substitutable national policy instruments. The important consequence is that only the joint interventions of labor income tax and the inflation target on the one hand and the consumption tax and money supply on the other hand are decisive for the national impact on the terms of trade and therefore the national influence on the equilibrium allocation. Hence, taking up the arguments developed in favor of policy coordination and seeking an international cooperation of either monetary or fiscal policy alone will only leave room for policymakers to still follow national interests by exploiting their monopolistic power on the terms of trade via the respective other policy instruments. As a consequence, potential gains from, say, international monetary policy coordination are squandered or
I.2. Outline of the Chapters

may even turn negative by letting the fiscal policy instruments be chosen non-cooperatively. A numerical example supports these findings.

In Chapter 3, I develop a model of a monetary union which is also set up in the tradition of the New Open Economy Macroeconomics (NOEM) but follows this time closely Obstfeld and Rogoff (1995, 1996). It is a dynamic general equilibrium model that depicts a monetary union consisting of two member states. The focus of this chapter lies on the analysis of federal fiscal transfer arrangements as already indicated by Peter Kenen (1969). As the underlying economic environment of Kenen’s proposition stems from Mundell (1961), the Obstfeld and Rogoff framework has been modified along two lines in order to better capture the main features of Mundell’s world within a modern macroeconomic framework: First, households provide different types of labor monopolistically. Final goods are produced by an aggregation technology over all different domestic types of labor. In this sense, technology shocks alter the aggregate labor productivity and subsequently shift marginal costs of producing market goods. As a consequence, since goods markets are assumed to be competitive and prices are flexible, temporary productivity shocks affect prices even though wages are rigid. Second, I introduce tradable and non-tradable goods. Therefore, I can describe demand shocks in addition to productivity shocks. This also allows to take into account the degree of economic integration as measured by the fraction of tradable consumption goods. I consider direct transfers among private sectors and indirect transfers among national fiscal authorities. I explore the properties of federal fiscal transfer schemes with regard to their capability to stabilize national consumption, production, and employment. The important result of this chapter, however, is that appropriately chosen federal fiscal arrangements provide perfect insurance against shocks affecting the member states of a monetary union adversely.

Each of the next three chapters presents its analysis as a self-contained unit. The Appendix to this thesis contains the derivation of the three models and the derivation of the results. All three sub-appendices are kept rather detailed in order to ease the reading.
Chapter 1

Optimal Monetary Policy in an Interdependent World

In the literature on the international dimension of monetary policy, the consensus is that gains from policy coordination are fairly small. Monetary policy is considered to be concerned with the stabilization of macroeconomic fluctuations only. Gains from policy coordination arise then from preventing strategic considerations regarding the use of these stabilization policies from unilaterally manipulating the terms of trade. However, as it has been emphasized by Lucas (2003), the gains from stabilizing macroeconomic fluctuations per se are generically quite limited. Instead, he argues that

"...there remain important gains in welfare ... from providing people with better incentives to work and to save, not from better fine tuning of spending flows."

In this paper, I take up his proposition and translate it into the context of an open economy. I demonstrate that gains from international monetary policy cooperation can be substantial when policymakers coordinate on the stimulation of labor and hence production rather than on the stabilization of exogenously driven fluctuations. To this end, I develop a simple dynamic stochastic two-country model with preset wages and cash-in-advance restrictions. National monetary authorities can affect the equilibrium allocation in this environment by conducting a general short-run monetary policy using both the inflation target as well as the actual money supply. On the one hand, the money supply policy affects the allocation by altering the nominal spending and thereby the nominal exchange rate ex post. A contraction of money supply then leads to an appreciation of the nominal exchange rate and the terms of trade. This equips monetary authorities with an effective policy instrument to stabilize economic fluctuations. On the other hand, the inflation target affects the allocation by changing the inflation expectations. Because inflation works as a tax on labor income, higher expected inflation leads households to claim higher nominal wages. This results in
an increase in the prices of goods and thereby causes an appreciation of the terms of trade.\textsuperscript{1}

Within this framework which allows the joint analysis of the inflation target and the actual money supply as combined tools of monetary policy, two important results emerge: First, the equilibrium and hence welfare effects of the inflation target policy are of first order whereas those of the money supply management are of second order. The intuition for this is simple: The money supply is effective only because of sticky wages through ex post deviations of the actual money supply from the expected one. Thus, money supply management only affects the variability of the equilibrium allocation. As a result, the ex ante equilibrium effects of money supply policies are thus of second order. By contrast, the inflation target changes the average labor supply and hence strikes a wedge between the marginal rate of substitution and the marginal rate of transformation. This inefficient wedge is present irrespective of whether wages are flexible or not and irrespective of whether the state of the world is uncertain. Consequently, the equilibrium implications are of first order. Second, regarding the inflation target policy, self-interested national policymakers indeed have an incentive to deviate from the globally optimal inflation target. From a global perspective, the optimal inflation target follows the Friedman principle and implements zero net nominal interest rates. It is optimal because it minimizes the wedge between the marginal rate of substitution and the marginal rate of transformation in all instances when wages are flexible and at least on average when wages are sticky. Independent and self-oriented national policymakers, however, strive for an appreciation of the terms of trade in order to improve the domestic labor-leisure trade-off in all instances when wages are flexible and at least on average when wages are sticky. As it is true for the money supply policy, deviations of inflation target from the globally optimal ones are always "beggar-thy-neighbor". The important conclusion to be drawn then is that as long as monetary authorities face the incentive to depart from the optimal monetary policy, gains from international arrangements or institutions that effectively prevent these strategic interactions are of first order. A simple numerical example demonstrates that international coordination of inflation target policy can amount to welfare gains four orders of magnitude larger than gains from coordinating stabilization policies through money supply management. Consequently, potential gains from policy coordination are not negligible.

\textsuperscript{1}Within a closed economy model, Ireland (1996) already argues that monetary policy can be conducted in a more general way by using the expected money growth rate (corresponds to the inflation target) and deviations from the expected money growth (corresponds to the actual money supply) rate as two distinct instruments. More recently, Adao et al. (2003) take up his approach and analyze optimal short-run monetary policy in a closed economy real business cycle model with monopolistic firms, a cash-in-advance restriction and preset prices.
The reason why it has become widely accepted that potential gains from policy coordinations must be quite limited might be best understood when looking at the literature from an historical perspective: Beginning with the seminal contributions by Hamada (1974), the first generation of game-theoretic models - represented most prominently by Hamada (1976), Oudiz and Sachs (1984), and Canzoneri and Gray (1985) - are based on traditional Keynesian models where policymakers are assumed to minimize an ad hoc motivated quadratic loss function that punishes deviations from given desired levels or blisspoints of the inflation rate, the increase in international reserves, or the output level. These models provide the theoretical rationale for international monetary policy coordination so as to overcome global inefficiencies induced by strategic considerations of independent monetary authorities. While there is no question about the merits of the aforementioned class of models for the purpose of studying international policy coordination, the assumptions of the policy objectives lead immediately to the result of limited gains for two reasons: First, the quadratic loss function itself directly reduces the problem to the minimization of deviations around the blisspoints. The consequence is that gains from coordinating monetary policy must be of second order. Second, these studies neglected the possibility that the blisspoints themselves are subject to strategic considerations. As it is demonstrated in this paper, the non-cooperatively set expected inflation rates and hence the non-cooperatively set nominal interest rates differ from the globally optimal ones and they hinge crucially on the macroeconomic interdependencies among the different countries. A proper notion of the desired levels of inflation or output conditioning on the international macroeconomic environment, however, requires a rigorous welfare foundation.

Such welfare foundations were provided with the advent of the ”New Open Economy Macroeconomics” (NOEM) that brought optimizing agents, monopolistic competition and nominal rigidities into dynamic general equilibrium models. This New Keynesian framework forms the basis of the second generation of policy coordination models as in Corsetti and Pesenti (2001), Obstfeld and Rogoff (2002a), and Devereux and Engel (2003). Nevertheless, even though the second generation models provide new important insights to the question of the needs of international policy coordination, the welfare gains are

---


3 Seminal contributions to this literature are Svensson and van Wijnbergen (1989) and Obstfeld and Rogoff (1995, 1996). For an excellent survey see Lane (2001).

similarly limited as in the first generation models. In contrast to the first generation models, however, the answer to this problem is now inherent to the specification of the conduct of policy itself. In one prominent class of models, as for example in Obstfeld and Rogoff (2002a) and Devereux and Engel (2003), it is assumed that monetary authorities follow policy rules that condition money supply deviations from any given initial stock of money on the realization of shocks. The average inflation rate induced by the mean growth rate of the money supply is implicitly assumed to be zero. In the other prominent class of models, as for example in Clarida et al. (2002) and Galí and Monacelli (2005), it is assumed that policymakers follow interest rate rules that condition on deviations of inflation rates and output from some given reference levels. Equivalent to the problem of exogenous blisspoints discussed above, both the average interest rate as well as the reference value for the inflation and output deviations are not further analyzed but implicitly fixed via the (log)linear approximation of the model about a zero inflation and nonstrategic steady state. Hence, in both classes of models, monetary policy solely focuses on stabilization issues and consequently the welfare gains of policy coordination are of second order.

Surprisingly, there are only very few contributions that consider non-stabilizing monetary policy interaction within a strategic setting. One exception is Cooley and Quadrini (2003) who study optimal interest rate policy in a two-country open economy model that is not a variation of the NOEM framework. Instead, they use a limited participation version where purchases of production intermediaries - partly imported - must be financed in advance. Consequently, the nominal interest rate has a distorting effect as it increases the cost of production. In this environment, self-interested monetary authorities face the incentive to increase the nominal interest rate in order to appreciate the terms of trade and thereby to expand domestic production. The policy competition leads to inflationary biases with inefficiencies that have sizable adverse welfare consequences. Another exception is Arseneau (2007) who studies the importance of the distortion caused by monopolistic competition for the optimal nominal interest rate policy. Similarly to the model here, he motivates money demand within a version of Corsetti and Pesenti (2001) by a cash-in-advance restriction. He also demonstrates that non-cooperative monetary authorities use the nominal interest rate to induce a domestic appreciation of the terms of trade that leads in equilibrium to sizable welfare losses as compared to the cooperative solution. Arseneau, however, concentrates on the interaction between optimal interest rates and the friction of monopolistic competition and thereby seeks to complement Cooley and Quadrini (2003) where production takes place in a perfectly competitive environment.

In contrast, in the present paper, the generalization of monetary policy conduct in an environment with nominal inertia facilitates the joint analysis of both the inflation target (or equivalently the average money growth rate) and the money supply management (or
1.1. The Model

The model economy consists of two identical countries which are denoted Home (H) and Foreign (F). Each country is populated by a continuum of households and both countries are of equal size one. Firms within a country produce two different consumption goods, one traded good that is demanded across borders, and one non-traded good that is demanded only within borders. In all there are thus four different goods. The goods markets are assumed to be perfectly competitive and goods price are fully flexible. The only productive factor is differentiated labor. Each household is a monopolistic supplier of a specific type of labor and it is identified by superscript \( i \).

1.1.1 Firms and Technologies

Technologies to produce the Home tradable (\( HT \)) and the Home non-traded (\( HN \)) goods are identical:

\[
Y_{j,s} = A_s \mathcal{L}_{j,s} \quad \text{with} \quad \mathcal{L}_{j,s} = \left( \int_0^1 L_{j,s}^i \frac{\theta-1}{\theta} \, di \right)^{\frac{\theta}{\theta-1}},
\]  

(1.1)

The notation I stick to throughout this paper is as follows: Superscripts denote where a variable belongs to, Foreign variables are distinguished by an asterisk \(^*\). Subscripts identify the characteristics of that variable, e.g. whether it’s a non-tradable or the Home tradable good.
where $\theta > 1$ and $j \in \{HT, HN\}$. The coefficient $A_s$ denotes the labor productivity and is subject to shocks. The associated wage level to employ a unit of aggregate labor is $W_s = \left( \int_0^1 W_s^{(1-\theta)} di \right)^{-\frac{1}{\theta}}$, where $W_i$ denotes the monopolistic money wage claimed by household $i$. Accordingly, the demand for specific type of labor provided by Home household $i$ is

$$L_i = \left( \frac{W_i}{W_s} \right)^{-\theta} L_s,$$

where $L_s$ is Home aggregate demand for labor. The foreigners share an identical aggregation technology and therefore the corresponding equations apply.

### 1.1.2 The Households

A Home household $i$ has preferences over consumption and labor effort as described by

$$U_i = E_i \sum_{s=t}^{\infty} \beta^{s-t} U_i^s,$$

where

$$U_i^s = \left( C_i^{1-\rho-1} - \frac{1}{p} L_i^{\nu} \right) \quad \text{with} \quad C_i^s = \frac{C_{T,s}^{\gamma} C_{HN,s}^{1-\gamma}}{\gamma(1-\gamma)(1-\gamma)},$$

$0 < \beta < 1$, $\rho > 0$, $\nu \geq 1$, and $0 \leq \gamma \leq 1$. The consumption index $C_i^s$ aggregates the Home non-tradable good $C_{HN,s}^i$ and tradable goods $C_{T,s}^i$ with unit elasticity of substitution. $C_{T,s}^i$ aggregates the Home tradable good $C_{HT,s}^i$ and the Foreign tradable good $C_{FT,s}^i$ with unit elasticity of substitution and equal shares, i.e. $C_{T,s}^i = 2C_{HT,s}^i 2C_{FT,s}^i$. Foreign households have the same preferences over tradable goods but differ with respect to their own non-tradable good. Hence, the corresponding equations apply.

### Individual Wealth and Cash Constraints

Households can trade a nominal bond with other households within a country. They cannot, however, trade any assets with households from abroad. The timing protocol when asset and goods markets open within a period follows Lucas (1982). At the beginning of a period $s$, household $i$ holds nominal wealth $W_i$. In the asset markets, household $i$ receives money transfers $X_i$, decides about the holding of domestic nominal bonds $B_i^s$ that repay in next

---

6This assumption is one of two possible stark assumptions that are sufficient for shutting down the current account which allows the derivation of a closed-form solution. In contrast to the other possible assumption where international financial markets are complete (compare eg. Devereux and Engel (2003)), the lack of international risk sharing preserves an important role for international monetary policy management: As Obstfeld and Rogoff (2002a) point out, optimal international monetary policy conduct requires to also take into account the need for international consumption risk sharing. In order to also capture the role of monetary policy for international consumption risk sharing, the extreme assumption of no international financial markets at all seems to be the more revealing one.
1.1. The Model

period \( R_s B_s^i \) at a gross nominal return \( R_s \), and about cash holdings \( M_s^i \).\(^7\) The asset market constraint reads

\[
M_s^i + B_s^i \leq W_s^i + X_s^i. \tag{1.5a}
\]

Thereafter, the goods markets open where purchases of consumption goods must not exceed initial cash holdings, ie.

\[
P_s C_s^i \leq M_s^i. \tag{1.5b}
\]

At the end of period \( s \), household \( i \) receives wage earnings \( W_s^i L_s^i \). Thus, the nominal wealth at the beginning of the next period \((s + 1)\) is

\[
W_{s+1}^i = M_s^i + R_s B_s^i - P_s C_s^i + W_s^i L_s^i. \tag{1.5c}
\]

**Optimal Decisions**

Households optimize their expected lifetime utility (1.4) by deciding on bond and cash holdings, consumption, and their monopolistic wages subject to the constraints (1.5a-1.5c), the demand for their specific type of labor (1.2), and subject to the constraint that they have to set wages one period in advance. In order to smooth consumption over time, household \( i \) demands domestic nominal bonds according to the intertemporal Euler equation

\[
\frac{1}{R_s} = \beta E_s \left( \frac{C_{s+1}}{C_s} \right)^{-\rho} \Pi_{s+1}^{-1}, \tag{1.6}
\]

where \( \Pi_{s+1} \) denotes the inflation in period \( s + 1 \). Individual optimization also yields the standard composition of consumption between the tradable goods basket and the non-tradable good and between Home and Foreign tradable goods. The corresponding Home consumption-based price indices are given by \( P_s = P_{T,s}^{\gamma} P_{HN,s}^{(1-\gamma)} \) and \( P_{T,s} = P_{HT,s}^{1/2} P_{FT,s}^{1/2} \). Overall consumption will be determined by the household’s cash holding. In particular, we will assume that the net nominal interest rate will be strictly positive and reaches zero only in the limit. Consequently, the cash constraint is binding and optimality implies that households use all their initial cash for consumption goods purchases.\(^8\)

\(^7\)Note that following Lucas (1982) directly would also imply households to purchase foreign cash within the asset markets. This, however, is completely equivalent to imposing only a single cash constraint since information is complete at the asset markets. I stick to the single cash constraint because it is assumed that households go to domestic retailers only who sell both Home and Foreign goods in domestic currency. This also facilitates the comparison to the standard NOEM approach using money-in-the-utility.

\(^8\)As it is well known, the cash-in-advance constraint with the Lucas timing convention of markets is binding if the net nominal interest rate is positive. In the sticky wage set up, a zero net nominal interest rate implies real indeterminacy as Carlstrom and Fuerst (1998) show in a comment to Ireland (1996) for a closed economy. Nevertheless, we follow Adao et al. (2003) and assume that the interest rate is positive but arbitrarily close to zero.
(Flexible Wages) When the household can set wages instantly, it will equate the real wage to a mark-up over the marginal rate of substituting consumption and labor (MRS) as implied by the condition
\[
\frac{W_i}{P_i} = \frac{\theta}{\theta - 1} R_s \frac{L_s^{i\nu - 1}}{C_s^{1 - \rho}}. \tag{1.7}
\]
The case when wages are flexible will serve as useful benchmark. Equation (1.7) reveals that households face two different incentives to claim real wages higher than the MRS would dictate: first, wage setters impose a monopolistic mark-up, and second, the cash-in-advance restriction leads the household to take into account that labor income is available for consumption only the period thereafter. This is to be evaluated by the nominal interest rate because the nominal interest rate reflects the opportunity cost of holding money as wealth. Consequently, higher nominal interest rates causes the households to claim higher money wages.

(Sticky Wages) When wages have to be posted one period in advance, optimal wage setting requires households to equate the expected marginal loss in utility implied by labor and the expected marginal gain in utility from the additional consumption purchases the period thereafter. Again, by making use of the Euler equation one gets
\[
\frac{W_i}{P_i} = \frac{\theta}{\theta - 1} E_{s-1} \left( \frac{L_s^{i\nu}}{\frac{1}{N_s} R_s C_s^{1 - \rho}} \right). \tag{1.8}
\]
as the optimal wage claim. Similar to the case of flexible wages, households impose a monopolistic mark-up and also take into account the effect of expected nominal interest rates. With preset wages, however, households cannot adjust their money wage claim to the realization of shocks. In contrast to the case of flexible wages where households can effectively control labor effort ex post, i.e., after shocks are realized, households are assumed to fully supply the amount of labor the firms demand at the posted money wage. By the identical structure of the Foreign households’ problem, they obtain equivalent optimality conditions.

1.1.3 Governments’ Budget Constraints

National monetary authorities change money supply by making direct money transfers to the households at the asset markets. The associated constraint for the Home authority reads
\[
\int_0^1 M_s^i \, di = \int_0^1 M_{s-1}^i \, di + \int_0^1 X_s^i \, di. \tag{1.9}
\]
The money supply will be set according to policy rules that are specified later in the discussion of monetary policy conduct. For the Foreign authority the corresponding equation applies.
1.2 Equilibrium Allocation

All households within a country are assumed to be identical except for the specific types of labor. This also includes that they start out with identical wealth and that they receive identical money transfers. Thus, by the symmetry of labor demand, all households take identical optimal decisions. We therefore drop superscript $i$.

1.2.1 Goods Prices and the Terms of Trade

Goods markets are assumed to be perfectly competitive. Since goods prices are flexible, they are set equal to the marginal costs. Consequently, prices of national tradable and non-tradable goods coincide and the identification of whether it is the tradable or non-tradable good can be saved. Goods prices are then

$$P_{H,s} = \frac{W_s}{A_s} \quad \text{and} \quad P_{F,s}^* = \frac{W_s^*}{A_s^*}, \quad (1.10)$$

respectively. The terms of trade are defined as the price of Home exports over the price of Home imports, ie. $ToT_s = \left( \frac{P_{H,s}}{P_{F,s}} \right)$, where $E_s$ denotes the nominal exchange rate. In terms of relative wage levels, we can restate the terms of trade as

$$ToT_s = \frac{A_s^*}{A_s} \left( \frac{W_s}{E_s W_s^*} \right). \quad (1.11)$$

By the consumption-based price indices follows for the real exchange rate that $RER_s = ToT_s^{(1-\gamma)}$.

1.2.2 Ex Post Equilibrium Allocation

As all households within a country take identical decisions and as they start out with identical initial wealth, they will ask and bid identical amounts of nominal bonds. Thus, there is no net trade in bonds and asset market clearing conditions require that households’ bond holding is zero in all states. Consequently, households hold only cash as wealth, ie. $W_s = M_{s-1}$ and $W_s^* = M_{s-1}^*$. Next, goods markets clearing of tradable goods necessitates nominal imports to equal nominal exports because nominal trade must be balanced as there are no payments through international financial markets. Moreover, since Home and Foreign households share the same preferences over tradable goods with unit demand elasticity, the law of one price holds for the composite of tradable goods. As a consequence, Home and Foreign consumption of tradable goods must be the same, ie. $C_{T,s} = C_{T,s}^*$. Making use of the optimal composition of tradable and non-tradable goods within households’ consumption baskets, ie. $C_{T,s} = \gamma T_{s}^T C_s$ and $C_{T,s}^* = \gamma T_{s}^T C_s^*$, respectively, reveals that - in terms of tradable goods - Home and Foreign consumption expenditures must be the same. Following Obstfeld and Rogoff (2002a), overall consumption expenditures are expressed in terms of
tradable goods consumption as

\[ Z_s = \frac{P_s}{P_{T,s}} C_s \quad \text{and} \quad Z^*_s = \frac{P^*_s}{P^*_{T,s}} C^*_s. \]  

(1.12)

The immediate equilibrium consequence then is \( Z_s = Z^*_s \). Furthermore, money market clearing and the binding cash-in-advance constraints determine households’ nominal expenditures. Taking ratios of the nominal consumption expenditures and using the goods market clearing implication of \( Z_s = Z^*_s \), one obtains the equilibrium nominal exchange rate to be solely determined by the ratio of Home and Foreign money supplies. In summary, independent of whether wages are flexible or preset a period before, the equilibrium entails

\[ Z_s = Z^*_s \quad \text{and} \quad E_s = \frac{M_s}{M^*_s}. \]  

(1.13)

It turns out to be very insightful to express the national variables in terms of their common and their different components as proposed by Aoki (1981). In particular, let subscript “\( w \)” denote the ”world” average component which is the geometric mean of Home and Foreign variables and let subscript ”\( d \)” denote the ”difference” component which is the ratio of Home over Foreign variables.\(^9\) The decomposition of Home and Foreign equilibrium consumption levels yields

\[ C_{w,s} = Z_s \quad \text{and} \quad C_{d,s} = ToT_s^{-\frac{1}{2}(1-\gamma)}. \]  

(1.14)

The world average consumption which is common to both Home and Foreign is clearly \( Z_s \).

As a consequence, in equilibrium, the difference between Home and Foreign consumption solely stems from the consumption of non-tradable goods. This is entirely captured by the real exchange rate and hence by the terms of trade. Consequently, the ex post equilibrium allocation is uniquely determined for given common consumption level \( Z_s \) and the terms of trade \( ToT_s \). Table 1.1 summarizes equilibrium consumption, output and labor.

**Flexible Wages**

When wages are flexible, it is straightforward to obtain

\[ Z_s = \left( \frac{(\theta-1) A_{w,s}}{R_{w,s}} \right)^{\frac{1}{\theta}} \quad \text{and} \quad ToT_s = \left( R_{d,s} A_{d,s}^{-\nu} \right)^{\frac{2}{\theta}}, \]  

(1.15)

where \( \chi = \nu - (1-\rho) > 0 \) and \( \chi' = \nu - (1-\rho)(1-\gamma) > 0 \). From (1.15) follows that the higher the world average nominal interest rates, the lower the common consumption \( C_{w,s} \). Higher nominal interest rates reflect the opportunity cost of keeping wealth as cash.

---

\(^9\)As a reminder, for Home and Foreign variables \( X \) and \( X^\ast \) the decomposition in levels is \( X = X_w X_d \) and \( X^\ast = \frac{X}{X_d} \) where \( X_w = (XX^\ast)^{\frac{1}{2}} \) and \( X_d = (X^\ast)^{\frac{1}{2}} \). The exponents are relative country sizes which in our case is \( \{\frac{1}{2}, \frac{1}{2}\} \).
1.2. Equilibrium Allocation

<table>
<thead>
<tr>
<th>Common World Components</th>
<th>Difference Components</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>$C_w = Z$</td>
</tr>
<tr>
<td></td>
<td>$C_d = T_oT^{-\frac{(1-\gamma)}{2}}$</td>
</tr>
<tr>
<td>Output</td>
<td>$Y_w = Z$</td>
</tr>
<tr>
<td></td>
<td>$Y_d = T_oT^{-\frac{1}{2}}$</td>
</tr>
<tr>
<td>Labor</td>
<td>$L_w = A_w^{-1}Z$</td>
</tr>
<tr>
<td></td>
<td>$L_d = A_d^{-1}T_oT^{-\frac{1}{2}}$</td>
</tr>
</tbody>
</table>

Table 1.1: Ex Post period equilibrium allocation for given $Z$ and $T_oT$.

Since the labor income cannot be spent in the period earned and households must keep it as cash, they take this into account and demand higher nominal wages. This effectively increases the wedge between the marginal rate of substitution between consumption and labor and the real wage claim. The consequence is an inefficiently low labor supply that results in a reduction of equilibrium output and consumption. The terms of trade, in turn, depend on relative nominal interest rates as these determine relative nominal wage levels and thereby relative goods prices. Because higher domestic nominal interest rates cause households to increase their wage claims, goods prices increase and alter the terms of trade. It is important to observe that it is the relative nominal interest rates that affect relative prices and the allocation. National money supplies determine only the national price levels and the nominal exchange rate according to equation (1.13). When wages are flexible, the nominal exchange rate does not matter for determining the real allocation. The role of the money supply changes, however, when wages are sticky.

Sticky Wages

Preset wages imply that goods prices are fully determined by the realization of productivity levels. The important implication is that the nominal exchange rate uniquely determines the terms of trade as by equation (1.11). Thus Home and Foreign consumption-based price levels are determined. Home and Foreign consumption levels are therefore determined by the respective money supplies through the cash-in-advance constraint. As a result, and in contrast to flexible wages, the ex post real allocation can only be altered by national money supplies. To be specific,

$$Z_s = \frac{A_{w,s}}{W_{w,s}}M_{w,s} \quad \text{and} \quad T_oT_s = \left( \frac{W_{d,s}}{A_{d,s}M_{d,s}} \right)^2.$$ (1.16)

When goods prices are effectively predetermined, world average consumption $Z_s$ can only be changed ex post by altering the common money supply which reflects in equilibrium a one-to-one change in real balances available for consumption purchases. The terms of trade,
in turn, can only be changed ex post by the nominal exchange rate which is determined by
the relative money supplies as in (1.13).

In fact, this monetary propagation mechanism resembles the standard equilibrium trans-
mission of monetary policy in NOEM, (compare eg. Corsetti and Pesenti (2001), Obstfeld
and Rogoff (2002a), or Devereux and Engel (2003)). In sharp contrast, however, and key to
the analysis in this paper, the money supply is not the only available monetary instrument
to affect the equilibrium allocation. For a more general short-run monetary policy conduct
it is important to realize that even though the ex post allocation can only be altered by
the actual money supply, the expected period inflation and hence the expected nominal
interest rates play a crucial role for the determination of the equilibrium allocation ex ante
because they affect the wage setting and hence the terms of trade ex ante. The argument
follows the same logic as in case of flexible wages: the expected inflation convey the expected
opportunity cost of keeping labor income as cash. Consequently, higher expected inflation
taxes on labor income lead to higher wage claims ex ante. Recall the optimal wage setting
condition (1.8).

1.2.3 Distribution of the Equilibrium Allocation

Uncertainty stems from productivity shocks that are assumed to be iid log-normal. Letting
lower case letters denote logs, productivity shocks have the following properties: \( Ea =
Ea^* = 0 \) and \( Var(a) = Var(a^*) = \sigma_a^2 \), where \( Ea = 0 \) is assumed for simplicity. In accordance
with the equilibrium variables, these shocks are expressed in terms of a "world" component
common to both countries and a "difference" component making up the gap in productivity.
Thus we have \( a_{w,s} = \frac{1}{2}(a_s + a_s^*) \) and \( a_{d,s} = \frac{1}{2}(a_s - a_s^*) \). The distribution of the decomposed
shocks implies in turn

\[
Var(a_s) = \sigma_a^2 = \sigma_{a_w}^2 + \sigma_{a_d}^2 \quad \text{since} \quad Cov(a_{w,s}, a_{d,s}) = \sigma_{a_{w,s},a_{d,s}} = 0.
\]

If monetary policies are stationary, iid productivity shocks imply that the households’ and
firms’ optimal decision rules are stationary. As a consequence, the equilibrium of the infinite
horizon setup is simply a repetition of the static version of a single period. Furthermore,
if the money supply is log-normal, too, the distribution of the equilibrium allocation turns
out to be jointly log-normal. Therefore, the solution to the stochastic general equilibrium

10Note that in stochastic environments ex post monetary policy conduct also influences ex ante wage
setting because ex post monetary interventions alter the equilibrium distribution that is relevant for optimal
wage setting. This effect has been emphasized by Obstfeld and Rogoff (2000, 2002a,b) and comes in addition
to the incentives to alter wages induced by the nominal interest rates. As I argue below, from a welfare
perspective, however, the distribution effects are of second order. In contrast, the effects of the nominal
interest rates will be of first order.
can be obtained in closed-form. For the rest of the paper, the time-subscripts are skipped for convenience.

1.3 Short-Run Monetary Policy

In studying optimal short-run monetary policy, I consider policy rules that consist of both the inflation target as well as the state-dependent actual money supply for each period. Monetary authorities are assumed to be able to perfectly commit to these policy rules. This is not only in line with the recent literature but it also reflects the experience that time inconsistency plays no longer a major role in the actual conduct of monetary policy in many OECD countries. Moreover, the policy rule that sets both the inflation target and the actual money supply attends an important feature: Although the commitment to standard money supply rules eliminates the use of surprise inflation, they implicitly preclude the use of a potentially desired inflation. Augmenting the money supply rule with an explicitly targeted inflation, however, is crucial to fully characterize optimal short-run monetary policy in open economies and potential gains from international monetary policy coordination.

The key insight of this paper revolves around the question of how policymakers exploit nominal frictions in different strategic settings in order to improve their respective resident’s economic well-being. On the one hand, labor being a credit good implies that inflation taxes labor income. Thereby, the monetary authority can influence workers’ wage setting through the inflation targeting policy. On the other hand, the nominal inertia places the money supply at monetary authorities’ disposal as an instrument by which it can alter households’ real balances directly and hence the actual allocation ex post. National monetary authorities can set the inflation target and the period money supply independently by steering the expected money growth rate and the state-dependent deviations from the announced money growth rate. The important difference between the two policy instruments is the way they affect the equilibrium allocation. The money supply management determines state-dependent spending flows. From an ex ante perspective, this corresponds to changes in the variability of the equilibrium allocation. The inflation target changes the incentives to workers’ wage setting and thereby the expected levels of the equilibrium allocation. Crucially then, from a welfare perspective, the inflation target policy is of first order whereas the money supply management is of second order.

There are yet two other non-monetary frictions in the model economy. The lack of international risk sharing is an important feature as it reveals important strategic interactions between national monetary authorities. The distortion created by monopolistic competition is a constant markup over competitive wages which only overlaps with all other economic
effects of interest but which yields no further insights. Therefore I abstract from the distortions created by monopolistic competition and follow Ireland (1996) by considering the limiting case of perfect competition where $\theta \to \infty$ and the wage markup is unity.$^{11}$

### 1.3.1 Inflation Target and the Money Supply as Two Distinct Policy Instruments

A central element of this paper is the observation that when monetary authorities set their policy rules, they dispose of two distinct policy instruments, the inflation target $\Pi_e$ and the state-dependent money supply $M$, by means of which they effectively alter the equilibrium allocation. To see that this is true, note that the equilibrium inflation can be stated in terms of consumption and money supply by the ratio of the cash-in-advance constraints in two consecutive periods. Expected inflation can then be written as

$$\Pi_e = \frac{E M'}{M} \left( \frac{E C'}{C} \right)^{-1} \exp\{-\sigma_{c,m}\}, \quad (1.17)$$

where a prime denotes the next period’s level. In the rational expectations equilibrium, the expected inflation is rising with the increase in expected money growth $\frac{E M'}{M}$ and with the decrease in expected consumption growth $\frac{E C'}{C}$. Expected inflation is also decreasing in the variability of future consumption and money supply. The claim is now the following: Monetary authorities have a degree of freedom to set separably the expected inflation as the inflation target and the actual money supply $M'$. This can be achieved by the appropriate and credible announcement of the future periods’ distribution of money supply. To be specific, the inflation target is set by means of the choice of the expected money growth, ie. the ratio $\frac{E M'}{M}$. The respective period’s actual money supply can then be implemented by means of the deviation $\frac{\Delta M'}{M}$ of the realized money growth from the expected one, ie. $\frac{\Delta M'}{M} = M' - \frac{E M'}{M}$. In the rational expectations equilibrium, the announced period money supply must be consistent in the sense that the deviations are zero in expectations ($E \Delta M' = 0$).

The important equilibrium relationship that implicitly provides the monetary policy with a degree of freedom to chose both the inflation target and the money supply is represented by the intertemporal Euler equation. As the nominal interest rate describes the opportunity cost of keeping cash and making consumption purchases tomorrow instead of spending cash for consumption purchases today, the Euler equation relates the current price level and current consumption to the expected future price level and to expected future consumption.

$^{11}$An alternative assumption is to introduce national fiscal stances that subsidize labor in order to offset the inefficient wage markup. As a matter of fact, in a companion paper I demonstrate that this is indeed part of the optimal monetary and fiscal policy in a two-country sticky wage model like the one at hand. However, it is not necessarily true that non-coordinated fiscal policy indeed sets labor income taxes to offset the monopolistic distortion. Instead, taxes are used to manipulate the terms of trade in exactly the same manner as it turns out to be the case for the nominal interest rate (see Evers (2007b)). Arseneau (2007) studies the role of monopolistic markups for nominal interest rate policy conduct in open economies in detail.
1.3. Short-Run Monetary Policy

This is best captured by a representation of the Euler equation as a modified Fisher relation that links the nominal interest rate to the real interest rate and the expected inflation:

\[
R = \tilde{R}^{\Pi^e} \exp\{\rho (\sigma_c^2 - \sigma_{c,m})\},
\]

(1.18)

where \(\tilde{R}\) denotes the gross real interest rate. Accordingly, the nominal interest rate is increasing in the expected inflation as consumption tomorrow becomes more expensive in expectations as compared to consumption today. The nominal interest rate is also increasing in the real interest rate because an increase in \(\tilde{R}\) reflects an increase in the marginal rate of substitution of consumption today for consumption tomorrow. As the relative desire for consumption tomorrow falls, the equilibrium opportunity cost of consumption tomorrow and thereby the nominal interest rate must rise. Moreover, the Fisher relation is modified as it takes into account the impact of the consumption and money supply variability on the nominal interest rate. Expressing the expected inflation in equation (1.17) in terms of the real interest rate, too, yields

\[
\Pi^e = \frac{EM'}{M} \left( \tilde{R}^\beta \right)^{\frac{1}{\rho}} \exp\left\{ \frac{(1 - * \rho)}{2} \sigma_c^2 - \sigma_{c,m} \right\},
\]

(1.19)

Suppose for the moment that the real interest rate is given. Then the expected next period’s money growth \(\frac{EM'}{M}\) selects for a given future money supply the expected inflation and hence chooses an explicit inflation target. By the Fisher relation it also follows that the inflation target is an instrument to affect the current period’s nominal interest rate. This implies that it can be used to affect the current equilibrium allocation. In turn, the actual money growth in the next period is going to affect the next periods allocation as depicted e.g. by \(\sigma_{c,m}\). Importantly, there is a subtle difference in the time when the inflation target on the one hand and the actual money supply on the other hand affect the allocation. This turns out to be the reason why the separability of inflation target holds true even if one takes into account the equilibrium effects of the monetary policy on the real interest rate. For a better illustration, it proves useful to consider the case of flexible wages and the case of sticky wages in more detail.

First, consider the case of flexible wages. From the two equations in (1.15) follows for given realizations of productivity shocks that the current allocation is determined by the Home and Foreign nominal interest rates. It also follows that expected consumption depends on expected nominal interest rates only. The immediate implication is that \(\sigma_{c,m} = 0\) as only changes in the future interest rates induce changes in future consumption. The reason is that changes in money supply are neutral because wages are flexible. For given future nominal interest rates and for given Foreign monetary policy, the current consumption level \(C\) and hence the real interest rate can only be altered by the nominal interest rate. By the Fisher relation, however, the result is that in equilibrium the current consumption level can be uniquely determined by the expected money growth \(\frac{EM'}{M}\) and hence by the inflation
target \( \Pi^e \). It is important to note that in case of flexible wages, only the expected money growth rate is relevant. There is a continuum of realized money supplies and associated price levels that are consistent with the equilibrium allocation. All what the rational expectations equilibrium requires is the consistency of expectations, namely that \( \frac{\Delta M'}{M'} \). As a result, monetary authorities dispose of two distinct policy instruments.

When wages are preset, things are only slightly more involved. The two equations in (1.16) imply that for given realizations of productivity shocks, the allocation depends on the preset wages and on Home and Foreign money supplies. Because wages are preset, ex post changes to the allocation can only be induced by the deviations from the expected money growth \( \frac{\Delta M'}{M'} \). As a consequence, the money supply management plays a crucial role in the determination of the real interest rates when wages are preset. The actual money supply in the current period affects the current consumption level whereas the distribution of state-dependent deviations from the expected money growth affects the expectation over the consumption level in the next period. Therefore, the real interest rate depends on both the current period’s money supply \( M \) and on the distribution of the deviations from the expected money growth \( \frac{\Delta M'}{M'} \). The important question, however, is whether each expected level of money growth and hence each level of the inflation target \( \Pi^e \) is consistent in the rational expectations equilibrium with a continuum of realized money supplies and distributions of future money supplies so that monetary authorities still dispose of two distinct policy instruments. The answer is yes and follows in principle the analog logic of the case when wages are flexible. The key insight again is that the expected money growth doesn’t affect the future allocation and it is in this sense allocatively neutral to the next period’s allocation. All that is relevant for the future allocation are the expected future nominal interest rates as they determine the wage setting (compare the optimal wage setting in equation (1.8)) and deviations from the expected money growth. By the same reasoning as above, for a given real interest rate controlled by the actual money supply and for given Foreign monetary policy, the expected money growth alters the current period’s nominal interest rate \( R \). In contrast to flexible wages, however, the nominal interest rate cannot alter the allocation ex post. With preset wages, however, they affect the nominal wages in expectation ex ante. Consequently, the consistency of the rational expectations equilibrium requires that the ex ante expected nominal interest rate to be on average the realized one. As a result, for any given future nominal interest rates, for any given consistent future deviations of money supplies from the expected levels with \( \frac{E\Delta M'}{M'} = 0 \), and for any given current period’s money supply \( M \), monetary authorities can set any inflation target \( \Pi^e \) by the appropriate choice of the expected money growth. Note that in contrast to flexible wages, each realized actual money supply \( M \) requires a unique choice of expected future money supply \( EM' \) to set the inflation target at its desired level. As a result, monetary authorities can exploit the inflation target and the money supply a two distinct monetary policy instruments.
The Nominal Interest Rate as a Monetary Policy Instrument

In principle, instead of arguing via the inflation target as the policy instrument, one can directly argue by means of the nominal interest rate because there is a one-to-one equilibrium relationship between the two variables. The implication is thus that the nominal interest rate can be effectively controlled by the monetary policy conduct. In fact, for reasons of presentations of the results and for better illustration of the economic intuition it proves useful to directly argue by means of the nominal interest rate as the respective policy instrument.\textsuperscript{12} The conclusion is that monetary authorities dispose of two independent policy instruments - the nominal interest rate and the money supply - no matter whether wages are flexible or preset.

Short-Run Monetary Policy Rule

National monetary authorities commit to a monetary policy rule that sets the nominal interest rate and the money supply. The money supply policy is assumed to follow a feedback rule that conditions the state-dependent levels of money supply on productivity shocks. The money supply rules comprise of the feedback coefficients \( \mu = \{ \mu_{aw}, \mu_{ad} \} \) and \( \mu^* = \{ \mu_{aw}^*, \mu_{ad}^* \} \) and they are of the form

\[
\hat{m} = \mu_{aw} \hat{a}_w + \mu_{ad} \hat{a}_d \quad \text{and} \quad \hat{m}^* = \mu_{aw}^* \hat{a}_w - \mu_{ad}^* \hat{a}_d,
\]

where variables with a hat denote deviations from their expected value. Accordingly, in (1.20), \( \hat{m} \) and \( \hat{m}^* \) denote the log-deviation from the expected money supply, i.e. \( \hat{m} = m - Em \) and \( \hat{m}^* = m^* - Em^* \). Recall that this specification indeed allows the policymakers to set both the nominal interest rate by choosing the expected values for money supply \( Em \) and \( Em^* \) and the state-dependent deviations \( \hat{m} \) and \( \hat{m}^* \) in order to react to productivity shocks. Note also that thereby the use of surprise inflation is ruled out.\textsuperscript{13} As a consequence, the Home

\textsuperscript{12}Moreover, in their analysis of a two-instrument short-run monetary policy, Adao et al. (2003) directly argue by means of the nominal interest rate and the money supply as the two distinct policy instruments. Facilitating the comparison to their analysis is the other reason to refer to the two policy instruments as the nominal interest rate and the money supply.

\textsuperscript{13}Two remarks to the specification of the monetary policy are in order: First, as concerning the period single nominal interest rate, it implies no restriction at all to set it in a non state-contingent way. In case of flexible wages, it can be demonstrated that even the ex post optimal nominal interest rate is independent of the actual realization of the state. In case of sticky wages, only the expected nominal interest rate matters as it was also argued in the foregoing discussion of the separability of monetary policy instruments. Hence, there is no requirement for state-dependent nominal interest rate setting, too. Second, as concerning the state-dependent money supply, using the standard feedback rule as eg. in Obstfeld and Rogoff (2002a) and Devereux and Engel (2003) would be to take the following AR(1) form: \( m_s = m_{s-1} + \mu_{aw,s} \hat{a}_{w,s} + \mu_{ad,s} \hat{a}_{d,s} \). This, however, implies \( Em_s = m_{s-1} \), which factually precludes the use of both, the nominal interest rate and the money supply as monetary policy instruments at the same time.
monetary policy rule is denoted by \{R, \mu\} and the Foreign monetary policy is denoted by \{R^*, \mu^*\}.

1.3.2 Policymakers’ Objective

The objective of national monetary authorities is to maximize their respective residents’ welfare. By the simplified iid structure of the model, Home policymaker’s problem reduces to choose \{R, \mu\} so as to maximize expected period utility \(EU\). The Foreign policymaker decides over \{R^*, \mu^*\} so as to maximize \(EU^*\). Making use of the equilibrium wage setting, Home and Foreign expected utility can be expressed as

\[
EU = E \left( \frac{1}{(1 - \rho)} - \frac{1}{\nu} \frac{1}{R} \right) (C)^{(1 - \rho)} \quad \text{and} \quad (1.21)
\]

\[
EU^* = E \left( \frac{1}{(1 - \rho)} - \frac{1}{\nu} \frac{1}{R^*} \right) (C^*)^{(1 - \rho)},
\]

respectively. It is important to observe that Home and Foreign policymakers’ objectives are symmetric except for the impact of the terms of trade. Recall from the discussion of the equilibrium allocation that the difference between the consumption levels is fully captured by the terms of trade (compare Table 1.1). Hence, deviations from the jointly optimal monetary policy will be solely on the grounds of strategically motivated manipulations of the terms of trade in the respective country’s own favor. Moreover, as the impacts of the terms of trade on Home and Foreign objectives are orthogonal, the incentives to strategically deviate from the socially optimal policy must necessarily be the kind of "beggaring-thy-neighbor".

Expressing the expected utility in terms of the closed-form solution of the equilibrium permits further insights into the short-run monetary policy conduct. In particular, this leads to a particularly convenient separation of the equilibrium implications of monetary policy that directly affect the average consumption level from the equilibrium implications of monetary policy that changes the variability of the allocation and hence consumption. Consider first the expression of expected utility for the flexible wage environment because it is embedded in the expression of the expected utility under sticky wages.

Flexible Wages

When wages are flexible, Home expected utility takes the form

\[
EU|_{\text{flex}} = \left( \frac{1}{(1 - \rho)} - \frac{1}{\nu R w R_d} \right) \frac{(1 - \rho)}{\chi} \frac{1}{R^*} \cdot \exp \{\Omega_{\text{flex}}\}.
\]

The term \(\Omega_{\text{flex}}\) summarizes the part of expected utility which depends on uncertainty only and it is given by

\[
\Omega_{\text{flex}} = \frac{(1 - \rho)^2 \mu^2}{2X^2 \sigma_{a_w}^2} \left( (1 - \gamma)^2 X^2 \sigma_{a_d}^2 \right).
\]
1.3. Short-Run Monetary Policy

In case of flexible wages, all variability of the allocation and hence consumption stems from exogenous changes in productivity. Importantly, the nominal interest rate determines the part of expected utility which doesn’t depend on uncertainty but it alters the mean level of expected utility. The consequence is that as nominal interest rate policy affects the first moment of expected utility, the policy implications are of first order.

Sticky Wages

When wages are sticky, Home expected utility can be decomposed such that it contains the expression for expected utility under flexible wages,

\[ EU_{\text{sticky}} = U_{\text{flex}}(R; R^*) \cdot \exp\{\Omega_{\text{sticky}}(\mu; \mu^*)\}. \tag{1.23} \]

Similarly to the above, \( \Omega_{\text{sticky}}(\mu, \mu^*) \) summarizes the second moment variance and covariance terms of expected utility, namely

\[
\Omega_{\text{sticky}}(\mu; \mu^*) = \frac{(1-\rho)\nu}{2} \left( \omega - \sigma_z^2 - \frac{\omega^2}{4X} \sigma_e^2 + \frac{(1-\rho)(1-\gamma)^2}{X} \sigma_{a,w} + (1-\rho)(1-\gamma)^2 \sigma_{a,d} \right) \\
+ \frac{\nu(1-\rho)(1-\gamma)}{2Y} \left( - (1-\gamma)X \sigma_{z,e} + \nu \sigma_{e,a,w} + (1-\rho)(1-\gamma) \sigma_{z,a,d} \right),
\]

where \( Z = \nu - (1 - \rho)(1 - \gamma)^2 \) and \( \omega \) is a constant independent of endogenous variables.\(^{14}\)

In contrast to flexible wages where all uncertainty stems from exogenous productivity disturbances, under preset wages an active state-dependent money supply management according to the feedback coefficients \( \{\mu, \mu^*\} \) entails endogenous uncertainty over the allocation. Crucially, the money supply policy affects the average level of consumption and hence the expected utility through the changes in the variability of the equilibrium allocation only. Consequently, and in contrast to the welfare implications of the nominal interest rate policy, as the money supply policy affects the equilibrium by altering second moments of the equilibrium distribution of the allocation, the welfare implications are thus of second order.

Because the importance of the different implications of the two monetary policy instruments on expected utility cannot be overemphasized, the discussion is summarized in the following proposition:

**Proposition 1.1** The welfare implications of short-run nominal interest rate policy are of first-order whereas the welfare implications of short-run money supply management are of second-order.

**Proof.**

See Appendix.

Proposition 1.1 shall constitute the backbone of the argument put forth in this analysis. By the discussion in the beginning, the literature on international monetary policy regimes has

\(^{14}\)It is defined as \( \omega = \frac{(1-\rho)\nu}{2X^2Y^2} \left( - \nu^2 Y^2 a_w^2 + (1 - \rho)(1 - \gamma)^2 X (Y^2 - \nu X) \sigma_{a,d}^2 \right) \).
largely focused on stabilization issues. Gains from policy coordination, however, are quantified to be fairly small. Relating to the discussion in Lucas (2003), gains from stabilization and thereby gains from international coordination of stabilization policies are generically quite limited as they are of second order. On the contrary, the nominal interest rate policy as discussed here means that monetary authorities exploit the inflation tax to govern the workers’ wage setting. In terms of Lucas, this denotes a supply side effect which is of first order. Consequently, from a quantitative perspective, welfare gains from international coordination of nominal interest rate policies are to be expected of an order of magnitude larger than welfare gains from coordinating monetary stabilization policy as it has been done in the past. A numerical example shall support this claim. Next, however, the analysis is continued with a discussion of the theoretical results of optimal coordinated and noncooperative international policy conduct.

1.4 Optimal Cooperative Monetary Policy

When national policymakers coordinate their respective monetary policies, they do so as to maximize the sum of the equally weighted Home and Foreign residents’ welfare.

1.4.1 The Optimal Nominal Interest Rate

The globally optimal interest rate policy is as follows.

**Proposition 1.2** The optimal nominal interest rate policy is to follow the Friedman rule, i.e.

\[ R_{Opt} = 1, \]

and \( R^{Opt} = R_{Opt} \) by symmetry.

**Proof.**

See Appendix.

The intuition for optimality of the Friedman rule can be best seen in the context of optimal taxation since it is an immediate implication of the optimal taxation principle (Diamond and Mirrlees (1971)) which postulates not to tax intermediate inputs. Recall that as labor income is available for consumption only the following period, the gross nominal interest rate reflects the intertemporal nominal cost of keeping labor income as cash that cannot be spent within the same period as when it is earned. This can then be understood as a tax on labor income and thus as a tax on an intermediate input to the production of goods. As a consequence, it is optimal to set the net nominal interest rate to zero and thereby to offset the implicit tax on labor input. Importantly, the optimality of the Friedman rule obtains for both environments, with flexible wages as well as sticky wages. When wages are flexible, a positive net nominal interest rate leads to a distortive wedge between the marginal rate
of substituting labor and consumption (MRS) on the one hand and marginal product of labor (MPL) on the other hand in all instances. When wages are sticky, labor is demand determined and the ratio between MRS and MPL is not necessarily at an inefficiently high level. However, expected utility is maximized when the distortion is minimized on average. Hence, the Friedman rule is optimal under sticky wages, too.\footnote{Compare also Chari et al. (1996), Chari and Kehoe (1999), Adao et al. (2003), or Kocherlakota (2005) for closed economy setups and Cooley and Quadrini (2003) and Arseneau (2007) in the context of open economies.}

1.4.2 The Optimal Money Supply

In contrast to the nominal interest rate, money supply is allocatively effective only when wages are sticky. In the discussion of monetary policy instruments, the bottom line was that the money supply management affects the variability of the equilibrium allocation. As an immediate consequence, the two relevant frictions that impose distortions to the equilibrium allocation which could be on target for the money supply are wage rigidity and the lack of international asset markets. The next proposition states the optimal feedback coefficients on aggregate and asymmetric productivity shocks.

**Proposition 1.3** The optimal money supply feedback rule follows

\[
\mu_{\alpha w}^{\text{Opt}} = \frac{(1 - \rho)}{\chi} \quad \text{and} \quad \mu_{\alpha d}^{\text{Opt}} = \frac{(1 - \rho)(1 - \gamma)^2}{\mathcal{Z}},
\]

where \(\mu_{\alpha w}^{\text{Opt}} = \mu_{\alpha d}^{\text{Opt}}\) and \(\mu_{\alpha w}^{\text{Opt}} = \mu_{\alpha d}^{\text{Opt}}\) by symmetry. Furthermore, with \(\mu_{\text{Flex}}\) as the feedback coefficients replicating the flexible wage equilibrium, it follows that

\[
\mu_{\alpha w}^{\text{Opt}} = \mu_{\alpha w}^{\text{Flex}} \quad \text{and} \quad \mu_{\alpha d}^{\text{Opt}} \begin{cases} > \mu_{\alpha d}^{\text{Flex}} & \text{if } \rho > 1 \text{ and } 0 < \gamma < 1, \\ < \mu_{\alpha d}^{\text{Flex}} & \text{if } \rho < 1 \text{ and } 0 < \gamma < 1, \\ = \mu_{\alpha d}^{\text{Flex}} & \text{if } \rho = 1 \text{ or } \gamma \in \{0, 1\}, \end{cases}
\]

where \(\mu_{\alpha d}^{\text{Flex}} = (1 - \rho)(1 - \gamma)\chi\).

**Proof.**

See Appendix. \(\blacksquare\)

In case of aggregate productivity shocks, the only distortion that matters stems from preset wages. The optimal money supply response is then to replicate the flexible wage allocation. To be more precise, in case of flexible wages it easy to see that the intra-temporal substitution elasticity of consumption and labor is \(\frac{1}{\nu}\). When wages are preset, labor is fully demand determined and hence uncoupled from the consumption decision. As a consequence, in order to mimic the optimal labor-consumption trade-off, the optimal money supply response to aggregate productivity shocks adjusts real balances so that
consumption and labor changes in the right proportion. To be specific, consider a positive aggregate productivity shock. If wages were flexible, households’ would raise nominal wages up to the point where real wages would equal the marginal rate of substituting labor and consumption (see equation 1.7). For $\rho > 1$ consumption and labor are substitutes and hence households increase consumption and also reduce labor in response to a rise in real labor income. Under sticky wages, this adjustment is not possible. The positive aggregate productivity shock leads ceteris paribus to a one-to-one drop in goods prices and thereby to a one-to-one increase in consumption whereas employment stays unaffected. As a result, the optimal response of monetary policy must be to dampen the increase of consumption by contracting the money supply. This leads to a reduction of equilibrium employment, too, and thereby the optimal consumption-labor ratio can be restored. In contrast, if $\rho < 1$, consumption and labor are complements and optimality requires a conjoint increase in labor and consumption. As a consequence, money supply must respond pro-cyclically. In case of log-utility, ie. $\rho = 1$, it is optimal that labor doesn’t respond to consumption fluctuations at all. The optimality of targeting the flexible wage allocation reproduces the findings of several recent contributions where wage or price rigidity prevents the equilibrium allocation from efficiency.\textsuperscript{16}

In case of idiosyncratic productivity shocks, the lack of international risk sharing comes in addition to the inefficiency caused by preset wages. Complete asset markets would enable Home and Foreign households to contract state-contingent payments in order to insure against all idiosyncratic risks. The consequence of perfect consumption risk sharing for the initially identical countries would be that for all goods the ratio of Home and Foreign marginal consumption utilities equals the ratio of the respective equilibrium goods prices (compare, for example, Backus and Smith (1993)). In particular, for the basket of tradable goods the implication is that the ratio of Home over Foreign marginal utilities must be unity, ie. $\frac{U^C_T}{U^C_T} = 1$. In terms of the equilibrium without asset markets, the ratio of marginal utilities of tradable goods consumption reads

$$\frac{U^C_T}{U^C_T} = T_0 T^{-(1-\rho)(1-\gamma)}.$$ 

Following, the ratio $\frac{U^C_T}{U^C_T}$ decreases in response to, for example, a terms of trade depreciation if $\rho > 1$ and the ratio increases if $\rho < 1$. The intuition is best captured in terms of substitutability and complementarity of consumption goods in the Edgeworth-Pareto sense.\textsuperscript{17}


\textsuperscript{17}Two goods are substitutes (complements) in the Edgeworth-Pareto sense if the marginal utility of one good is decreasing (increasing) with the consumption of the other good. For open economy setups, see also Svensson (1987) and Corsetti and Pesenti (2001).
If $\rho > 1$, the different consumption goods are substitutes. A terms of trade depreciation caused by a positive Home productivity shock leads households to substitute the costlier Foreign goods for the cheaper Home goods. Thereby, Home households consume more Home tradables and non-tradables whereas Foreign households consume less Foreign tradables and non-tradables. In equilibrium, however, it is that Home and Foreign households consume the same amount of tradables, ie. $C_T = C_T^*$. As a consequence, the difference in marginal utilities of tradable goods consumption necessarily stems from the difference in consumption of non-tradables: Home households consume too much tradables relative to non-tradables as risk sharing would imply, Foreign households consume too less tradables relative to non-tradables as risk sharing would imply. The appropriate money supply change to offset this effect is to make Home households pay more for Foreign tradable goods and thereby to make them paying more for overall tradable goods and to make Foreign households pay less for Home tradables and thereby to make them paying less for overall tradable goods. Hence, to attenuate the lack of risk sharing, optimal money supply necessitates an depreciating of Home nominal exchange rate. For $\rho < 1$, when consumption goods are complements, the according logic applies. In this case, however, it is that Home households consume too less tradables relative to non-tradables whereas Foreign households consume too much tradables relative to non-tradables than under perfect risk sharing. The optimal money supply management therefore leads to an appreciation of Home nominal exchange rate in order to make Home households pay less for Foreign goods and to make Foreigners pay more for Home goods.

To see that there is in general a conflict of closing the domestic gaps between the marginal rate of substituting consumption and labor on the one hand and closing the international gap between Home and Foreign marginal consumption utilities on the other hand, suppose that Home and Foreign monetary authorities target the flexible wage allocation and implement $\mu_{ad}^{\text{flex}} = \mu_{ad}^{\star \text{flex}}$. This policy response, however, never fully offsets the impact of asymmetric productivity shocks on the terms of trade, ie. $\dot{a}_d - \nu \dot{Y}_a \dot{a}_d$. Consequently, by the logic developed above, monetary authorities face an incentive also to attenuate the implications of the lack of international consumption risk sharing. Moreover, the trade-off between targeting the two distortions is characterized by a more active response when money supply management targets the flexible wage allocation than it is optimal. To prevent a repetition of arguments, however, a discussion is left to the reader. The importance of this trade-off is also discussed in Obstfeld and Rogoff (2002a).

18There are three special cases where no such trade-off exists: When i) $\rho = 1$, the intertemporal and the intratemporal substitution elasticities coincide and hence risk is fully diversified via goods consumption in market equilibrium; when ii) $\gamma = 1$, only tradable goods are consumed and hence $C_T = C_T^*$ directly implies perfect consumption risk sharing; and when iii) $\gamma = 0$, no tradable goods are demanded at all and therefore the only inefficiency that is prevailing is wage stickiness.
1.5 Noncooperative Monetary Policy

In the discussion of national policymakers’ objectives, the central message is captured by Proposition 1.1: The nominal interest rate policy has first-order welfare implications whereas the money supply policy implications are of second order. Consequently, the losses from not coordinating monetary policy are of first-order when national monetary authorities face the incentive to deviate from the jointly beneficial Friedman rule (Proposition 1.2). In this section it is shown that self-oriented national policymakers follow the Friedman rule if and only if the two countries are closed. As long as there are trade linkages between the two countries, the incentives to manipulate the terms of trade cause the policymakers to unilaterally deviate from the optimal cooperative solution. Thereby, the gains from cooperation that are forgone if national monetary authorities act independently are of first order.

1.5.1 The Nominal Interest Rate

The following proposition establishes that the nominal interest rate in a Nash equilibrium of the policy setting game in general differs from the cooperative nominal interest rate.

Proposition 1.4 The unique Nash equilibrium of non-cooperatively set nominal interest rates implies

\[ R^{Nash} = 1 + \frac{\gamma X}{Y + (1 - \gamma)X}, \]

and \( R^{*Nash} = R^{Nash} \) by symmetry. Furthermore,

\[ R^{Nash} > R^{Opt} \quad \text{for} \quad \gamma \neq 0 \quad \text{and} \quad R^{Nash} = R^{Opt} \quad \text{iff} \quad \gamma = 0. \]

Proof.
See Appendix.

The intuition for this finding is straight forward: Policymakers face the incentive to improve their respective households’ consumption-labor trade-off. If there are any trade linkages between the two countries, ie. if \( \gamma > 0 \), the non-cooperative solution prescribes the policymakers to induce workers to claim higher wages by raising the nominal interest rate and consequently the implicit tax on labor. For instance, given Foreign wages and goods prices, an increase in Home wages imply an appreciation of the Home’s terms of trade. In equilibrium, this causes a fall in labor demand: first, higher wage claims lead to a direct fall in Home labor demand. Second, the increase in the Home’s relative prices induces a reduction of the demand for Home goods. As a consequence, both Home labor demand and labor income falls. Foreign households, however, have to give more of their goods in exchange for Home goods. This yields ceteris paribus a higher Home consumption-labor ratio and thereby higher welfare. As a result, domestic policymakers “beggar-thy-neighbor” by inducing the
Foreign households to work more and thus to worsen Foreign consumption-labor trade-off. In the symmetric equilibrium, the consequence are higher distortive Home and Foreign nominal interest rates that reduce aggregate output and thereby aggregate consumption as labor is supplied at an inefficiently low level. Again, this holds true for both environments: for flexible wages in all instances and under sticky wages on average. Importantly, the incentive to unilaterally deviate from the globally optimal Friedman rule is increasing with the degree of openness. That is, the larger the economic interdependence through trade linkages between the two countries, the higher is the incentive to ”beggar-thy-neighbor” and to unilaterally manipulate the terms of trade by increasing the domestic interest rate. As the welfare implications are of first-order, the consequences of the failure to cooperate on the nominal interest rates become more severe the more interdependent the countries are.

1.5.2 The Money Supply

The next Proposition shows the equilibrium money supply when national policymakers act independently.

**Proposition 1.5** The unique Nash equilibrium of non-cooperatively set money supply rules is

\[ \mu_{Nash}^{aw} = \frac{(1 - \rho)}{\chi} \quad \text{and} \quad \mu_{Nash}^{ad} = (1 - \rho)(1 - \gamma) \left( \frac{\chi + (1 - \gamma)\gamma}{\chi^2} \right), \]

and \( \mu_{Nash}^{\ast} = \mu_{Nash}^{aw} \) and \( \mu_{Nash}^{\ast} = \mu_{Nash}^{ad} \) by symmetry. Furthermore,

\[ \mu_{Nash}^{aw} = \mu_{Opt}^{aw} \quad \text{and} \quad \mu_{Nash}^{ad} = \begin{cases} < \mu_{Opt}^{ad} & \text{if } \rho > 1 \text{ and } 0 < \gamma < 1, \\ > \mu_{Opt}^{ad} & \text{if } \rho < 1 \text{ and } 0 < \gamma < 1, \\ = \mu_{Opt}^{ad} & \text{if } \rho = 1 \text{ or } \gamma \in \{0, 1\}. \end{cases} \]

**Proof.**

See Appendix.

In case of aggregate productivity shocks, the Nash solution does not differ from the optimal solution. Aggregate shocks shift consumption expenditures in a way that is common to both countries. Therefore, the policy targets coincide. Why, however, is it that national policymakers do not use aggregate productivity shocks as a stochastic anchor to manipulate the terms of trade? Indeed, they face the incentive to do so. Nevertheless, the only friction that prevents from individual optimality is wage rigidity. Therefore, the only incentive to manipulate the terms of trade is to achieve the optimal consumption-labor trade-off. Hence, the optimal policy response to aggregate productivity shocks also solves the Nash
problem.

This is no longer the case for asymmetric productivity shocks. Recall that the optimal cooperative money supply management faces a trade-off between targeting the domestic gap between the marginal rate of substituting consumption and labor and the international gap between Home and Foreign marginal consumption utilities. Proposition 1.5 shows that as long as imperfect international risk sharing is a matter of concern, i.e. as long as $\rho \neq 1$ or $0 < \gamma < 1$, both single national monetary authorities unilaterally deviate from the jointly optimal response and thereby from the optimal trade-off. The reason is again that self-oriented policymakers try to rather close the domestic gap to improve the domestic consumption-labor trade-off. In fact, in the non-cooperative Nash equilibrium, authorities set money supply to react more actively to idiosyncratic shocks than it is globally efficient. Hence, from the discussion of the optimal trade-off follows that non-cooperative policymakers attach more value to targeting the domestic gap at the expense of a widening of the international gap. The immediate consequence is that the unilateral deviation “beggars-thy-neighbor”.

1.6 A Numerical Example

A numerical example shall illustrate the quantitative importance of international monetary policy cooperation and evaluate the different welfare implication of the nominal interest rate policy and the money supply management. To keep the comparison to the literature simple, I take parameters values from Obstfeld and Rogoff (2002a): $\sigma_a = 0.01$ and $\nu = 1$. Moreover, two possible trade scenarios are considered: a low-trade scenario ($\gamma = 0.2$) which corresponds to an import over GDP ratio of 10% and high-trade scenario ($\gamma = 0.6$) which corresponds to an import over GDP ratio of 30%.

The gains from international monetary policy coordination are reported in Table 1.2. For varying values of $\rho$, three numbers are stated: $\xi$ denotes the necessary percentage increase in consumption so that households are indifferent between international monetary policy coordination and independent policy conduct. By the structure of the model, the overall measure can be further decomposed into $\xi_R$ for nominal interest rate policy and $\xi_M$ for money supply management.

Independent of the degree of openness $\gamma$, welfare gains from coordinating the nominal interest rate are decreasing in $\rho$. For larger values of $\rho$, the inter-temporal substitution elasticity is decreasing. Consequently, as households attach greater importance to actual consumption, the higher non-cooperative nominal interest rate and hence the intertemporal costs of holding labor income as cash doesn’t preponderate too much. Importantly, the gains are, however, increasing in the extend to which the two countries are linked through trade. Clearly, the more important the terms of trade are for domestic households, the more prone are domestic policymakers to manipulate relative nominal goods prices in order to make
1.7 Conclusion of Chapter 1

Within the large body of the literature on international monetary policy coordination, the broad consensus is that gains from policy coordination are small if not negligible. This view is corroborated by theoretical considerations that focus on the coordination of international

<table>
<thead>
<tr>
<th>Low-trade scenario ($\gamma = 0.2$)</th>
<th>High-trade scenario ($\gamma = 0.6$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Different values for $\rho$</td>
<td>Different values for $\rho$</td>
</tr>
<tr>
<td>$\rho = 0.5$ $\rho = 1$ $\rho = 2$ $\rho = 4$ $\rho = 8$</td>
<td>$\rho = 0.5$ $\rho = 1$ $\rho = 2$ $\rho = 4$ $\rho = 8$</td>
</tr>
</tbody>
</table>

Welfare Measure (compensating % change in consumption)$^a$

<table>
<thead>
<tr>
<th>$\xi^R$</th>
<th>1.674</th>
<th>0.537</th>
<th>0.155</th>
<th>0.042</th>
<th>0.011</th>
<th>11.566</th>
<th>5.831</th>
<th>2.378</th>
<th>0.811</th>
<th>0.243</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\xi^M$</td>
<td>0.151</td>
<td>0.062</td>
<td>0.158</td>
<td>0.229</td>
<td>0.007</td>
<td>0.006</td>
<td>0.019</td>
<td>0.037</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\xi$</td>
<td>1.827</td>
<td>0.537</td>
<td>0.217</td>
<td>0.200</td>
<td>0.240</td>
<td>11.574</td>
<td>5.831</td>
<td>2.384</td>
<td>0.830</td>
<td>0.280</td>
</tr>
</tbody>
</table>

Table 1.2: Gains from International Monetary Policy Coordination.

$^a$ Following Lucas (1987, 2003), $\xi$ denotes the percentage compensation of consumption so that $U((1 + \xi)C^A, L^A) = U(C^B, L^B)$, where A and B are two different policies. Moreover, the decomposition of the equilibrium welfare yields the measure $\xi^R$ for the welfare implication of different nominal interest rates and $\xi^M$ for different money supply managements.

national residents better off. In contrast, welfare gains from coordinating the money supply management are increasing in absolute deviation of $\rho$ from unity because thereby the consequences of the lack of international risk sharing are more pronounced. Gains are, however, smaller in the high-trade scenario because international consumption risk sharing is achieved to a larger extend through international trade on goods markets and consequently the relative importance of attenuating the lack of international asset markets is decreasing. Notably, as already indicated by Proposition 1.1, since the nominal interest rate effects are of first order, welfare gains are of first order. In the high-trade scenario, gains from coordinating the nominal interest rate are up to 4 orders of magnitude larger than gains from coordinating macroeconomic stabilization through money supplies. These results are consistent on the one hand with the broad literature on policy coordination that concentrated on stabilization issues as in Obstfeld and Rogoff (2002a), Pappa (2004), and Canzoneri et al. (2005), and on the other hand with the very recent considerations of the optimal nominal interest rate policy in open economies in Cooley and Quadrini (2003) and Arseneau (2007).

1.7 Conclusion of Chapter 1

Within the large body of the literature on international monetary policy coordination, the broad consensus is that gains from policy coordination are small if not negligible. This view is corroborated by theoretical considerations that focus on the coordination of international
monetary stabilization policies. While all these contributions deserve their very merits for revealing important insights, the finding that the quantitative importance is fairly small is not surprising: stabilization policies target the variability of the allocation and monetary policy focuses on short-run demand side management. They are hence generically of second order. However, to plagiarize Lucas (2003), the "potential for welfare gains from better long-run, supply side policies exceeds by far the potential from further improvements in short-run demand management".

The contribution of the present paper is to take up this proposition and introduce it into the context of international monetary policy coordination. The arguments made are formalized within a simple dynamic stochastic two-country model with preset wages and cash-in-advance restrictions. In this environment, monetary authorities can manipulate the terms of trade by conducting a general short-run monetary policy using both the inflation target and the money supply. On the one hand, money supply affects the allocation and the terms of trade ex post by altering relative nominal spending and thereby the nominal exchange rate. In this respect, monetary policy is used in the traditional way so as to stabilize macroeconomic fluctuations by fine tuning spending flows. On the other hand, the inflation target effectively controls the nominal interest rate. Thus, it affects the allocation and the terms of trade ex ante by altering the households’ wage setting conditions and thereby goods prices. In this respect, monetary policy changes the incentives to work and might cause inefficiencies on the supply side. It is demonstrated that the resulting welfare implications of inflation target policy are of first order whereas the welfare implications of money supply management are of second order.

The important consequence - and the central message of this analysis - is that gains from coordinating money supply management are generically of second order if they focus on stabilization issues. In contrast, gains from preventing excessively high inflation rates and hence nominal interest rates resulting from self-interested strategic manipulations of the terms of trade are of first order and hence expected to be of higher orders of magnitude. A numerical example of the simple model already indicates that welfare gains from globally optimal monetary policy conduct might be substantial. The present analysis of a more general monetary policy conduct in interdependent economies hence leads to the conclusion that gains from policy coordination might have been to a large extent underestimated.
Chapter 2

Optimum Policy Domains in an Interdependent World

Throughout the last couple of decades the world has experienced a strong and steady increase in the economic interdependence among national economies. Accordingly, national macroeconomic policies are also subject to a steady increase in mutual interdependence. As a consequence, because conflicting national policy objectives might lead to international disagreements, the necessity of the international coordination of macroeconomic policies has become a central postulation within both the public as well as the academic debate: Countries should coordinate macroeconomic policies in order to incorporate externalities of national policies on other countries and - more importantly - to overcome inefficiencies arising from strategic considerations to exploit the international transmission of national macroeconomic policies in one country’s own favor. In the international macroeconomics literature, these game-theoretic arguments formed the basis for the theoretical rationale in favor of policy coordination. Following the natural separation of national macroeconomic policymaking into monetary and fiscal policy, the academic literature evolved along two major strands: Beginning with Hamada (1974, 1976), the larger body of the literature focuses on the analysis of monetary policy.\(^1\) The role of fiscal policy is largely ignored in these models. The other strand of the literature as in Hamada (1986), Kehoe (1987), and Chari and Kehoe (1990) analyzes fiscal policy where the international monetary policy regimes are taken as given.\(^2\) By uncoupling the analysis of the international monetary policy

\(^1\)The most prominent contributions to the early stage of the literature are Oudiz and Sachs (1984), Rogoff (1985), and Canzoneri and Gray (1985). More recent contributions include Obstfeld and Rogoff (2000, 2002a), Clarida et al. (2002), Devereux and Engel (2003), Cooley and Quadrini (2003), Benigno (2001), Benigno and Benigno (2003, 2006), Pappa (2004), Liu and Pappa (2005), Gali and Monacelli (2005), Corsetti and Pesenti (2005), and Evers (2007a), where this list is far from complete. Overviews are provided by Cooper (1985), Canzoneri and Henderson (1991), Persson and Tabellini (1995), and Canzoneri et al. (2005).

domain and the fiscal policy domain, however, a crucial question cannot be addressed: How does the international coordination of only a part of national macroeconomic policies change the strategic behavior of the independently conducted remaining part of national policies. Put differently, how does independent national fiscal policy, for example, shift in response to the international coordination of monetary policy?

In this paper, I seek to close the gap in the literature and ask how the strategic incentives shift in one policy field where authorities still act independently when national policies move to coordination within the other policy field. In particular, I argue that the analysis of international policy coordination requires to include both the monetary side and the fiscal side because either monetary or fiscal policy coordination alone does not suffice to extract gains from international coordination of national macroeconomic policies. The intuition for this proposition is straightforward: The necessity of international policy coordination is based on the fact that national policy entities can exert monopoly power on macroeconomic variables in general and the terms of trade in particular. Crucially then, both monetary and fiscal policy can be used to strategically manipulate the terms of trade. Hence, the coordination of a single policy stance through an international agreement still leaves room for national authorities to still unilaterally manipulate the terms of trade by means of different policy instruments. As a consequence, potential gains from, say, international monetary policy coordination are squandered or may even turn negative by letting the fiscal policy instruments be chosen non-cooperatively.

The economic setup to address this question builds on the standard framework of the New Open Economy Macroeconomics as in Corsetti and Pesenti (2001) and Obstfeld and Rogoff (2002a) and augments the model in Evers (2007a) with fiscal policy. It is a simple stochastic two-country general equilibrium model without capital. Goods prices are assumed to be perfectly flexible, but workers have to set monopolistic wages one period in advance. Furthermore, households make their consumption decision in face of a cash-in-advance restriction. In this environment, monetary authorities can affect the terms of trade by conducting a general short-run monetary policy resorting to the credibly announced expected future money supply and state-dependent, expectationally consistent deviations from the expected money supply: By means of the announcement of the expected future money supply, monetary authorities can directly control the nominal interest rate. This

enables the monetary authority to alter the worker’s wage setting condition and thereby the goods prices ex ante. The period’s actual money supply alters the nominal exchange rate and thus the terms of trade ex post.\footnote{Ireland (1996) already recognized that monetary policy can be conducted by using both the expected money growth rate and the state-dependent deviations from the expected level. Adao et al. (2003) take up his point and analyze general short-run monetary policies where the authors directly argue by means of the nominal interest rate as controlled by the expected level of money supply and the actual money supply as the state-dependent deviation from the expected level. I follow Adao et al. for convenience and argue directly in terms of the nominal interest rate and the money supply, too. That monetary authorities indeed dispose of two distinct policy instruments and that they control the nominal interest rate and the money supply independently is demonstrated in Evers (2007a) and in Chapter 1 of this thesis where I introduce the generalization of the short-run monetary policy conduct into a standard NOEM framework as in Obstfeld and Rogoff (2002a) and Devereux and Engel (2003).} Fiscal authorities, on their part, can influence the terms of trade ex ante by using distortionary taxes on labor income and ex post by changing distortionary taxes on consumption. The key property of the different policy instruments is that the labor income tax and the nominal interest rate on the one hand and the consumption tax and money supply on the other hand are perfectly substitutable national policy instruments. To be specific, when authorities want to exert, say, a positive impact on the workers’ wage setting so as to raise the expected terms of trade ex ante, they can do so either by increasing the nominal interest rate which is implicitly achieved by raising expected inflation and comes along with an increase in expected inflation tax on labor income. The same positive impact on the expected terms of trade can be induced directly by an increase in the labor income tax. Both interventions cause the workers to ask higher nominal wages. This, in turn, implies an increase in goods prices and leads ceteris paribus to an appreciation of the terms of trade. Factually, only the compound effect of national policy intervention matters for the consequences on the workers’ optimal wage setting. The same is true for the ex post interventions to the nominal exchange rate. Changes in the money supply induce changes in relative international nominal goods demand and hence in the nominal exchange rate. In fact, the same movement in nominal spending can be achieved by adjusting the consumption tax. Thus, the identical ex post innovation to the nominal exchange rate and thereby to the terms of trade can be attained by fiscal policy.

The important consequence is that the joint monetary and fiscal policy conduct determines the national impact on the terms of trade. Hence, taking up the arguments developed in favor of policy coordination and seeking an international cooperation of either monetary or fiscal policy alone will only leave room for policymakers to still follow national interests by exploiting their monopolistic power on the terms of trade via the respective other policy instruments.

In the next section, the model is described and the equilibrium conditions are derived. I discuss the equilibrium allocation and its distribution in Section 2.2. In this section, it
is also shown that respective national monetary and fiscal policy instruments are perfect
substitutes and derive the national policymakers’ objective in closed form. In Section 2.3,
I consider optimal public policy coordination and the Nash equilibria of independently set
national policy interventions. The analysis of cooperating monetary authorities under fiscal
independence is carried out in Section 2.4. In Section 2.5 I give a numerical example in order
to assess the relevance of policy coordination. In Section 2.6 I conclude. The derivations of
the equilibrium and the of the results are delegated to an Appendix.

2.1 The Model

In the model, the world consists of two countries, denoted as Home (H) and Foreign (F).
Each country produces two types of consumption goods, one that is traded with foreigners,
and one that is demanded only within the country. In all, there are thus four goods. These
goods are produced with labor as the only input factor. Furthermore, goods are traded in
perfectly competitive markets and at perfectly flexible prices. Both countries are populated
by a continuum of households with size one. Each household is characterized by a specific
variety of labor of which it is the monopolistic supplier. Households can choose their wages
individually. However, they have to be set one period in advance. In order to identify a
particular household, the household will be indexed by a superscripted $i$.4

2.1.1 Firms

Within countries, technologies to produce the tradable and the non-tradable goods are
assumed to be identical:

$$Y_{j,s} = A_s L_{j,s} \quad \text{with} \quad L_{j,s} = \left( \int_0^1 L_{j,s}^\frac{\theta-1}{\theta} dt \right)^\frac{\theta}{\theta-1},$$

(2.1)

where $\theta > 1$. For both sectors, a typical firm $j$ producing either the Home tradable good
($HT$) or the Home non-tradable good ($HN$) employs labor that is composed according to
a CES aggregator over all domestic varieties of labor. The aggregate productivity of labor
$A_s$ in (2.1) is subject to shocks. The associated demand for a specific type of labor is

$$L_s^i = \left( \frac{W_s^i}{W_s} \right)^{-\theta} L_s,$$

(2.2)

where $L_s$ is Home aggregate demand for labor. $W_s^i$ denotes the monopolistic money wage
claimed by household $i$ and $W_s = \left( \int_0^1 W_s^{(1-\theta)} dt \right)^\frac{1}{1-\theta}$ defines the aggregate home wage
level. Foreigners share an analogous aggregation technology and therefore the corresponding
equations apply.

4Superscripts denote where a variable belongs to, foreign variables are distinguished by an asterisk *. Subscripts identify the characteristics of that variable, e.g. whether it’s the home non-tradable or the foreign tradable good.
2.1. The Model

2.1.2 Households

Households within a country have identical preferences over consumption and labor effort. They are described by

\[ U_i^t = E_t \sum_{s=t}^{\infty} \beta^{s-t} u_i^s, \quad \text{where} \]

\[ u_i^s = \left( \frac{C_i^{1-\rho} - 1 - \frac{1}{\nu} L_i^s}{1 - \rho} \right) \]

with

\[ C_i^s = C_{iT,s}^\gamma C_{iHN,s}^{1-\gamma} \]

where \( 0 < \beta < 1, \rho > 0, \nu \leq 1, \) and \( 0 \leq \gamma \leq 1. \) The real consumption index \( C_i^s \) is given by a CES aggregator over the Home non-tradable good \( C_{iHN,s}^s \) and a composite of tradable goods \( C_{iT,s}. \) The elasticity of substitution is equal to one. The composite of tradable goods \( C_{iT,s} \) is given by a CES aggregator over the Home tradable good \( C_{iHT,s}^s \) and the Foreign tradable good \( C_{iFT,s}^s, \) where the elasticity of substitution is equal to one, too. Foreign households have the same preferences over tradable goods but differ with respect to their own non-tradable good.

Asset Markets

Households can trade nominal bonds with other households within borders. However, households cannot trade any assets internationally. It turns out that incomplete international risk sharing has an important implication: in contrast to most other contributions that assume households to have access to a full set of state-contingent claims, Obstfeld and Rogoff (2002a) and Evers (2007a) point out that the lack of private consumption risk sharing indeed leads to the non-optimality of replicating flexible wage and price allocation. In particular, they identify the optimal nominal exchange rate management to face the trade-off between replicating the flexible wage allocation and the efficient consumption risk sharing. It is this trade-off on which national policymakers will seek to manipulate the terms of trade through ex post market interventions to improve domestic welfare that is absent under complete asset markets.

Individual Budget and Cash Constraints

Household \( i \) starts out in period \( s \) with nominal wealth \( W_i^s \). First, the asset markets open. Household \( i \) receives money transfers \( X_i^s \), decides about nominal domestic bond holdings \( B_i^s \) that repay \( R_s B_i^s \) at a gross nominal return \( R_s \) in next period, and about cash holdings \( M_i^s \). The asset market constraint reads

\[ M_i^s + B_i^s \leq W_i^s + X_i^s. \] (2.4a)

Thereafter, the goods markets open. Purchases of consumption goods that are taxed at a rate \( t_{C,s} \) must not exceed initial cash holdings, ie.

\[ (1 + t_{C,s}) P_s C_i^s \leq M_i^s. \] (2.4b)
At the end of the period, household $i$ receives net wage earnings $(1 - t_{L,s})W^i_sL^i_s$, where $t_{L,s}$ denotes a proportional tax on labor income, and a lump-sum transfer $T^i_s$ that rebates the receipts of consumption and labor income taxes. Thus, the nominal wealth at the beginning of the next period is

$$W^i_{s+1} = M^i_s + R^i_sB^i_s - (1 + t_{C,s})P^i_sC^i_s + (1 - t_{L,s})W^i_sL^i_s + T^i_s.$$  \hfill (2.4c)

**Optimal Decisions**

The household’s problem is to maximize its expected lifetime utility (2.4) by deciding over bond and cash holdings, consumption, and their monopolistic wages subject to the constraints (2.4a-2.4c), the demand for their specific type of labor (2.2), and subject to the constraint that they have to set wages one period in advance. Optimal bond holdings implies the intertemporal Euler equation,

$$\frac{1}{R^s} = \beta E^s_s \left( \frac{C^{s+1}_{s+1}}{C^s} \right)^{-\rho} \frac{P^s_s(1 + t_{C,s})}{P^{s+1}_{s+1}(1 + t_{C,s+1})}. \hfill (2.5a)$$

The net nominal interest rate is assumed to be strictly positive and reaches zero only in the limit. Consequently, the cash constraint is binding and optimality implies that the household uses all its initial cash for consumption goods purchases. Individual optimization yields for any person $i$ the standard composition of consumption between the tradable goods basket and the non-tradable good and between Home and Foreign tradable goods. The corresponding Home consumption-based price indices are given by $P^s_s = P^\gamma_T^s P^{{(1-\gamma)}}_{HN,s}$ and $P_T^s = P^{1/2}_{HT,s} P^{1/2}_{FT,s}$. The optimal money wage claim is constrained to be set one period in advance. Optimality requires the money wage posted for period $s$ to be set such that the expected marginal utility loss implied by labor effort equals the expected marginal utility from consumption in period $s+1$ that additional labor income in $s$ allows but which cannot be spent before $s+1$. Making use of the intertemporal Euler equation, we end up with

$$W^i_s = \frac{\theta}{\theta - 1} \frac{E^{s-1}_s \left( L^{i\nu}_s \right)}{E^{s-1}_s \left( \frac{(1-t_{L,s})}{(1+t_{C,s})R^s_s} \frac{L^i_s}{P^i_s C^i_s} \right)}$$ \hfill (2.5b)

as the optimal wage claim. For Foreign households, the corresponding equations apply.

**2.1.3 Governments’ Budget Constraints**

At the beginning of a period, national governments make money transfers to the households. At the goods markets, they collect state-contingent proportional consumption taxes. At the
end of the period, national governments collect state-contingent labor income taxes and rebate the receipts of all taxes lump-sum to the households. The two associated constraints for the Home government read

\[
\int_0^1 M_i^s di = \int_0^1 M_{i-1}^s di + \int_0^1 X_i^s di \quad (2.6)
\]

and

\[
t_{C,s} \int_0^1 C_i^s di + t_{W,s} \int_0^1 W_i^s L_i^s di = \int_0^1 T_i^s di,
\]

respectively. Money supply and state-dependent tax rates will be set according to policy rules that we specify later in the discussion of monetary and fiscal policy conduct. The Foreign policy authorities share the same budget constraints.

### 2.2 Equilibrium in Closed Form

All households within a country are identical except for their own special type of labor. Specifically, they are assumed to start out with identical initial nominal wealth. Hence, as these households face ex ante the same optimality conditions and since there is no asymmetric redistribution of wealth among households within a country on behalf of the governments at all, households will take identical actions. For the rest of the analysis the superscript \( i \) is dropped.

#### 2.2.1 Equilibrium Prices and the Terms of Trade

By perfectly competitive goods markets, prices for home and foreign goods are

\[
P_{HT,s} = P_{HN,s} = \frac{W_s}{A_s} \quad \text{and} \quad P_{FT,s} = P_{FN,s} = \frac{W_s^*}{A_s},
\]

respectively. In (2.7), \( W_s \) and \( W_s^* \) denote the preset Home and Foreign wage levels which cannot be adjusted to any period innovations. Therefore, the realization of the productivity shocks fully determines goods prices when wages are set the period before. The terms of trade, \( ToT_s \), are defined as the price of home imports in terms of home exports, i.e. in money prices \( ToT_s = \left( \frac{P_{HT,s}}{E_s P_{FT,s}} \right) \), where \( E_s \) denotes the nominal exchange rate. In terms of relative wage levels, we can restate the terms of trade as

\[
ToT_s = A_s^* \left( \frac{W_s}{E_s W_s^*} \right).
\]

The real exchange rate, \( RER_s \), is then by the consumption-based price indices \( RER_s = ToT_s^{(1-\gamma)} \). The implication of effectively predetermined prices is that only the nominal exchange rate alters the current terms of trade and thereby relative consumption spending.

#### 2.2.2 Ex Post Equilibrium Allocation

As a consequence, given monetary and fiscal policies, the ex post realized equilibrium allocation is uniquely determined by the cash-in-advance constraints and the market clearing
conditions for goods and financial assets. To be more precise, because the current account must be balanced (as there is no international trade in financial assets) and Home and Foreign households equally split their expenditures between the Home and the Foreign tradable goods, the market clearing of tradable goods implies $P_{H,s} Y_{HT,s} = E_s P_{F,s}^* Y_{FT,s}^*$, ie. both countries earn the same revenue on tradable goods production. Moreover, producer currency pricing and identical preferences over tradable goods imply that the law of one price holds for tradable goods baskets. Consequently, home and foreign households will consume the same amount of tradable goods in equilibrium, ie. $C_{T,s} = C_{T,s}^*$. If we follow Obstfeld and Rogoff (2000) and express overall consumption expenditures in terms of tradable goods consumption, ie.

$$Z_s = \frac{P_s}{P_{T,s}} C_s \quad \text{and} \quad Z_s^* = \frac{P_s^*}{P_{T,s}^*} C_s^*, \quad (2.9)$$

we can use Home and Foreign optimal divisions of tradable and non-tradable goods consumption to find the equilibrium ratio of overall Home and Foreign consumption spending in terms of tradable goods to be unity, ie. $Z_s = Z_s^*$. Next, asset market clearing requires nominal bonds to be in zero net supply. Since households are identical in wealth, there will be zero net trade in nominal bonds. Consequently, households’ nominal wealth consists of equilibrium cash holdings only. Furthermore, because nominal consumption spending is given by the cash-in-advance constraint, goods market clearing and the equalized trade balance requires the nominal exchange rate to adjust so that consumption spending in terms of tradable goods is equalized across countries. Since the equilibrium nominal consumption spending, in turn, depends on the money supply and the consumption taxes, it turns out to be very useful to express money supply and taxes on consumption expenditures as a compound policy intervention.

**Definition 1 (National public policy intervention to ex post consumption spending)**

Let $I_{C,s}$ and $I_{C,s}^*$ denote the respective compound national public policy intervention to goods markets via the cash-in-advance constraint with

$$I_{C,s} = \frac{M_s}{1 + t_{C,s}} \quad \text{and} \quad I_{C,s}^* = \frac{M_s^*}{1 + t_{C,s}^*}, \quad (2.10)$$

The ratio of Home and Foreign nominal consumption spending which are determined by the cash holdings therefore delivers the nominal exchange rate. With Definition 1 it follows that

$$Z_s = Z_s^* \quad \text{and} \quad E_s = \frac{I_{C,s}}{I_{C,s}^*}, \quad (2.11)$$

With a uniquely determined nominal exchange rate, consumer prices $P_s$ and $P_s^*$ and the real exchange rate are given. Consequently, by the cash-in-advance constraint, Home and Foreign equilibrium consumption levels are given. Hence, goods market clearing determines
2.2. Equilibrium in Closed Form

Home | Foreign
--- | ---
Consumption | \( C_s = Z_s(ToT_s)^{-\frac{1}{2}(1-\gamma)} \)

\( C^*_s = Z_s(ToT_s)^{\frac{1}{2}(1-\gamma)} \)

Output | \( Y_s = Z_s(ToT_s)^{-\frac{1}{2}} \)

\( Y^*_s = Z_s(ToT_s)^{-\frac{1}{2}} \)

Labor | \( L_s = A^{-1}_sZ_sToT_s^{\frac{1}{2}} \)

\( L^*_s = A^{-1}_sZ_sToT_s^{\frac{1}{2}} \)

Table 2.1: The ex post period equilibrium allocation given monetary and fiscal policy.

the equilibrium employment levels because labor is fully demand determined. Table 2.1 summarizes Home and Foreign ex post equilibrium levels of consumption, output and labor. As a result, the ex post equilibrium allocation is uniquely determined for given common consumption level \( Z_s \) and the terms of trade \( ToT_s \). It is important to observe that the differences between Home and Foreign consumption and output are entirely captured by the terms of trade. Consequently, the impact of a shift in the terms of trade on Home and Foreign variables are orthogonal. It proves useful to follow the method proposed by Aoki (1981) to express these variables in terms of world averages and differences. Accordingly, let subscript ”w” denote the ”world” average level of a variable which is the geometric mean of Home and Foreign variables and let subscript ”d” denote the ”difference” component which is the ratio of Home over Foreign variables.\(^6\) The equilibrium consumption spending in terms of the tradable goods \( Z_s \) and the terms of trade \( ToT_s \) can then be written as

\[
Z_s = \frac{A_{w,s}}{W_{w,s}}I_{C_{w,s}} \quad \text{and} \quad ToT_s = \left( \frac{W_{d,s}}{A_{d,s}I_{C_{d,s}}} \right)^2.
\] (2.12)

2.2.3 Ex Post Public Policy Intervention to Consumption Spending: Money Supply and Consumption Taxes

The innovations to the ex post equilibrium allocation can be summarized by the deviations of spending \( Z_s \) and the terms of trade \( ToT_s \) from their expected values. Letting small letters denote logs, ie. \( x = \log X \), and letting a hat denote the deviation from the expected level, ie. \( \hat{x} = x - \bar{x} \), it follows that

\[
\hat{z}_s = \hat{a}_{w,s} + \hat{i}_{C_{w,s}} \quad \text{and} \quad \hat{tot}_s = -2\left( \hat{a}_{d,s} + \hat{i}_{C_{d,s}} \right).
\] (2.13)

\(^6\)For a Home variable \( X \) and a Foreign variable \( X^* \), the decomposition in levels is \( X = X_w X_d \) and \( X^* = \frac{X_w}{X_d} \) where \( X_w = (XX^*)^{\frac{1}{2}} \) and \( X_d = (\frac{XX^*}{X_w})^{\frac{1}{2}} \). The exponents are relative country sizes which in our case are identical.
When wages are predetermined and given the realization of productivity shocks, the ex post allocation can only be altered by changes in Home and Foreign money supplies or consumption tax rates as captured by $\hat{i}_{Cw,s}$ and $\hat{i}_{Cd,s}$. The world consumption spending $Z_s$ can only be changed ex post by altering the average money supply or the average consumption tax rate which reflects in equilibrium a one-to-one change in real balances available for consumption purchases. The terms of trade, in turn, can only be changed ex post by the nominal exchange rate which is determined by relative money supplies and relative consumption tax rates as in (2.11). Hence, the terms of trade can only be moved by $\hat{i}_{Cd,s}$ ex post.

Importantly, the effects of changes in the national money supply and the consumption tax rate on the ex post allocation are perfectly equivalent. In particular, an expansion of Home money supply leads to an increase in cash holdings dedicated to consumption purchases and hence to an increase in consumption spending. An equivalent change in net spending is achieved, however, by a reduction of the consumption tax rate. The same applies for nominal exchange rate and therefore for the terms of trade. A relative increase in Home money supply over Foreign money supply causes a depreciation of the nominal exchange rate. The determination of the nominal exchange rate by relative money supplies is standard in the NOEM literature (compare Obstfeld and Rogoff (1995, 1996)). However, an equivalent manipulation to the nominal exchange rate can be achieved by lowering Home consumption taxes relative to Foreign consumption taxes since this increases relative Home consumption spending relative to Foreign consumption spending, too. Goods market clearing then requires the nominal exchange rate to depreciate, as in case of a money supply increase. As a result, the following holds true:

**Lemma 2.1** The money supply and the consumption tax are perfectly substitutable national policy instruments. Relevant for the consumption spending and the exchange rate determination is only the compound intervention to consumption spending.

Consequently, since the ex post innovations depend only on productivity shocks and the ex post policy interventions to spending, the variance and covariance terms of the equilibrium allocation can be solved for once the joint distribution of productivity shocks and policy interventions is specified. This allows the explicit calculation of the ex ante expected levels of the equilibrium allocation and hence the preset nominal wage levels.

### 2.2.4 Distribution of the Ex Post Equilibrium Allocation

Productivity shocks are assumed to be iid log-normal with the following properties: $Ea = Ea^*$ and $Var(a) = \sigma_a^2 = \sigma_{a^*}^2 = Var(a^*)$. For ease it is assumed that $Ea = 0$. In terms of world and difference components it follows for the variances and covariances that

$$Var(a_s) = \sigma_a^2 = \sigma_{aw}^2 + \sigma_{ad}^2 \quad \text{since} \quad Cov(a_{w,s}, a_{d,s}) = \sigma_{aw,s,ad,s} = 0.$$
2.2. Equilibrium in Closed Form

By *iid* productivity shocks and given that public policies are stationary, the optimal choices of households and firms are stationary. As a consequence, the equilibrium of the infinite horizon setup is simply a repetition of a single period. For the rest of the paper, we therefore leave out time-subscripts for convenience. Moreover, if Home and Foreign public policy interventions to consumption spending $i_C$ and $i^*_C$ and productivity shocks are jointly log-normal, the distribution of the ex post equilibrium allocation turns out to be log-normal, too.

### 2.2.5 Ex Ante Public Policy Intervention to Wage Setting: Nominal Interest Rate and Labor Income Tax

In equilibrium, households within a country choose the same wage levels. Therefore, Home and Foreign equilibrium wage levels $W$ and $W^*$ are given by the aggregate versions of individual optimal wage setting as in (2.5b) and its Foreign counterpart. Recall that the nominal interest rate on the one hand and the labor income tax on the other hand affect the households’ optimal wage setting in the same way. Similar to the first definition, we express the nominal interest rate and the labor income tax relative to the consumption tax as a compound policy variable.

**Definition 2 (National public policy intervention to ex ante wage setting)** Let $I_{W,s}$ and $I^*_{W,s}$ denote the respective compound national public policy intervention to wage setting with

$$I_W = \frac{1 + t_C}{1 - t_W} R \quad \text{and} \quad I^*_W = \frac{1 + t^*_C}{1 - t^*_W} R^*.$$  \hfill (2.14)

The aggregate Home and Foreign wage levels are thus determined by

$$W = \frac{\theta}{\theta - 1} E\left(\frac{L^\nu}{1 \frac{L}{t_W} p C^\sigma}\right) \quad \text{and} \quad W^* = \frac{\theta}{\theta - 1} E\left(\frac{L^*\nu}{1 \frac{L^*}{t^*_W} p^* C^*_\sigma}\right).$$  \hfill (2.15)

Consequently, the equilibrium wage levels can then be explicitly solved for in terms of the distribution of the ex post equilibrium allocation. The determination of the ex post allocation, however, yields an important implication for nominal interest rates and the distorting labor income taxes: These two policy instruments are completely ineffective if it is to alter the equilibrium allocation ex post. Consequently, the endogenous variability of consumption spending and the terms of trade are independent of Home and Foreign nominal interest rates and labor income taxes. Nevertheless, they do have an important impact ex ante because they change the environment for the households to set nominal wage claims as they shift expected levels of consumption spending and the terms of trade.

In particular, Home and Foreign aggregate wage levels in (2.15) can be used to express the expected consumption spending and the terms of trade by means of the expected levels of the respective policy choices and the variances and covariances of endogenous and exogenous...
variables. Accordingly, the expected consumption spending measured by tradable goods reads

\[ E_z = -\frac{1}{\lambda} \left( i_{W_w} + \ln \frac{\theta}{\theta - 1} + \Sigma_z \right) \]  

(2.16)

where \( \lambda = \nu - (1 - \rho) > 0 \). The addend \( \Sigma_z \) collects the endogenous and exogenous variance and covariance terms of the equilibrium allocation\(^7\), ie.

\[ \Sigma_z = \frac{1}{2} (\nu^2 - (1 - \rho)^2) \sigma_z^2 + \frac{1}{8} (\nu^2 - (1 - \rho)^2 (1 - \gamma)^2) \sigma_{tot}^2 - \frac{\nu^2}{2} (2 \sigma_{z,a_w} - \sigma_{tot,a_d}) + \frac{\nu^2}{2} (\sigma_{a_w}^2 + \sigma_{a_d}^2). \]

In equilibrium, expected consumption spending can be raised ex ante either by lowering expected world nominal interest rates or by cutting the expected world labor income taxes. The expected terms of trade are given by

\[ E_{tot} = \frac{2}{\gamma} (i_{W_d} - \Sigma_{tot}) \]  

(2.17)

where \( \gamma = \nu - (1 - \rho)(1 - \gamma) > 0 \), and

\[ \Sigma_{tot} = \frac{1}{2} (\nu^2 - (1 - \rho)^2 (1 - \gamma)) \sigma_{z,tot} + \frac{\nu^2}{2} (2 \sigma_{z,a_d} - \sigma_{tot,a_w}). \]

The terms of trade depend on differences in both the nominal interest rates and the labor income taxes. A relative rise of the expected Home nominal interest rate or Home labor income taxes leads to higher Home wage claims and hence higher Home goods prices. Consequently, the expected terms of trade rise.

It is important to observe that similar to the public policy interventions to the ex post spending the effects of expected national nominal interest rate policy and the labor income taxation on the equilibrium allocation ex ante are perfectly equivalent. For instance, monetary policy can be used to increase the expected Home nominal interest rate by means of an (credible) announcement to increase the money supply in the following periods. Since labor income becomes available for consumption only with a delay of one period, higher inflation imposes a tax on labor income and leads to higher nominal wage claims ex ante.

\(^7\)Observe that there is no loss of generality to treat the levels of intervention \( i_W \) and \( i^*_W \) to be non-stochastic because the compound national public policy intervention to the optimal wage setting cannot alter the realization of the allocation ex post. All that matters are expected levels of \( i_W \) and \( i^*_W \), not the distribution around expected levels. Note also that because the consumption taxes will be set state-contingent, labor taxes must be set so as to be perfectly negatively correlated with the consumption taxes in order to keep the ratio constant at its desired level.
The equilibrium consequences are on the one hand that ceteris paribus world average consumption spending decreases because the expected inflation tax on labor income worsens Home households’ incentive to supply labor and therefore production becomes more expensive. On the other hand, higher Home wage levels improve the expected terms of trade because Home goods become more expensive relative to Foreign goods. The same shift in Home households’ wage setting and hence Home wage level, however, can be achieved by an appropriate change in the labor income tax because this has the identical distorting effect on the aggregate wage level. As a result, higher expected nominal interest rates and higher labor income taxes have identical implications on the equilibrium allocation as it is established in the following lemma:

**Lemma 2.2** The nominal interest rate and the labor income tax are perfectly substitutable national policy instruments. Relevant for the wage level determination is only the compound intervention to households’ wage setting.

Lemma 2.1 and Lemma 2.2 form the backbone of this analysis. There are two ways in which public policy can be used to affect the equilibrium allocation: ex post by market interventions and ex ante by giving incentives for households to change their wage setting. In either way, both monetary and fiscal policy instruments have a counterpart that constitutes a perfect substitute.\(^8\) As an immediate consequence, it follows that if national policymakers face the incentive to exploit their monopoly power on the terms of trade, they will do so either ex ante by changing the conditions for households to set wages or ex post by market interventions in order to manipulate the nominal exchange rate. Considering international policy coordination to surmount inefficiencies arising from non-cooperatively set public policies, either monetary or fiscal, policymakers must take into account that the compound monetary and fiscal policy intervention is decisive, not a single policy instrument. As we will see, this has important implications for the choice of international policy domains: For instance, suppose national monetary authorities coordinate and set nominal interest rates and money supply to the globally optimal level but they myopically consider the fiscal stance not to be responsive to the change in monetary policy conduct whatsoever. Self-oriented national fiscal policies then will determine the degree of interventions by choosing their target level and thereby render monetary policy coordination completely gainless. Before starting with the analysis of the different policy domains, however, first the policymakers’ objectives are derived in closed form.

---

\(^8\)The equivalence result in Lemma 2.1 resembles a recent finding in Adao et al. (2006) who also demonstrate that a state-dependent consumption tax might be used as an alternative expenditure-switching instrument in a monetary union where the flexibility of the nominal exchange rate is sacrificed. The equivalence of the nominal interest rate and the labor income tax as policy instruments is well known in closed economy setups (compare eg. Chari and Kehoe (1999)) but it has been so far ignored in the analysis of open economies.
2.2.6 The Objectives in Closed Form

National authorities aim at maximizing their respective residents’ expected utility. By making use of the equilibrium wage levels in equation (2.15) and aggregate budget constraints it follows that Home and Foreign expected period utility can be written as

\[
E_u = \left( \frac{1}{1-\rho} - \frac{(\theta-1)}{\nu \theta} \frac{1}{I_W} \right) EC^{1-\rho} \quad \text{and} \quad (2.18)
\]

\[
E_{u^*} = \left( \frac{1}{1-\rho} - \frac{(\theta-1)}{\nu \theta} \frac{1}{I_{W^*}} \right) EC^{*1-\rho}.
\]

Recall that in equilibrium the only difference between Home and Foreign consumption stems from the terms of trade. Consequently, if Home or Foreign authorities deviate from the jointly optimal policy, they will do so only as they face the incentive to manipulate the terms of trade in their own favor. In this case, independently and strategically conducted monetary and fiscal policy must be "beggar-thy-neighbor".

Expected utility written in terms of the equilibrium distribution of consumption permits a particularly convenient separation of the public policy interventions to the wage setting that only affect the expected consumption level from the public policy interventions to consumption spending that alter the variability of consumption. To be specific, Home expected utility can then be written as

\[
E_u = \left( \frac{1}{1-\rho} - \frac{\theta-1}{\theta \nu} \frac{1}{I_W} \right) \left( \frac{\theta-1}{\theta} \frac{1}{I_{W^*}} \right) \frac{(1-\rho)}{4} \frac{\nu X}{\Omega(i_C; i_{C^*})} \cdot \exp \left\{ \frac{\nu (1-\rho) X}{2X} \left( i_C; i_{C^*} \right) \right\}
\]

\[
\equiv \bar{u}(I_W; I_{W^*}) \cdot \exp \left\{ \frac{\nu (1-\rho) X}{2X} \left( i_C; i_{C^*} \right) \right\}. \quad (2.19)
\]

In (2.20), the term \( \bar{u}(I_W; I_{W^*}) \) depicts the part of expected utility that is independent of consumption variability and which can only be controlled through ex ante policy interventions to households’ wage setting. The term \( \Omega(i_C; i_{C^*}) \), in turn, summarizes the part of expected utility which depends on uncertainty only and which is a function of ex post public policy interventions to consumption spending. It is given by

\[
\Omega(i_C; i_{C^*}) = -\lambda \sigma_z^2 - \frac{1}{2} \lambda \sigma_{tot}^2 + \frac{(1-\gamma)}{2\nu \lambda} \sigma_{z,tot}^2 + \nu (2\sigma_{z,a} - \sigma_{tot,a})
\]

\[
+ \frac{(1-\gamma)}{2\nu} \lambda \left( 2\sigma_{z,a} - \sigma_{tot,a} \right) + \nu \lambda \left( \sigma_{a+w}^2 + \sigma_{a+d}^2 \right)
\]
where \( Z = \nu - (1 - \rho)(1 - \gamma)^2 > 0 \).

It is important to observe that the ex ante interventions differ from the ex post interventions in the order of magnitude with respect to their welfare implications. As already emphasized in Evers (2007a), ex post interventions are stabilization responses to deviations of productivity levels. Enhancing expected utility thus entails that risk aversion enforces the minimization of fluctuations in consumption and labor. These welfare effects are, however, generically of second order (compare the discussion by Lucas (2003)). Hence, gains from coordinating ex post stabilization policies are of second order. On the other hand, ex ante interventions to the wage setting are of first order because the distortions resulting from policies drive an inefficient wedge between the optimal labor-leisure trade-off on average. Consequently, gains from coordinating ex ante wage setting distortions are of first order.

### 2.3 International Cooperation of Public Policies

Monetary and fiscal policymakers are assumed to be able to perfectly commit to the policies they announce. In case of ex post market interventions that will alter the nominal exchange rate, monetary policy follows a money supply feedback rule that conditions on productivity shocks. They are given by

\[
\hat{m} = \mu_{aw}\hat{a}_w + \mu_{ad}\hat{a}_d \quad \text{and} \quad \hat{m}^* = \mu_{aw}^*\hat{a}_w - \mu_{ad}^*\hat{a}_d.
\]

Fiscal policy follows a similar feedback rule for consumption taxes, namely

\[
\hat{\tau}_C = \tau_{C,aw}\hat{a}_w + \tau_{C,ad}\hat{a}_d \quad \text{and} \quad \hat{\tau}_C^* = \tau_{C^*,aw}\hat{a}_w - \tau_{C^*,ad}\hat{a}_d,
\]

where \( \tau_C = \log(1 + t_C) \) and \( \tau_C^* = \log(1 + t_C^*) \). The compound public policy reactions to productivity shocks can therefore be written as

\[
\hat{i}_C = i_{C,aw}\hat{a}_w + i_{C,ad}\hat{a}_d \quad \text{and} \quad \hat{i}_C^* = i_{C,aw}^*\hat{a}_w - i_{C,ad}^*\hat{a}_d,
\]

where \( i_{C,aw} = \mu_{aw} - \tau_{aw}, \ i_{C,aw}^* = \mu_{aw}^* - \tau_{aw}^*, \ i_{C,ad} = \mu_{ad} - \tau_{ad}, \) and \( i_{C,ad}^* = \mu_{ad}^* - \tau_{ad}^* \).

---

\(^9\)Foreign expected utility can be stated as

\[
\bar{E}_u^* = \bar{u}^*(I_W; I_{W}) \exp \left\{ \frac{\nu(1 - \rho)}{2\lambda} \Omega^*(i_C; i_C^*) \right\}, \quad \text{where}
\]

\[
\bar{u}^* = \left( \frac{1}{(1 - \rho)} - \frac{\theta - 1}{\theta\nu} \frac{1}{I_W} \right) \left( \frac{\theta - 1}{\theta} \frac{1}{I_{W}} \right) \left( \frac{1 - \gamma}{1 - \gamma} \right), \quad \text{and}
\]

\[
\Omega^*(i_C; i_C^*) = -\lambda \sigma_z^2 - \frac{Z}{4} \sigma_{tot}^2 - \frac{(1 - \gamma)}{Y} \lambda^2 \sigma_{z, tot} + \nu(2\sigma_{z, aw} - \sigma_{tot, ad})
\]

\[
- \nu \frac{(1 - \gamma)}{Y} \lambda(2\sigma_{z, ad} - \sigma_{tot, aw}) + \nu \lambda(\sigma_{w}^2 + \sigma_{d}^2).
\]
In case of ex ante policy interventions to the optimal wage setting, the monetary policy is to set the expected gross nominal interest rate. Implicitly, this is achieved by the choice of the mean money supply for the next period, i.e. by \(E_m\) and \(E_m^*\). Fiscal policy chooses the labor income tax rates which must be perfectly negatively correlated to the consumption taxes so that the ratio of both is at the desired level.

### 2.3.1 Optimal Public Policy Coordination

The analysis begins with the consideration of single national policymakers who choose both the monetary and the fiscal policy instruments, i.e. there is a single Home authority that chooses \(\{I_W, i_{C,a,w}, i_{C,a,d}\}\) and a single Foreign authority that chooses \(\{I_W^*, i_{C,a,w}^*, i_{C,a,d}^*\}\). When single national policymakers coordinate their respective policy interventions, they do so as to maximize the joint welfare of Home and Foreign residents.

#### Ex Ante Intervention to Wage Setting

The first proposition establishes the globally optimal policy intervention to ex ante wage setting:

**Proposition 2.1** The optimal public policy intervention to the wage setting is

\[
I_{W}^{Opt} = \theta - \frac{1}{\theta},
\]

and \(I_{W}^{Opt} = I_{W}^{Opt}\) by symmetry.

**Proof.**

See Appendix.

The optimal public policy to the wage setting is to offset both distortions that prevent ex ante efficient wages: first, the markup of monopolistic suppliers of specific labor types, and second, the inflation tax caused by the cash-in-advance distortion. The combination of the fiscal and monetary policy instrument to achieve this is indeterminate. However, the most prominent combination to achieve the optimal level of ex ante intervention to the wage setting is to use labor income taxes to offset monopolistic distortions and to run the nominal interest rates according to the Friedman rule.

#### Ex Post Intervention to Consumption Spending

The next proposition states the optimal ex post intervention to spending:

**Proposition 2.2** The optimal ex post public policy intervention to consumption spending follows

\[
i_{C,a,w}^{Opt} = \frac{(1 - \rho)}{\lambda} \quad \text{and} \quad i_{C,a,d}^{Opt} = \frac{(1 - \rho)(1 - \gamma)^2}{Z},
\]

\(^{10}\)See also the detailed discussion in Evers (2007a).
2.3. International Cooperation of Public Policies

and $i^{*\text{Opt}}_{C,a_w} = i^{Opt}_{C,a_w}$ and $i^{*\text{Opt}}_{C,a_d} = i^{Opt}_{C,a_d}$ by symmetry.

**Proof.**

See Appendix. □

When both countries are hit by a common productivity shock, the only friction that matters are preset wages. Indeed, optimal public policy then replicates the flexible wage allocation by closing the gap between the marginal utility of consumption and the marginal disutility of labor in each instance. Under flexible wages, $(1-\rho)\nu$ determines the efficient intra-temporal elasticity of substituting labor for consumption.\(^{11}\) Under rigid wages, however, labor is fully demand determined and hence uncoupled from the consumption decision. Optimal public policy must therefore be used to imitate the optimal trade-off between consumption and labor. A positive aggregate productivity shock and subsequently lower goods prices lead to an increase in consumption spending. Without intervention, equilibrium employment would stay unaffected. Following, if $\rho > 1$, the optimal policy rule corrects the inefficiently high consumption level by counter-cyclically reducing nominal consumption spending up to the level that equates marginal utility of consumption and marginal disutility of labor. If $\rho < 1$, optimality requires a positive reaction of labor to an increase in consumption. Hence, the optimal policy rule has then to be pro-cyclical. Only in case of log-utility, it is in fact optimal having labor not to respond to consumption fluctuations at all.

Things are different when the two countries are hit by fully asymmetric productivity shocks. Then, in addition to rigid wages, the incomplete risk sharing places the policymakers to tackle another source of inefficiency. In particular, efficient risk sharing requires the ratio of Home over Foreign marginal utilities of tradable goods consumption to be unity.\(^{12}\) In general, however, closing the domestic gap between the marginal rate of substituting consumption and labor and the marginal rate of transformation by replicating the flexible wage allocation leaves an international gap between Home and Foreign marginal utilities of consumption. Hence, there exists a trade-off between closing domestic and international gaps. To be more precise, consider an asymmetric productivity shock that increases Home productivity over Foreign, say, which implies the terms of trade and thereby the real exchange rate to depreciate. Consequently, relative Home over Foreign nominal consumption spending increases by one-to-one to the fall in the real exchange rate. This leaves employment unaffected. In case of $\rho > 1$, to improve upon the gap between the marginal utility of

\[^{11}\] Optimal wage setting implies $EL^\nu = EC^{(1-\rho)}$. As this has to hold for each instance under flexible wages, we get $L^\nu = C^{(1-\rho)}$. Hence, the implied intra-temporal elasticity of labor with respect to consumption is $(1-\rho)/\nu$.

\[^{12}\] This follows as an immediate implication of (i) marginal utilities of any shared consumption good must equal relative prices and (ii) in case of tradable goods PPP of tradable goods imply that marginal utilities of tradable consumption goods must be identical across countries. Hence, $U_{C,T}^{H}/U_{C,T}^{F} = 1$ under complete financial markets and initially identical countries (compare Backus and Smith (1993)).
consumption and marginal disutility of labor effort, Home national authority has to counter-
cyclically decrease consumption by either increasing consumption taxes or reducing money
supply. Foreign national authority counter-cyclically increases consumption to oppose the
negative foreign productivity shock. Thereby, the nominal exchange rate appreciates but
by less than necessary to offset the direct effect of the productivity shock on the terms of
trade. As a consequence, the terms of trade and the real exchange rate still deteriorate even
when national authorities target the flexible wage allocation. The immediate implication for
Home and Foreign marginal utilities of consumption is that Home marginal utility is lower
than Foreign. To see this, note that in the equilibrium without international asset markets,
the ratio of marginal utilities of tradable goods consumption reads

\[ \frac{U_{C_T}}{U_{C_T}^*} = T_0 T^{-(1-\rho)(1-\gamma)}. \]

Thus, Home households consume too much tradable goods whereas Foreign households con-
sume to few tradable goods as efficient risk sharing would require. If households were able
to perfectly contract state contingent payments in international financial markets, Home
households would have to make a transfer to Foreign households to equate marginal utilities
of tradable goods consumption. However, by the lack of international financial markets,
consumption risk sharing can only be achieved by nominal exchange rate management. In
fact, an implicit Home transfer to Foreign can be attained by a depreciation of Home cur-
rency: the Home tradable good becomes cheaper for Foreign residents whereas the Foreign
tradable good becomes dearer for Home residents. Therefore, Home residents pay more in
exchange for Foreign goods which indeed comes along with a reproduction of a transfer. As
a result, from a global perspective it is not optimal to nationally target the flexible wage
equilibrium. As the story goes exactly the symmetric way for \( \rho < 1 \), it turns out that the
optimal market interventions to asymmetric productivity shocks are less responsive than
simply targeting the flexible wage allocation. Hence, in general, optimal policy responses
to asymmetric shocks face the trade-off between dispelling domestic inefficiencies caused by
wage rigidity and the inefficiency caused by the lack of international risk sharing. Note that
it is this trade-off only that renders the optimal policy conduct to be second-best.14

2.3.2 Non-Cooperative Public Policy: Nash

Next, I consider what public policies look like when the single Home and Foreign national
policymakers conduct public policy independently. Recall the discussion of the respective
welfare functions that policymakers seek to maximize. The only difference between Home

---

13In fact, one can easily show that \( i_{C_{a,d}}^{flex} = \frac{(1-\rho)(1-\gamma)}{i} \) indeed replicates the flexible wage allocation in case
of asymmetric shocks and one finds that \(-1 < i_{C_{a,d}}^{flex} \). Moreover, the terms of trade than still changes by
\( \hat{\tau}_{tot} = \frac{1}{2} \hat{u_a}. \)

14Compare also the discussion in Obstfeld and Rogoff (2002a) and Evers (2007a) on this trade-off.
and Foreign residents’ welfare stems from the terms of trade. Hence, national policymakers deviate from the optimal policy only on the grounds of manipulating the terms of trade to their own national benefits.

**Ex Ante Intervention to Wage Setting**

**Proposition 2.3** The (unique) Nash equilibrium of non-cooperatively set public policy interventions to the wage setting yields

\[ I_{W}^{\text{Nash}} = \frac{\theta - 1}{\theta} \left( 1 + \gamma X + (1 - \gamma)X \right), \]

and \( I_{W}^{\ast \text{Nash}} = I_{W}^{\text{Nash}} \) by symmetry. Furthermore,

\[ I_{W}^{\text{Nash}} > I_{W}^{\text{Opt}} \text{ if } \gamma \neq 0 \quad \text{and} \quad I_{W}^{\text{Nash}} = I_{W}^{\text{Opt}} \text{ if } \gamma = 0. \]

**Proof.**

See Appendix.

From this proposition follows that non-cooperatively set policy interventions to workers’ wage setting will be strictly larger than the optimal one unless the two countries are in autarky, i.e. \( \gamma = 0 \). By the discussion of the equilibrium wages, higher nominal interest rates or labor income taxes entail higher wage claims ex ante. Why, then, has the national policymaker the incentive to induce wage claims to be inefficiently higher than the optimal level? The answer is to improve upon the consumption-labor trade-off. Given the other country’s wage level, an increase in domestic wages implies an increase in labor income and consequently in expected consumption the next period. However, in equilibrium, higher relative wages imply higher terms of trade and a reduction in domestic goods demand. Furthermore, higher relative wages also induce a direct fall in labor demand. Hence, aggregate labor demand and labor income falls. The net effect on the consumption-labor ratio and thus on welfare, however, is positive because households substitute to the relatively cheaper foreign goods. Thus, domestic policymakers ”beggar-thy-neighbor” by exporting labor effort and importing more consumption goods at relatively higher domestic wages. As a result, as both Home and Foreign national authorities behave symmetrically in equilibrium, higher Home and Foreign inflation and labor income tax induce inefficiently high nominal wage claims that reduce in turn aggregate world output and world consumption. Only in case of no trade, i.e. when the two countries are closed economies (\( \gamma = 0 \)), does the positive net substitution effect vanish and the optimal policy intervention to wage setting indeed constitutes a Nash equilibrium.

**Ex Post Intervention to Consumption Spending**

**Proposition 2.4** The (unique) Nash equilibrium of non-cooperatively set public policy interventions to consumption spending yields
\[ i_{Nash}^{C,a_w} = \frac{(1 - \rho)}{X} \quad \text{and} \quad i_{Nash}^{C,a_d} = (1 - \rho)(1 - \gamma) \left( \frac{X + (1 - \gamma)Y}{2Y + (1 - \gamma)X^2} \right), \]

where \( i_{Nash}^{C,a_w} = i_{Nash}^{C,a_d} \) and \( i_{Nash}^{C,a_d} = i_{Nash}^{C,a_d} \) by symmetry. Furthermore,

\[
\begin{align*}
\begin{cases}
  < i_{Opt}^{C,a_d} & \text{if } \rho > 1 \text{ and } 0 < \gamma < 1, \\
  > i_{Opt}^{C,a_d} & \text{if } \rho < 1 \text{ and } 0 < \gamma < 1, \\
  = i_{Opt}^{C,a_d} & \text{if } \rho = 1 \text{ or } \gamma \in \{0, 1\}.
\end{cases}
\end{align*}
\]

**Proof.**

See Appendix.

Indeed, when the two countries are hit by a common aggregate productivity shock, it is not surprising that the Nash solution does coincide with the optimal response: The only source of frictions are preset wages. Hence, as the impact of the shock is identical, the policy targets coincide.

In contrast, when the two countries are hit by idiosyncratic shocks, the optimal response does not constitute a Nash equilibrium of the policy rule setting game in general. If national policymakers decide on their respective policy interventions non-cooperatively, Proposition 2.4 implies that as long as the lack of complete international financial markets matters, i.e. as long as \( \rho \neq 1 \) or \( 0 < \gamma < 1 \), both single national authorities face the incentive to unilaterally react more actively to idiosyncratic shocks than it is globally efficient. Why? The answer is again to improve upon the domestic labor-leisure trade-off. Put differently, national policymakers regard it beneficial to close the domestic gap between marginal utility of consumption and marginal disutility of labor at the cost of increasing the international gap between Home and Foreign marginal utilities of consumption. By the discussion above, optimal public policy responses to idiosyncratic productivity shocks trade off gains from replicating the flexible wage allocation and the gains from international consumption risk sharing. In particular, the optimal policy rule moves away from replicating the flexible wage equilibrium because the efficient consumption risk sharing requires a less active response to shocks in order to implicitly compensate for the lack of state-contingent transfers within complete international financial market. For instance, when \( \rho > 1 \) and an asymmetric shock increases Home productivity and reduces Foreign productivity, the optimal policy rule induces the nominal exchange rate to fall by less than under replicating the flexible wage equilibrium in order to lessen the appreciation of the terms of trade. That is, the Home tradable good must be cheaper for Foreign residents and Foreign tradable good must be dearer for Home residents than under replicating the flexible wage equilibrium. The consequence is an implicit transfer of tradable goods from Home to Foreign that increases
demand for Home labor and reduces demand for Foreign labor. From a national perspective, however, it pays to deviate from the optimal policy response since given the other national policymaker sticks to the optimal rule, the gains from reducing the domestic marginal utility gap by altering the terms of trade outweighs the loss implied by the widening of the international gap between marginal consumption utility. Specifically, Home reduces labor effort by cutting consumption spending in order to further bridge the domestic gap by either expanding money supply or lowering consumption taxes. For $\rho > 1$, the consumption level itself falls by less than the labor effort (compare the discussion of Proposition 2.2). The reason is that a cut in Home consumption spending appreciates Home currency and improves the terms of trade. Hence, Home residents’ substitute for the then cheaper Foreign good. The Home currency appreciation in turn inflicts a beggar-thy-neighbor effect on Foreign because it directly reduces Foreign consumption and increases Foreign labor effort. The analog story holds true for $\rho < 1$.

As a consequence, deviations from the optimal policy responses to idiosyncratic productivity shocks result in an amplification of the response of the nominal exchange rate and thereby of the terms of trade. From a global perspective, this amplification of the terms of trade fluctuations harms both Home and Foreign residents since this leads to higher consumption fluctuations. The result is that international risk sharing and thus international consumption smoothing has worsened.

2.4 Monetary Cooperation and Fiscal Independence

Next, the analysis of decentralized public policies follows. In this case, national policy-making is separated between two distinct authorities: a monetary policy authority that decides on the nominal interest rate and the money supply rule and a fiscal policy authority that decides on consumption and labor income taxes. I focus on the case when monetary authorities coordinate their respective policy instruments in order to maximize the sum of Home and Foreign residents welfare whereas the fiscal authorities implement their respective policies independently of each other so as to maximize the domestic residents’ welfare only.

In this section, it is shown that for all instances when there is an incentive for self-oriented national authorities to deviate from both the optimal ex ante intervention to the wage setting (compare Proposition 2.3) and the optimal ex post market intervention rules (compare Proposition 2.4), monetary policy coordination inevitably leaves room for independently operating national fiscal authorities to exploit their power on the terms of trade. In particular, by the two lemmata in Section 2.2, monetary and fiscal policy instruments are perfect substitutes with regard to the respective interventions. Only
the compound action of domestic monetary and fiscal policy accounts for the effect on private households behavior. Hence, coordinating monetary authorities seek to implement the optimal levels of public policy interventions. Independently acting fiscal authorities, however, undermine monetary authorities endeavor by enforcing a deviation of the optimal public policy intervention levels by the motives as discussed in the foregoing section.

**Ex Ante Wage Setting Intervention**

The following proposition states that in case of ex ante intervention to the wage setting gains from monetary policy coordination are fully carried off by the fiscal sides:

**Proposition 2.5** The (unique) Nash Equilibrium between the cooperating monetary authorities and the two symmetric independent fiscal authorities entails fiscal dominance, i.e. the net nominal interest rates are tied down to zero whereas the wage taxes are set to the non-cooperative level.

**Proof.**

See Appendix.

Since the net nominal interest rates cannot become negative, the two fiscal authorities will force the cooperating monetary authorities to implement zero net nominal interest rates as then fiscal authorities exerts full control over Home and Foreign wage setting. As a result, in the Nash equilibrium with symmetric fiscal actions, Home and Foreign fiscal authorities will implement the non-cooperative levels of public policy intervention \( I_{W}^{Nash} \). The importance of this result cannot be overemphasized: As argued above, the magnitude of welfare implications of ex ante intervention to the wage setting is of first-order. Therefore, gains from policy coordination are of first-order. As a consequence, by Proposition 2.5, potentially large gains from policy coordination can only be realized if the fiscal policy is included in the international policy cooperation.

**Ex Post Market Intervention**

In case of common productivity shocks, the only distortion that matters ex post are preset wages. Hence, there is no basis on which the cooperating monetary authorities and the two self-oriented fiscal policymakers might dispute. As a consequence, we find the following result.

**Proposition 2.6** The (continuum of) Nash Equilibria between the cooperating monetary authorities and the two symmetric independent fiscal authorities entails the optimal national compound policy responses to aggregate shocks.
2.4. Monetary Cooperation and Fiscal Independence

Proof.
See Appendix.

In fact, since both domestic monetary and fiscal authority want to implement the optimal feedback rule of ex post public intervention $i^{Opt}_{C,a_w} = i^{Opt}_{C,a_w}$, all combinations of monetary and fiscal coefficients that lead to the optimal feedback rule are equilibria. Hence, all what domestic public policymakers face is a coordination problem of which combination to choose.

In case of idiosyncratic shocks, however, national fiscal authorities face an incentive to enforce a deviation from optimal policy responses on the grounds that they want to improve upon the domestic gap between the marginal consumption utility and the marginal disutility of labor effort at the cost of worsening international consumption risk sharing. The next proposition shows that as long as the need for a risk sharing arrangement matters, self-oriented national fiscal authorities will indeed effectively deteriorate optimal public policy responses to idiosyncratic shocks.

**Proposition 2.7** Only if $\rho = 1$ or $\gamma \in \{0, 1\}$, the (continuum of) Nash equilibria between the cooperating monetary authorities and the two symmetric independent fiscal authorities in pure strategies entail optimal national compound policy responses to idiosyncratic shocks. Otherwise, if $\rho \neq 1$ and $0 > \gamma > 1$, the (continuum of) Nash equilibria that exhibit mixed strategies lead to a (unique) distribution of the compound public policy intervention that inevitably leads to welfare losses as compared to the optimal response to asymmetric shocks.

Proof.
See Appendix.

Obviously, when the goals of cooperating monetary authorities and the two distinct national fiscal authorities coincide and all policymakers seek to close the domestic gaps induced by the wage rigidity, the Nash equilibria must consist of optimal responses to asymmetric shock. This is the case either if the countries are closed ($\gamma = 0$) or if international financial markets are redundant ($\rho = 1$ or $\gamma = 1$). Again, as in case of aggregate productivity shocks, policymakers still face a coordination problem that implies the Nash solution to be non-unique. On the other hand, if the goals do not coincide, cooperating monetary policymakers want to implement the optimal policy responses $i^{Opt}_{C,a_d}$ and $i^{Opt}_{C,a_d}$. Given the action taken by the monetary authorities, the equilibrium best responses of national fiscal authorities are to choose consumption taxes so as to implement the Nash intervention rules $i^{Nash}_{C,a_d}$ and $i^{Nash}_{C,a_d}$. Hence, it follows from the perfect substitutability of national policy instruments (Lemma 2.1) that there cannot exist a Nash equilibrium in pure strategies. Nevertheless, there exist Nash equilibria in mixed strategies. As a result, the mixed strategies equilibria must entail welfare losses as compared to the globally optimal response to asymmetric productivity shocks because on average the national responses are more active as it is optimal. To
evaluate the relevance of these theoretical results, I consider a numerical exercise in the
next subsection.

2.5 Gains and losses from Policy Cooperation with and without Fiscal Independence

The losses that fiscal independence implies for international monetary policy coordination are assessed on the basis of a numerical simulation of the model. To alleviate comparisons and since the model is similar in structure to theirs, I adopt the parameters chosen by Obstfeld and Rogoff (2002a). To be specific, I assume that $\sigma_{aw} = \sigma_{ad} = 0.01$ and that $\nu = 1$. The value for $\theta$ is chosen to be $\theta = 7.66$. This implies a monopolistic markup over marginal costs of 15%. Two possible scenarios are considered: a low-trade scenario ($\gamma = 0.2$) which corresponds to an import over GDP ratio of 10% and high-trade scenario ($\gamma = 0.6$) which corresponds to an import over GDP ratio of 30%. The impacts of different public policy arrangements are measured by their % change in consumption that is required to compensate the representative household to be indifferent between either two policy environments. $\xi$ denotes the percentage compensation of consumption so that $U((1 + \xi)C^A, L^A) = U(C^B, L^B)$, where $A$ and $B$ are two different policies. Moreover, the structure of the model allows an easy decomposition of this measure into the two relevant components: effects of the ex ante policy interventions to the wage setting ($\xi^{lw}$) and effects of the ex post policy interventions to the consumption spending ($\xi^{lc}$).

Table 2.2 reports the gains from international monetary policy coordination when national fiscal authorities act independently. There are no gains from coordinating monetary policy interventions to households’ optimal wage setting as Proposition 2.5 shows: The Fiscal authorities dominate the cooperating monetary authorities by tying the nominal interest rate to zero and implementing the competitive level of public policy interventions. In case of ex post interventions, only asymmetric shocks lead to a divergence of policy goals. As shown in Proposition 2.7, as long as risk-sharing matters, the unique Nash equilibrium is in mixed strategies. Surprisingly, in the low-trade scenario, coordinating monetary policy actually leads to welfare losses. For independently acting fiscal authorities forces policymakers to mix their responses to asymmetric shocks, circumstances arise where the compound ex post policy interventions are either less responsive than optimal or more responsive than it would be nationally desirable. Welfare losses in expected utility by having both these circumstances outweigh the gains from reacting at least sometimes optimally to asymmetric shocks. Furthermore, these losses are increasing in the absolute deviation of $\rho$ from unity because thereby the consequences of policy disturbances on expected utility become more pronounced. The same effect persists in the high-trade scenario. The welfare implications are, however, smaller because the inefficiency arising from consumption risk
sharing plays a less important role in case of higher trade integration. Recall that the elasticity of substituting the Home tradable good for the Foreign tradable good is always one. Hence, relative price movements can be better dealt with by households substituting for the relatively cheaper tradable good the larger the share of tradable consumption goods in the overall consumption basket is.

Welfare gains from fiscal policy coordination when monetary authorities already cooperate are presented in Table 2.3. In contrast to the first exercise, there are gains from choosing the ex ante intervention to households’ wage setting cooperatively. The figures in the first row show that these gains from fiscal cooperation are substantial. They are decreasing in $\rho$ even though the inter-temporal inefficiency caused by the non-optimal policy intervention is increasing simply because the inter-temporal elasticity of substitution is decreasing in $\rho$, too. Thus, consumption patterns are less sensible to disturbances to the optimal wage-setting. The gains are, however, increasing in the degree of openness because the national policymakers are more prone to manipulate the terms of trade ex ante as the relative importance of the terms of trade is naturally increasing in Home as well as Foreign shares of tradable goods in overall consumption. Corresponding to the results in the first exercise, the gains from cooperation of the fiscal authorities’ ex post markets interventions is increasing in $\rho$ but decreasing for higher trade integration as consumption risk sharing occurs stronger through trade itself.

In this numerical exercise, two result are worth being emphasized: First, the results show
that monetary policy cooperation may lead to welfare losses when independent fiscal policy-makers follow national interests only. Second, welfare gains from international public policy coordination - and in particular fiscal policy coordination when monetary authorities already cooperate - are quite large. Both points are in sharp contrast to the existing literature on international monetary policy coordination: First gains from cooperation are fairly small as they solely stem from ex post market interventions and exchange rate responses to shocks (with the exception of Cooley and Quadrini (2003), Arseneau (2007) and Evers (2007a)). Second, the crucial role of fiscal policy responses to monetary cooperation is fully neglected.

### 2.6 Conclusion of Chapter 2

The goal of this paper has been to analyze the interplay of monetary and fiscal policy domains in a world where countries are linked through trade. In particular, it is questioned that one can analyze the gains and losses of international arrangements that promote the coordination of either monetary or fiscal policy without taking into account the response of the remaining independent and non-cooperatively conducted public policy to this arrangement. This claim is addressed in a simple stochastic two-country sticky-wage model with a cash-in-advance restriction. In this environment, monetary authorities can affect the terms of trade by conducting a general short-run monetary policy using both the nominal interest rate and the money supply. Fiscal authorities can also affect the terms of trade by using distortionary taxes on labor income and consumption. It turns out that labor income taxes
and nominal interest rates are perfectly substitutable national policy instruments when policymakers affect workers’ optimal wage setting ex ante. The consumption tax and the money supply are perfectly substitutable national policy instruments when policymakers alter the consumption spending ex post. As a consequence, I find that international monetary policy coordination requires fiscal policy coordination, too, in order to fully skim off the gains from international policy coordination. Moreover, the numerical exercise suggests that letting the fiscal authorities act independently when monetary authorities cooperate might even lead to welfare losses.

Against this background, I adopt a critical stance on considering monetary policy coordination without taking into account the implications for fiscal policy conduct. The main argument is that as long as national monetary and fiscal authorities share the same objectives - in our case the respective residents’ welfare - they do also share the same incentives to manipulate the terms of trade in their country’s favor. Hence, as both national authorities do dispose of policy instruments to effectively alter the terms of trade, coordination of monetary policy in order to overcome strategic incentives of self-oriented national monetary policymakers - as it is argued in the literature - still leaves playground for fiscal policymakers to exploit their own monopoly power. The numerical example demonstrates that gains from monetary policy coordination are negligibly small or even negative (Table 2.2) even though gains from policy coordination in general might be large (Table 2.2 + Table 2.3). Therefore, the conclusion is that if one takes the game-theoretic arguments seriously - as one definitely should - the optimum policy domain where international coordination is promising must span both the monetary policy as well as the fiscal policy.
Chapter 3

Federal Fiscal Transfers in Monetary Unions: A NOEM Approach

The creation of a monetary union, as is often proposed by its advocates, enhances trade among member states since it reduces transaction costs and leads to higher economic, political and social integration. But creating a monetary union also means that its member states sacrifice their monetary policy autonomy. Furthermore, the member states abandon the flexibility of exchange rates that adjusts regionally asymmetric shocks among member states.

Building on the seminal work by Mundell (1961), the literature beginning with Kenen (1969) has argued that monetary unions must be embedded in adequate federal fiscal institutions that provide insurance against asymmetric shocks among the member states. This also forms the consensus in the debate over EMU. In a report, Delors (1989) argued that the lack of exchange rate flexibility would cause tensions within the monetary union that may even lead to a breakdown of the union if no such adjusting institution was installed. Delors recommended binding limits on national budget deficits and the coordination of national fiscal policies so as to establish a union-wide arrangement absorbing asymmetric shocks among the member states. The latter recommendation, however, was daffed aside in the Maastricht Treaty and only the limits on national budget deficits and public debts were anchored in the Stability and Growth Pact. To emphasize his concern about the missing of a fiscal arrangement dealing with asymmetric shocks, Feldstein (1997) claimed that

"...on balance, a European monetary union would be an economic liability. The gains from reduced transformation costs would be small and might, when looked at from the global point of view, be negative. At the same time, EMU would increase cyclical instability, raising the cyclical unemployment rate."

65
It is even more astonishing that the Maastricht Treaty paid so little attention to an adjusting arrangement as more than 20 years before the Delors Report, the so-called MacDougall Report [1977], a study of the feasibility of EMU, already put forward the creation of a central or federal fiscal arrangement that would automatically redistribute taxes or transfers among the member states in order to absorb the effects of asymmetric shocks. In particular, it suggested that a system of built-in stabilizers should work through a federal or central budget that collects taxes from a prospering state and pays transfers to a state in recession. These transfers could either be among national governments or directly among private sectors, i.e. households and firms.

The focus of this paper is to analyze federal fiscal transfers that are aimed to reduce economic fluctuations caused by temporary asymmetric productivity and preference shocks. We explore the properties of different transfer schemes with regard to their capability to stabilize national consumption, production and employment. Specifically, we consider the two types of transfers suggested by the MacDougall Report and which are implemented in some existing federal states: Federal transfers among the households of different member states and intergovernmental transfers that include payments between national fiscal authorities only. The difference in how the two transfer schemes affect the allocation is key to the analysis: Whereas federal transfers among households change the current income of households directly and affect the allocation through choices made by the households, intergovernmental transfers change national public consumption and the allocation through the goods markets. Intergovernmental transfers affect households' current income indirectly through changes in goods and subsequently labor demand.

In case of preference shocks that shift the demand from goods produced in one region to goods produced in another region, intergovernmental transfers simply shift back the demand effect through a change in public consumption such that production and employment remain unaltered. This implies no change in labor income and thus no fluctuations in consumption, employment and welfare at all.

In contrast, if the member states are hit by asymmetric productivity shocks, changes in consumer prices lead to a further income effect and changes in the real interest rates prompt households to save. This comes in addition to the labor income effects caused by changes in demand. In this case, neither transfer scheme stabilizes consumption and employment fluctuations at the same time. A combination of federal transfers among households and intergovernmental transfers, however, will do. Whereas the intergovernmental transfers keep employment and thus labor income constant, the federal transfers among households redistribute private savings and the income effect due to the change in consumer prices. Consequently, both consumption and employment are stabilized. As a result, we find that federal fiscal arrangements as suggested by the MacDougall Report indeed provide perfect
insurance in the sense that fluctuations induced by asymmetric shocks can be completely prevented.

In light of the many times expressed concerns about the lack of a federal fiscal stabilization mechanism, it is surprising that researchers have rarely analyzed federal fiscal transfer mechanisms within a theoretical framework. The few theoretical approaches to federal fiscal institutions that employ federal transfers among states forming a monetary union are Kletzer and Buiter (1997), Kletzer (1999), and Kletzer and von Hagen (2001). The main part of the literature on federal fiscal institutions stabilizing economic fluctuations in monetary unions evolves around the empirical relevance of such institutions in existing unions. This academic debate was initiated by the contributions of Sala-i-Martin and Sachs (1991) and von Hagen (1992). They estimated the effects of federal fiscal transfer payments to offset asymmetric shocks in the U.S to be between 10% and 40%. An overview on the subsequent literature is provided by Kletzer and von Hagen (2001) and Mélitz and Zumer (2002).

Closest to our model is Kletzer and von Hagen (2001). They consider two regions forming a monetary union within a dynamic general equilibrium framework with rigid wages. Each region produces a final good for consumption using a set of intermediate input factors produced with labor. The regions are exposed to asymmetric productivity shocks. Kletzer and von Hagen show that asymmetric productivity shocks have no effects on consumption and employment as long as the economic structures of the two member states are similar. Only if the economic structures are sufficiently different, asymmetric productivity shocks cause fluctuations. In that case, a redistributive federal transfer scheme can stabilize either consumption, or production, or employment, but not two of them at the same time.

We extend their analysis in several aspects. First, we introduce preference shocks. This allows us to study shifts in demand directly and to revisit Mundell’s original example. Second, we stress the importance of trade as a means of cushioning asymmetric demand and supply shocks. Specifically, we consider the degree of economic integration and structural differences of the member states to gain further insights into effectiveness and functioning of the different transfers. Third, we use welfare criteria to judge the insurance property of the fiscal arrangements. Fourth, we show that a combination of the two federal fiscal transfers will always provide perfect insurance.

Our model is set up in the tradition of the New Open Economy Macroeconomics (NOEM) and follows closely Obstfeld and Rogoff (1995, 1996)\(^1\). This allows us to embed the different federal fiscal transfer schemes into a dynamic general equilibrium setting with nominal rigidities. The great convenience of this approach is that we can jointly analyze immediate and

---

\(^1\)See also Lane (2001) and more recently Lane and Ganelli (2003) for excellent overviews.
long-run consequences of federal fiscal arrangements on consumption, production, employment and the current account in depth. This enables us to gain insights into the functioning and effectiveness of the different transfer schemes in response to asymmetric shocks. To conform the model to our purposes, we extend the model by Obstfeld and Rogoff along two lines. First, in our model, households provide different types of labor monopolistically. Final goods are produced by an aggregation technology over all different domestic types of labor. In this sense, technology shocks alter the aggregate labor productivity and subsequently shift marginal costs of producing market goods. This is in stark contrast to Obstfeld and Rogoff who model productivity as a preference parameter. As a consequence, since goods markets are assumed to be competitive in our model, temporary productivity shocks affect prices even though wages are rigid as it is emphasized in Obstfeld and Rogoff (2000). This enriches the dynamics of the model immensely. The underlying view is that wages are much more rigid than prices. Furthermore, it seems implausible that productivity shocks even though only transitory in nature have no impacts on prices at all. Second, we introduce tradable and non-tradable goods.\footnote{Hau (2000) extends the original model by Obstfeld and Rogoff for tradable and non-tradable goods.} Therefore, we can describe the demand shock initially considered by Mundell (1961) as a shift of demand from tradable goods produced in one state to tradable goods produced in the other state. This also allows us to take into account how economic integration as measured by the fraction of tradable consumption goods affects functioning and effectiveness of different transfer schemes.

The rest of this paper is structured as follows. In the next section, we set up our model of the monetary union. In Section 3.2, we consider the solution of the log-linearized model. We portray the transmission of demand and supply shocks as well as changes in policies in depth. In Section 3.3, we turn to the analysis of the different transfer schemes. We conclude in Section 3.4.

### 3.1 The Model

We consider a monetary union that consists of two regions, referred to as "home" (H) and "foreign" (F). Each region is populated by a continuum of households with measure one. Each household within a region provides a specific type of labor which is used as an input factor for the production of two goods, a tradable and a non-tradable good. Altogether we have four different goods, two tradable and two non-tradable ones. We assume the goods markets to be perfectly competitive, but labor is supplied monopolistically. Wages are sticky and set one period in advance. The only asset households trade across regions is a nominal bond. There are two local governments, one within each region, a federal government and a central bank. Local governments collect lump-sum taxes from their inhabitants to finance government expenditures. In addition, local governments receive or pay transfers within
3.1. The Model

the federal budget. The federal government acts as a balance sheet for transfers among households of the different regions and between the two local fiscal authorities.

The notation we stick to throughout this paper is as follows: Superscripts denote where a variable belongs to, foreign variables are distinguished by an asterisk *, and union wide aggregates by $MU$. Subscripts identify the characteristics of that variable, e.g. whether it’s a non-tradable or the home tradable good. Households and their specific input factors are denoted by $i \in [0, 1]$.

3.1.1 Goods Production

The technologies to produce the tradable and the non-tradable goods are identical within the region:

$$y_{j,s} = a_s \left( \int_0^1 l_{j,s}(i) \frac{\theta + 1}{\theta} di \right)^{\frac{\theta}{\theta - 1}}, \quad (3.1)$$

where $\theta > 1$, for $j \in \{H, N\}$ being either the home tradable good $H$ or home non-tradable good $N$. The CES technology simply aggregates all different types of labor $i$ within a region which are substitutable at a constant elasticity $\theta$. The aggregate productivity of labor $a_s$ is subject to shocks. Foreigners share the same aggregation technology over foreign types of labor and with a different productivity level. Factor cost minimization implies

$$l_{j,s}(i) = \frac{1}{a_s} \left( \frac{w_s(i)}{W_s} \right)^{-\theta} y_{j,s}, \quad (3.2)$$

to be the optimal demand for labor input of quality $i$ at wage $w_s(i)$ and an overall wage level

$$W_s = \left( \int_0^1 w_s(i)^{\frac{1}{\theta}} di \right)^{1-\frac{1}{\theta}}. \quad (3.3)$$

With perfectly competitive goods markets, the prices for home tradable and non-tradable goods are

$$P_{H,s} = P_{N,s} = \frac{W_s}{a_s}, \quad (3.4)$$

respectively. Total aggregate output of the home tradable good is $Y_{H,s}$ and of the home non-tradable good $Y_{N,s}$. For the foreign region the corresponding expressions apply.

3.1.2 Households

Households within the same region have identical preferences over consumption, real balances and labor effort. They are described by

$$U_t(i) = \sum_{s=t}^{\infty} \beta^{s-t} \left( lnC_s(i) + \chi ln \left( \frac{M_s(i)}{P_s} \right) - \frac{1}{2} L_s(i)^2 \right), \quad (3.5)$$
where $0 < \beta < 1$, and $\chi > 0$. The real consumption index for person $i$, $C_s(i)$ is given by

$$C_s(i) = \left( \gamma \frac{1}{\rho} C_{T,s}(i)^{\frac{\rho-1}{\rho}} + (1 - \gamma) \frac{1}{\rho} C_{N,s}(i)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \quad (3.6)$$

$0 < \gamma < 1$, and is identical for all persons within a region. Following our notation, $C_{N,s}(i)$ denotes $i$'s consumption of the home non-tradable good. $C_{T,s}(i)$ is a real consumption index over the two tradable goods $C_{H,s}(i)$ and $C_{F,s}(i)$ with

$$C_{T,s}(i) = \left( \alpha \frac{1}{\rho} C_{H,s}(i)^{\frac{\rho-1}{\rho}} + (1 - \alpha) \frac{1}{\rho} C_{F,s}(i)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \quad (3.7)$$

$0 < \alpha < 1$. Foreign households have the same preferences over tradable goods but differ with respect to their own non-tradable good. Individual optimization yields for any person $i$ the following composition of the home real consumption index between the tradable commodities index and the non-tradable good

$$C_{T,s} = \gamma \left( \frac{P_{T,s}}{P_s} \right)^{-\rho} C_s, \quad \text{and} \quad C_{N,s} = (1 - \gamma) \left( \frac{P_{N,s}}{P_s} \right)^{-\rho} C_s, \quad (3.8)$$

and between home and foreign tradable commodities

$$C_{H,s} = \alpha \left( \frac{P_{H,s}}{P_{T,s}} \right)^{-\rho} C_{T,s}, \quad \text{and} \quad C_{F,s} = (1 - \alpha) \left( \frac{P_{F,s}}{P_{T,s}} \right)^{-\rho} C_{T,s}. \quad (3.9)$$

The parameters $\gamma$, $\alpha$, and $\rho$ will play a crucial role in our analysis. By our assumption of CES consumption indices, $\gamma$ determines together with the price ratio of tradable and non-tradable goods the composition of tradable and non-tradable goods in the overall consumption basket. In fact, as $\gamma$ reaches zero, households demand only non-tradable goods and no trade in goods will occur. As $\gamma$ reaches unity, both foreign and home households prefer tradable goods only. Since we assume that preferences over tradable-goods are identical for home and foreign households, preferences coincide. Therefore, $\gamma$ captures the difference in preferences. This implies that $\gamma$ also displays differences in price levels and real interest rates, too. On this account we will interpret $\gamma$ to indicate the degree of economic integration of the two member states.\(^3\) The parameter $\alpha$ gives the consumption composition of the two tradable goods. Since home and foreign households share identical tastes with respect to the composition of the two tradable commodities, we can consider a shift of demand from one region to the other. This is Mundell’s original case when he analyzed optimum currency areas. The elasticity of substitution $\rho$ determines in combination with $\gamma$ and $\alpha$ how private households’ demand responds to changes in relative prices. In particular, in our setting $\rho$ accounts for the expenditure switching effect following changes in relative prices.

\(^3\)In a common sense, economic integration suggests also factor mobility. In fact, following our assumption of production processes, even though labor is completely mobile, it is only demanded domestically. Thus, viewing $\gamma$ as the degree of economic integration seems plausible.
The corresponding home consumption-based price indices are
\[
P_s = \left( \gamma P_T^{(1-\rho)} + (1 - \gamma) P_N^{(1-\rho)} \right)^{\frac{1}{1-\rho}},
\]
and
\[
PT,s = \left( \alpha P_H^{(1-\rho)} + (1 - \alpha) P_F^{(1-\rho)} \right)^{\frac{1}{1-\rho}}.
\] (3.10)

Home household \(i\)’s optimization problem is to maximize (3.5) subject to its budget constraint
\[
B_{s+1}(i) + M_s(i) + P_s C_s(i)
\leq (1 + i_s) B_s(i) + w_s(i) L_s(i) + M_{s-1}(i) + T_{loc,s} + T_{hh,s},
\] (3.11)
where \(i_s\) is the nominal interest rate, and subject to the labor demand for household \(i\)’s specific variety of labor
\[
l_s(i) = \frac{1}{a_s} \left( \frac{w_s(i)}{W_s} \right)^{-\theta} \left( Y_H,s + Y_N,s \right).
\] (3.12)

The transfers in (3.11) consist of the usual net lump-sum transfer from the local fiscal authority, \(T_{loc,s}\), and the private transfer from foreign households to domestic households, \(T_{hh,s}\) (read ’household to household’), that is part of the federal fiscal arrangement. Transfers are identical for all households within a region. Therefore we skip indices.

The optimality conditions are the standard consumption Euler equation
\[
C_{s+1}(i) = C_s(i)\beta(1 + r_{s+1}),
\] (3.13)
optimal money demand
\[
\frac{M_s(i)}{P_s} = \chi C_s(i) \frac{1 + i_{s+1}}{i_{s+1}},
\] (3.14)
and the optimal wage setting
\[
w_s(i) = \frac{\theta}{\theta - 1} L_s(i) C_s(i) P_s.
\] (3.15)

The nominal interest rate is given by
\[
1 + i_{s+1} = (1 + r_{s+1}) \frac{P_{s+1}}{P_s}.
\] (3.16)

Note that with a common currency the nominal interest rate is the same in both regions. Of course, the real interest rates differ as purchasing power parity need not hold. Optimality conditions for foreign households are accordingly.
3.1.3 Local Governments and Federal Fiscal Arrangements

Local governments distribute lump-sum transfers to local households $T_{loc,s}$ and purchase government expenditures, $G_s$. Local governments only consume local non-tradable goods and public consumption is purely dissipative. In addition, local governments receive or pay intergovernmental transfers, $T_{gg,s}$ (read ‘government to government’), through the federal budget and obtain payments from the central bank, $T_{cb,s}$. The local governments’ budget constraints are

$$T_{loc,s} + P_{N,s}G_s = T_{gg,s} + T_{cb,s}$$
and $$T_{loc,s}^* + P_{N,s}^*G_s^* = T_{gg,s}^* + T_{cb,s}^*,$$

(3.17)

The payments by the central bank are the seignorage revenues from issuing new money. Consequently, the payments sum up to

$$T_{cb,s} + T_{cb,s}^* = (M_{s}^{MU} - M_{s-1}^{MU}),$$

(3.18)

where $M_{s}^{MU}$ denotes the union wide money supplied by the common central bank. In our model, the federal government serves only as a balance sheet. The two constraints for transfers among private households across regions and for transfers between the two local governments are

$$T_{hh,s} + T_{hh,s}^* = 0$$
and $$T_{gg,s} + T_{gg,s}^* = 0,$$

(3.19)

respectively.

3.1.4 Market Clearing, Terms of Trade, and the Real Exchange Rate

All households within a region are identical. In particular, they have identical initial wealth and thus they choose the same actions. As a consequence, per capita levels coincide with national aggregate levels since each of two regions is of size one.

In equilibrium, the markets for tradable and non-tradable goods clear. For the home tradable good we thus have that total production $Y_{T,s}$ must meet the sum of total home demand for the home tradable good, $C_{H,s}$, and total foreign demand of the home tradable good, $C_{H,s}^*$, i.e.

$$Y_{T,s} = C_{H,s} + C_{H,s}^*.$$  

(3.20)

4In principle, national public consumption could have been assumed to be useful. However, we do without this assumption as we will consider only the temporary deviations of allocations. Furthermore, since we evaluate the stabilization property of different transfer schemes, we want to concentrate on effects on consumption and employment only.
3.1. The Model

We can substitute the optimal fraction of total consumption expenditures that is allotted to consumption of the home tradable good in (3.8) and (3.9). Total home and foreign production of tradable goods in terms of home and foreign total consumption levels are

\[ Y_{T,s} = \alpha \gamma \left( \frac{P_{H,s}}{P_s} \right)^{-\rho} \left( C_s + RER_s^{-\rho} C_s^* \right) \]

and

\[ Y_{T,s}^* = (1 - \alpha) \gamma \left( \frac{P_{F,s}}{P_s} \right)^{-\rho} \left( C_s + RER_s^{-\rho} C_s^* \right), \]  \hfill (3.21)

respectively. The real exchange rate, \( RER_s \), is defined as the price of a foreign consumption basket in terms of the home consumption basket, ie.

\[ RER_s = \left( \frac{P_s}{P_s^*} \right). \] \hfill (3.22)

The terms of trade, \( ToT_s \), are defined as the price of home imports in terms of home exports, ie. in money prices

\[ ToT_s = \left( \frac{P_{H,s}}{P_{F,s}} \right) = \left( \frac{a^*_s W_s}{a_s^* W^*_s} \right), \quad \text{and} \quad RER_s = ToT_s^{(1-\gamma)}. \] \hfill (3.23)

The second equation in (3.23) stems from the definition of consumption-based price indices and simply states that in case of no trade in goods, ie. if \( \gamma = 0 \), the real exchange rate and the terms of trade must coincide, and if households consume tradables only (\( \gamma = 1 \)), the real exchange rate must be unity, since then preferences are identical and the law of one price holds. This is another motive for interpreting \( \gamma \) as measure of economic integration.

Together with the market clearing conditions for non-tradable consumption goods, \( Y_{N,s} = C_{N,s} + G_s \), we arrive at the total aggregate demand for home goods, namely

\[ Y_s = (1 - \gamma (1 - \alpha)) \left( \frac{P_{H,s}}{P_s} \right)^{-\rho} C_s + \alpha \gamma \left( \frac{P_{H,s}}{P_s} \right)^{-\rho} RER_s^{-\rho} C_s^* + G_s, \] \hfill (3.24)

and for the total aggregate demand for foreign goods we get

\[ Y_s^* = (1 - \alpha) \gamma \left( \frac{P_{F,s}}{P_s} \right)^{-\rho} C_s + (1 - \alpha \gamma) \left( \frac{P_{F,s}}{P_s} \right)^{-\rho} RER_s^{-\rho} C_s^* + G_s^*. \] \hfill (3.25)

Written as above, the composition of total aggregate demand has a straightforward interpretation. Take for example home aggregate demand. In (3.24) the first terms denotes home private consumption. It is overall home private consumption evaluated at relative prices of home goods \( \left( \frac{P_{H,s}}{P_s} \right)^{-\rho} C_s \) less the fraction \( \gamma (1 - \alpha) \) private households demand from abroad. In turn, the second term denotes the fraction of home goods in foreign private consumption. The last term is domestic public consumption which is completely home-biased.

The asset market clearing conditions for the bond and the money markets are

\[ B_s + B_s^* = 0, \quad \text{and} \quad M_s + M_s^* = M_{MU}^s. \] \hfill (3.26)
Together with home current account identity we get the global equilibrium

\[ Y_{s}^{MU} = C_{s}^{MU} + G_{s}^{MU}. \]  

(3.27)

It states that aggregate income within the union is equal to the union-wide aggregate public and private consumption.

In order to analyze the effects of unanticipated temporary productivity and demand shocks, we take log-linear approximations of the model around a completely symmetric steady state. In this initial steady state we have \( \bar{B} = \bar{B}^{*} = 0, \bar{G} = \bar{G}^{*}, \bar{a} = \bar{a}^{*}, \) and \( \bar{\alpha} = \frac{1}{2}. \) We delegate all derivations to the appendix. Instead, we directly turn to the solution of the log-linearized model.

3.2 Solution of the log-linearized Model

Our model exhibits nominal rigidities as households have to set their wages one period in advance. As a consequence, when defining the equilibrium conditions, we have to distinguish between long-run equilibrium conditions, i.e. the long-run flexible wage equilibrium conditions, and the short-run equilibrium conditions with fixed wages.

The long-run equilibrium is given by the per capita versions of the Euler condition (3.13), optimal money demand (3.14), Fisher parity (3.16), optimal labor supply, i.e. the wage setting equation (3.15), aggregate production \( Y_{s} = a_{s}L_{s}, \) aggregate demand (3.24), the price level (3.10), all the foreign counterparts of these conditions, home budget constraint (3.11) and the transversality condition, and market clearing for the nominal bonds and for money (3.26).

In the short-run, however, households cannot adjust their wages to unanticipated shocks. With preset nominal wages, employment will be completely determined by the domestic labor demand. Thus, in the short-run, the equality of the marginal utility derived from consuming additional income and the disutility from labor need not hold. Consequently, the set of short-run equilibrium conditions is the same as the set of long-run equilibrium conditions but without (3.15) and its foreign counterpart.

Since we consider unanticipated temporary shocks to productivity and to the preferences for tradable consumption goods, changes in the long-run equilibrium result only from the short-run current account imbalances. As households want to smooth their consumption over time, they disperse the effects of the shocks over time. Consequently, the households have to adjust their net foreign asset positions accordingly. It is instructive to begin the analysis of the transmission of temporary shocks by looking at the impact of changes in the net foreign asset positions on long-run consumption, production, and employment.
3.2. Solution of the log-linearized Model 75

3.2.1 Long-run implications of productivity and demand shocks

The aggregate long-run changes in the monetary union are the population weighted sum of national changes for consumption, production and labor. By assuming the two regions to be completely symmetric initially, the log-linearization around the symmetric steady state implies locally that national deviations exactly offset each other, i.e., we have that $\hat{C}^{MU} = \hat{Y}^{MU} = \hat{L}^{MU} = 0$. Consequently, domestic and foreign deviations are exactly of opposite signs. We therefore discuss the effects on domestic variables only.

Consumption

The shift of home private steady state consumption to the new steady state level in response to a shift in the net foreign assets reads

$$\hat{C} = \frac{1 + \rho (2 - \gamma)}{2 (\rho (2 - \gamma) - (1 - \gamma))} \delta \hat{B} = \Lambda_{\hat{C}, \hat{B}} \hat{C},$$

(3.28)

where $0 < \Lambda_{\hat{C}, \hat{B}}$. An increase in home net foreign assets increases steady state income by the annuity $\delta \hat{B}$. Higher permanent income leads to a higher consumption level. This, in turn, causes higher wage claims and, subsequently, an increase in prices of domestic final goods for two reasons: First, the opportunity cost of labor supply increases. Second, the demand for labor rises due to the higher consumption level. The foreign households perceive exactly the opposite effect. Consequently, the terms of trade rise and cause an expenditure switching effect. Whether or not this expenditure switching will dampen the effect of higher permanent income depends on the degree up to which consumption goods are substitutable.

Production and Employment

As the mechanism described above works through the overall goods demand and optimal wage setting, the relation between consumption and production is

$$\hat{Y} = \frac{(1 - \gamma) - \rho (2 - \gamma)}{1 + \rho (1 - \gamma)} \hat{C} = \Lambda_{\hat{Y}, \hat{C}} \hat{C},$$

(3.29)

with $-1 \leq \Lambda_{\hat{Y}, \hat{C}} \leq 0$.

---

On the notation: Long-run percentage deviations from the steady state levels are marked by $\hat{X}$, short-run percentage deviations by $\check{X}$. Deviations of variables that are zero in steady state are expressed in terms of steady state consumption expenditures, e.g. $\check{B} = \frac{\delta n}{\bar{P} \bar{C}}$. To alleviate the economic interpretation of the transmission processes, we introduce notation for the constant terms that stem from the log-linear approximations. Each constant is dubbed $\Lambda_{X, Y}$ and its two indices indicate the effect of a change in the former on the latter. I.e. $\Lambda_{X, Y}$ is the constant for $X = \Lambda_{X, Y} Y$.

We assume throughout the analysis that the elasticity of substitution between two goods suffices $1 - \gamma < \rho$. This is quite innocuous as empirical studies suggest and much less restrictive than the assumption in the NOEM literature that $1 < \rho$ when consumption goods are supplied monopolistically to ensure well-behaved demands.
From these considerations we deduce that an increase in home per capita net foreign assets leads to a change in production and employment by

$$\hat{Y} = \hat{L} = -\frac{\delta}{2} \hat{B}. \quad (3.30)$$

### 3.2.2 Short-run implications of productivity and demand shocks

Now, we consider the short-run implications of unanticipated temporary innovations of the terms of trade, of the demand for tradable goods, of federal transfers among households, and of intergovernmental transfers and portray the transmission process in detail. The union-wide aggregates of consumption, production, and employment are invariant to fully asymmetric shocks. They are only affected by a change in the union-wide money supply and the proximate change in the nominal interest rate common to both regions. This reason leads us to consider only completely asymmetric shocks that bear no aggregate effects since the common component in shocks can be targeted by adjusting the union-wide money supply. An immediate consequence is, as above, that foreign per capita variables change by exactly the opposite magnitude.

**Consumption**

In order to ease the illustration of the impact of shocks on the consumption level, we separate the direct equilibrium response induced by the change in exogenous variables from the indirect equilibrium effects induced by the change in the consumption level itself. The short-run implication of the domestic consumption level to the different exogenous innovations can be expressed by

$$\hat{C} = \Lambda_{C,DI} \left( \frac{1}{2} \Lambda_{DI,\tilde{T}T} \tilde{T} + \Lambda_{DI,\alpha} \hat{\alpha} + \hat{T}_{hh} + \hat{T}_{gg} \right), \quad (3.31)$$

$$0 < \Lambda_{C,DI}. \quad \text{In the equation above, the terms in brackets depict the change in income due to direct equilibrium responses to exogenous innovations. We define this income as 'consumption-disposable-income' (subscripted by } DI \text{) as it denotes the part of income that is dedicated to consumption purchases only. It will be explored in more detail below.} \Lambda_{C,DI} \text{ captures the equilibrium feedback effect of a change in consumption on consumption-disposable-income. It consists of the net income effect caused by the change in the consumption level and the savings effect in response to a change in the consumption level today as households smooth their consumption path.}$$

---

7In the following expression we have already incorporated the assumption that national fiscal authorities cannot adjust their national lump-sum taxes in the short-run, i.e. $\hat{T}_{loc} = \hat{\tilde{T}}_{loc} = 0$, and that transfers within the federal fiscal arrangement must net out, i.e. $\hat{T}_k - \hat{T}^*_k = 2T_k$ with $k = \{hh, gg\}$. Regarding concerns about keeping national taxes temporarily fixed, this assumption serves only to distinguish the two different transfer schemes since we want to concentrate on the role and effectiveness of different federal fiscal transfers. Otherwise, intergovernmental transfers might be redistributed to households via regional lump-sum taxes and is thus equivalent to federal transfers among households.
(Innovations to the Terms of Trade) The response of contemporaneous home consumption-disposable-income to changes in the terms of trade can be separated into a net income effect and a savings effect:

\[ \Lambda_{\hat{D}I,\hat{ToT}} = (1 - \rho(2 - \gamma)) \gamma - \Lambda_{\hat{S},\hat{RER}}(1 - \gamma). \]  

(3.32)

The first part in (3.32) denotes the net income effect induced by a change in relative prices. It is the difference between the change in labor income and the change in consumption prices. The non-tradable goods do not appear as the labor income effect is exactly offset by the change in private and public expenditures for non-tradables which leaves labor demand unchanged. Clearly, if no good is traded, the net income effect is zero. For the opposite case, when all goods are traded, the net effect obviously depends on the elasticity of substitution among home and foreign tradable goods. The second part in (3.32) depicts the change in wealth, i.e., savings, in response to the innovations to the terms of trade that alter the real exchange rate, namely

\[ \Lambda_{\hat{S},\hat{RER}} = \Lambda_{\hat{B},\hat{C}} \frac{\gamma}{1 + (1 - \gamma)\Lambda_{\hat{Y},\hat{C}}} + \frac{\chi}{1 - \beta} = \Lambda_{\hat{B},\hat{C}} \Lambda_{\hat{C},\hat{RER}} + \Lambda_{\hat{M},\hat{RER}}. \]  

(3.33)

where \( 0 \leq \Lambda_{\hat{B},\hat{C}} \Lambda_{\hat{C},\hat{RER}} \equiv \Lambda_{\hat{B},\hat{RER}} \leq \frac{1}{\gamma} \). Two effects are collected in (3.33): First, an increase in the real exchange rate causes a differential of domestic and foreign real interest rates that prompts the households to alter their net foreign assets. Second, a shift in the real exchange rate and thus in the ratio of the two national price indices implies a change in relative money holdings.

(Shift in Demand) Suppose that home and foreign households temporarily prefer home tradable goods to foreign tradable goods. This shock simply shifts consumption-disposable-income by the additional labor income \( \Lambda_{\hat{D}I,\hat{\alpha}} = \gamma \) since the total production of home private consumption goods increases by exactly that ratio of tradable goods.

(Federal Transfers among private households) Obviously, changes in federal transfers among private households of the two member states of the union directly shift the disposable income without deterioration since they directly enter private households’ budget.

(Intergovernmental Transfers) Intergovernmental transfers change relative public consumption between the two regions as they cannot be directly redistributed to domestic households through local lump-sum transfers. However, intergovernmental transfers shift labor income between the regions indirectly by altering the demand for domestic and foreign non-tradable goods. Note that the two transfers in (3.33) affect consumption-disposable-income equally as we express all steady state deviations relative to steady state consumption expenditures. Nevertheless, the two transfers differ with respect to their impact on production and thus labor demand as we will see below.
The important lesson to be learned here is that as both transfers, the federal transfers among private households and the intergovernmental transfers, and the private wealth adjustments in response to unanticipated changes in relative prices affect the consumption level equally through consumption-disposable-income. Thus, to keep the consumption level unaltered, transfers have to target the wealth adjustments in response to unexpected deviations of the real exchange rate, $\Lambda_{S,RER}(1 - \gamma)$, but not equilibrium net savings. Equilibrium net savings also include adjustments in wealth due to changes in the consumption level in order to smooth the consumption path over time.

Production and Employment

In the short-run, employment is completely determined by labor demand since wages are preset. Labor demand itself is driven by two forces: It is increasing in domestic goods demand and it is decreasing in labor productivity. Deviations of the employment level are given by

$$\dot{L} = \dot{Y} - \dot{a}.$$  \hfill (3.34)

Total home production responds to shocks and policy innovations by

$$\dot{Y} = \frac{1}{2} \Lambda_{Y,TT} \dot{ToT} + \Lambda_{Y,a} \dot{a} + \Lambda_{Y,yy} \dot{T}_{yy} + \Lambda_{Y,th} \dot{T}_{hh}.$$  \hfill (3.35)

(Innovations to the Terms of Trade) Asymmetric productivity shocks affect output in the short-run only through the effect of the shift in the terms of trades because output markets are perfectly competitive and wages cannot be adjusted,

$$\Lambda_{Y,TT} = \Lambda_{C,TT} (1 - \gamma)(1 - \bar{g}) - \rho(2 - \gamma) \gamma (1 - \bar{g}) - \bar{g},$$  \hfill (3.36)

where $\Lambda_{C,TT} = \Lambda_{C,DI} \Lambda_{DI,TT}$. We encounter three effects in (3.36): First, as described above, the innovations to the terms of trade shift domestic consumption-disposable-income and thus overall domestic consumption by $\Lambda_{C,TT}$. Consequently, the demand for domestic non-tradable goods changes by its fraction of overall consumption expenditures. Second, the shift in the terms of trade leads to an expenditure switching effect for the relatively cheaper tradable good. This is the only effect on tradable goods demand since there is no aggregate change in the demand for tradable goods in response to changes in aggregate consumption expenditures as opposed to the case for non-tradable goods. Recall that foreign overall consumption changes by exactly the opposite magnitude of domestic overall consumption. By assuming that domestic and foreign households prefer domestic and foreign tradable goods alike, the change in domestic demand for domestic tradable goods offsets the change in foreign demand for the domestic tradable good. What remains is the consumption expenditure switching effect only as this effect is common to both domestic and foreign demand. As the first two effects stem from the private households reaction, they have to be evaluated by the initial steady state fraction of private consumption in output $(1 - \bar{g})$. The third effect depicts
the change in national government consumption. Public consumption changes exactly by its initial fraction of output, namely $\bar{g}$, as national budgets must balance.

\textbf{(Shift in Demand)} The preference shocks shift home total production by

\[ \Lambda_{Y, \bar{a}} = \left( \gamma + \Lambda_{C, \bar{a}} (1 - \gamma) \right) (1 - \bar{g}). \] (3.37)

Clearly, since $\bar{a} \gamma$ denotes the fraction of the home tradable good in private consumption, an increase in that fraction home and abroad changes total production directly by $2(1 - \bar{g})\bar{a} \gamma$. In addition, this creates a labor income effect which implies an increase in domestic non-tradable goods consumption depicted by the second term in (3.37).

\textbf{(Federal Transfers among private households)} Federal transfers among private households affect production only through the change in private consumption,

\[ \Lambda_{Y, \bar{T}_{hh}} = \Lambda_{C, \bar{D}I} (1 - \gamma)(1 - \bar{g}). \] (3.38)

Federal transfers among private households lead to an increase only in the demand for non-tradable goods. By the same token as the innovations to the terms of trade lead to no impact of changes in consumption-disposable-income on net tradable goods demand, federal transfers among households have no impact on the net tradable goods demand. The net effect on tradable goods demand is zero because households in one region pay the transfers that households within the other region receive. As a consequence, the effect of federal transfers vanishes as we reach the full-trade scenario. If $\gamma = 1$, half of home private consumption is produced at home, half of it is imported from abroad. But the same is true for the foreign households. Since the transfers must balance, these opposing effects cancel out. Federal ‘household to household’ transfers cannot correct output.

\textbf{(Intergovernmental Transfers)} The important difference between federal transfers among households and intergovernmental transfers is the direct effect of changing public consumption on demand, namely $(1 - \bar{g})$:

\[ \Lambda_{Y, \bar{T}_{gg}} = \Lambda_{C, \bar{D}I} (1 - \gamma)(1 - \bar{g}) + (1 - \bar{g}). \] (3.39)

Since production is completely demand determined in the short-run, intergovernmental transfers shift production and employment in the first instance. But as changes in production and employment result in changes in labor income and thus consumption-disposable-income, the effect of private consumption deviations comes in addition. This has two immediate consequences. First, recall that both transfers are expressed as deviations relative to the steady state consumption expenditures. It follows that intergovernmental transfers require a smaller transfer volume to achieve the same correction in output and employment.
fluctuations. Second, in sharp contrast to federal transfers among private households, intergovernmental transfers never lose bite to correct demand. Thus, even if we are in the full trade scenario, intergovernmental transfers can still correct output and employment because they directly affect demand.

**Net Foreign Asset Positions**

The adjustment of net foreign asset positions can be obtained from the adjustment in consumption and production. The current account imbalances are solely motivated by the changes of private households’ savings. To prevent us from repeating the effects, we skip the discussion.

### 3.2.3 Welfare

We analyze the life-time welfare effects according to Obstfeld and Rogoff (1995) and concentrate on the terms depending on consumption and employment. For simplicity we assume that households only barely utilize liquidity services from holding real money balances and take $\chi$ in (3.5) to be sufficiently small.\(^8\) In our analysis of the different federal fiscal arrangements, we approximate home residents’ life-time utility as

$$dU^R = \left( \hat{C} - \hat{L}^2 \hat{L} \right) + \frac{\beta}{1 - \beta} \left( \hat{C} - \hat{L}^2 \hat{L} \right).$$  \hspace{1cm} (3.40)

In order to see the welfare implications of stabilizing consumption and production, and thus employment, we rewrite (3.40) as

$$dU^R = \left( 1 + \frac{\beta}{1 - \beta} \left( 1 - \hat{L}^2 \Lambda_{\hat{Y},\hat{C}} \right) \Lambda_{\hat{C},\hat{C}} \right) \hat{C} - \hat{L}^2 \hat{L}. \hspace{1cm} (3.41)$$

While current employment fluctuations affect life-time utility only contemporaneously on consumption and employment, consumption fluctuations disturb residents’ welfare via three channels: Besides the direct contemporaneous effect, the desire to smooth consumption over time prompts a change of future consumption level and subsequently a change in employment. From (3.41) follows immediately that welfare fluctuations are stabilized and thus perfect insurance is provided if and only if both consumption fluctuations and employment fluctuations are deleted. Furthermore, since we only consider asymmetric shocks without aggregate risk, $\hat{C}^{MU} = \hat{Y}^{MU} = \hat{L}^{MU} = 0$, foreign welfare fluctuations follow $dU^{*,R} = -dU^R$. As a consequence, using a second-order approximation, the sum of national welfare deviations reads

$$dU^{MU,R} = dU^R + dU^{*,R} = -\left( 1 + \frac{\beta}{1 - \beta} \left( 1 - \hat{L}^2 \Lambda_{\hat{Y},\hat{C}} \right) \Lambda_{\hat{C},\hat{C}} \right) \hat{C}^2 - \hat{L}^2 \hat{L}^2. \hspace{1cm} (3.42)$$

\(^8\)Obviously, if we had motivated money holding by a cash-in-advance constraint, this restriction would have been obsolete. In the limit, if $\chi \rightarrow 0$, we enter a cashless economy as recently employed in open economy models by Benigno and Benigno (2003) and Benigno (2004).
Thus, from a union-wide perspective it is indeed optimal to completely delete fluctuations in consumption and employment resulting from unanticipated demand and productivity shocks.\footnote{Note that it is in principle constrained optimal to delete consumption and employment fluctuations as monopolistic distortions on labor markets still prevail. However, as it is standard in the New Keynesian literature, this could be corrected by redistributing proportional labor income taxes lump-sum.}

\section*{3.3 Federal Fiscal Arrangements}

We now turn to the policy analysis itself. The aim of this analysis is to compare different arrangements of fiscal federal transfers whose function is to stabilize fluctuations of consumption and employment within the member states of the monetary union. It is useful for reasons of comparison to consider first the stabilization of demand shocks and to analyze then stabilization of productivity shocks.

\subsection*{3.3.1 Stabilizing Demand Shocks}

Demand shocks disturb the composition of private tradable goods consumption. They have no impact on current prices because wages are fixed. Nevertheless, a shift in demand affects overall consumption level since it changes income disposable on consumption by changing labor income.

\subsubsection*{Stabilizing Consumption}

As the two transfer schemes affect consumption-disposable-income in (3.31) equally, consumption stabilization require identical transfer volumes

\[ \hat{T}_{hh} = \hat{T}_{gg} = -\gamma \hat{\alpha}. \]  

(3.43)

A shift in demand towards home tradable goods increases home households’ labor income by $\gamma \hat{\alpha}$. Both transfers simply redistribute this income effect. Nevertheless, the two transfer schemes differ with respect to their effects on employment. The federal transfers among households lead to fluctuations in employment by

\[ \hat{L}_{T_{hh}} = (1 - \bar{g}) \gamma \hat{\alpha}. \]  

(3.44)

The federal transfers among households neglect the direct effect of changes in demand due to the contemporaneously different composition of tradable goods consumption. Only consumption-disposable-income has been redistributed and thus the overall consumption level is stabilized.
Since the intergovernmental transfers work through goods markets, they redistribute income by redistributing demand and earnings. As a consequence, we get
\[ \hat{L}_{T_{gg}} = 0. \] (3.45)
As opposed to the direct transfers among private households, the intergovernmental transfers correct fluctuations through the change in demand for non-tradable goods. To correct demand directly means to correct labor income and hence consumption-disposable-income. Overall fluctuations are thus stabilized through transfers between national fiscal authorities. As a result, the intergovernmental transfers provide both member states perfect insurance against demand shocks in the monetary union.

**Stabilizing Employment**

Since the above also holds true for stabilizing consumption, it must be true for stabilizing employment. We have that
\[ \hat{T}_{gg} = -\gamma \hat{\alpha}. \] (3.46)
Transfers among households are
\[ \hat{T}_{hh} = \hat{T}_{gg} - \frac{(1 - \bar{g}) (1 - \gamma)}{(1 - \gamma) \Lambda_{C,D}} \gamma \hat{\alpha}. \] (3.47)
Consequently, transfers among private households cannot stabilize both consumption and production fluctuations at the same time. Moreover, these transfers face the problem that for states to be strongly engaged in trade, the volume of the transfer may grow very large. As direct transfers to households are always spent on domestic as well as on foreign tradable goods, they flow back as additional demand. The implication is that by ever redistributing the additional earnings, the transfers among households amplify the effect of demand shocks. Production and employment cannot effectively be stabilized.\(^{10}\)

**Perfect Insurance through Federal Transfer Scheme**

From the above we can directly deduce that intergovernmental transfers alone provide perfect insurance. We summarize these findings in our first result:

**Proposition 3.1** In case of preference shocks that shift demand from the tradable good produced in one region to the tradable good produced in the other region, transfers between fiscal authorities can always stabilize fluctuations in consumption, output and employment at the same time. Intergovernmental transfers thus eliminate welfare fluctuations and provide perfect insurance.

\(^{10}\)It is important to qualify this statement against the background of our assumption that wages are temporarily fixed. The amplifying effect of federal transfers among households implies that the impact of small demand shocks becomes too large so that both the assumption of fixed wages and thus demand determined labor supply as well as the accuracy of linearization are no longer justifiable.
3.3.2 Stabilizing Productivity Shocks

Let us next study the stabilization of productivity shocks. As we only consider completely asymmetric productivity shocks, we set $\hat{a} = -\hat{a}^*$. Due to sticky wages, productivity shocks alter marginal costs of producing tradable and nontradable goods and cause a shift in the terms of trade by $ToT = -2\hat{a}$.

### Stabilizing Consumption

By the reason that the two transfers affect private consumption identically, they coincide in volume if they target consumption fluctuations. We get

$$\hat{T}_{hh} = \hat{T}_{gg} = \left((\rho(2 - \gamma) - 1) \gamma + \Lambda S_{\text{RER}}(1 - \gamma)\right) (-\hat{a}).$$  \hspace{1cm} (3.48)

In order to stabilize the consumption level, both transfers correct the change in consumption-disposable-income that is due to the shift in productivity and relative prices. In contrast to demand shocks, however, transfers have to redistribute the income effect due to the change in consumption prices and the savings response to changes in the real exchange rate, too. This comes in addition to the labor income effect whose influence on consumption-disposable-income works equivalent to the effect of a demand shock. To gain more insight about what these transfers do to stabilize consumption fluctuations, it proves useful to look at the two polar cases of full economic integration and no economic integration at all. In case of full economic integration, when $\gamma = 1$, the consumer price levels in the two regions coincide as preferences do. As a consequence, domestic and foreign households experience the same incentive to adjust wealth in response to price level deviations because the real interest rates coincide. In equilibrium, no direct adjustment to changes in prices occur.\(^\text{11}\) Furthermore, the income effect of consumer price deviations vanishes since the changes in domestic and foreign prices offset each other. Transfers then redistribute only the changes in labor income. In case of no economic integration, when $\gamma = 0$, a shift in the terms of trade induces the households only to adjust current consumption. In equilibrium, there is no effect on savings at all, neither on net foreign assets nor on money holding. The reason has to be seen in two completely different effects: First, households do not adjust their net foreign assets, and second, as no foreign assets are traded, and neither are goods, the balance of payments implies that money holdings remain unchanged. Since the labor-leisure trade-off

\(^{11}\)Changes in net foreign assets as well as changes in money holdings occur then in equilibrium only as to smooth the consumption deviations that are induced by the labor income effect. In case of Cobb-Douglas preferences over tradable goods, i.e. $\rho = 1$, the deviation in consumption-disposable-income is zero because the expenditure switching effect exactly offsets the labor income effect. No adjustments in savings are made whatsoever the shock to the terms of trade is. Cole and Obstfeld (1991) were the first to make the point that financial markets are redundant if preferences over goods are Cobb-Douglas. More recently, the redundancy was employed in the context of NOEM for instance by Obstfeld and Rogoff (2000) and more prominently by Corsetti and Pesenti (2001).
is not binding in the short-run, households have to accommodate their consumption exactly to the change in prices.\textsuperscript{12} Consequently, stabilizing consumption then requires the transfers to relieve the tension of money holdings caused by the balance of payments.

Recall from (3.32) that the sign of the savings response to changes in the real exchange rate is always positive and that the sign of the net income effect depends on whether domestic and foreign tradable goods are substitutes or complements in consumption.\textsuperscript{13} If tradable goods are substitutes in consumption, a positive shock to domestic productivity that lowers the terms of trade causes a positive net income effect. The increase in labor income due to the expenditure switching of tradable goods and the gain of the consumption price effect is larger than the loss in labor income due to the drop in labor demand directly caused by the shock. Consequently, the first term in (3.48) is positive, too. Since the transfers redistribute the effects on consumption-disposable-income, transfers are always negative if any trade in goods occurs or zero otherwise. In contrast, if the two tradable goods are complements, the surprising case might occur that the region hit by a positive productivity shock receives transfers from the region hit by a negative shock. This becomes most obvious in the full trade scenario where savings effects are shut off. Complementarity implies that domestic households respond to the negative change in labor income by reducing consumption of both domestic and foreign tradable goods. Foreign households respond to their positive change in earnings by increasing consumption of both domestic and foreign tradable goods. As a consequence, the expenditure switching to the domestic tradable good never offsets the change in home income. Equivalently, the foreign net income effect is never negative. Thus, to stabilize consumption the ‘unlucky’ region that has perceived a negative productivity shock has to pay the transfers. In summary we arrive at

**Proposition 3.2** In case of productivity shocks, stabilizing consumption requires the two transfer schemes to be the same size. Transfers are always negative for a region having perceived a positive productivity shock if goods are substitutes. Transfers might be positive if goods are complements and regions are close to full economic integration.

To see the difference between the two transfer schemes, we have to consider employment. Employment changes under federal transfers among households by

\[
\hat{L}_{T,h} = (1 - \rho(2 - \gamma)\gamma)(1 - \bar{g})(-\hat{a}). \tag{3.49}
\]

\textsuperscript{12}Note that \(A_{C,D,1}A_{D,1,T,\varphi^T}|_{\gamma=0} = -1.\)

\textsuperscript{13}Two goods are said to be substitutes in consumption if the marginal utility from consuming one good is decreasing in consumption of the other good. The two goods are said to be complements if the marginal utility is increasing. A sufficient condition for substitutability is that the intratemporal elasticity of substitution is greater than the intertemporal elasticity of substitution. In our log-utility case it is the case if \(\rho > 1.\) If \(\rho < 1,\) then the two goods are complements in consumption. See, e.g., Svensson (1985) and Corsetti and Pesenti (2001).
As the federal transfers among private households correct only the shift in national consumption levels, the immediate impact of productivity shocks on labor demand, namely the expenditure switching effect and the change in public consumption on goods demand as well as the direct effect of the productivity shock on labor demand itself remain.

The reaction of employment when the intergovernmental transfers are implemented can be stated as

\[
\hat{L}_{T_{gg}} = \left(1 + \Lambda_{\hat{\bar{S}}, \hat{R}_{ER}}\right)(1 - \gamma)(1 - \bar{g})(-\hat{a}.
\]

Since the transfers are the same size, the resulting employment fluctuations differ exactly by the direct transfer induced shift in public expenditures evaluated in terms of private consumption. The direct shift in public expenditures, however, has two effects: First, as it was the case when stabilizing demand shocks, it completely offsets the direct labor demand effects. Second, it induces a new source of employment fluctuations, namely the redistributed price income effect and the savings response to changes in the real interest rate. The ranking of employment fluctuations implied by the two transfers is ambiguous. In particular, in case of trade autarky, i.e. no economic integration at all, households would like to adjust money holdings but the balance of payments prevents money flows. Transfers that stabilize consumption fluctuations compensate for the desired flows in money holdings. But since intergovernmental transfers have to correct the desired money flows through a change in labor income, this additional element in national public demand for non-tradable goods leads to an additional fluctuation in employment. Thus, intergovernmental transfers lead to larger employment fluctuations than federal transfers among private households. In the full trade scenario, households do not adjust wealth in response to the change in the real exchange rate. The only resulting effects on employment are the direct effects of expenditure switching, of national public consumption, and of labor productivity. But these direct labor demand effects are exactly the effects targeted by the intergovernmental transfers so that no further change in consumption-disposable-income remains. As a result, and reminiscent of demand shocks, employment is fully stabilized.

**Proposition 3.3** In case of productivity shocks, stabilizing consumption fluctuations implies for employment, that transfers between national fiscal authorities dominate federal transfers among private households if

\[
|1 - \rho(2 - \gamma)\gamma| > \left(1 + \Lambda_{\hat{\bar{S}}, \hat{R}_{ER}}\right)(1 - \gamma).
\]

They lead to smaller fluctuations in employment and in welfare.

The intuition behind this finding is straightforward: Since federal transfers among private households are incapable to capture the direct labor demand effect, \(|1 - \rho(2 - \gamma)\gamma|\), that leads to fluctuations in employment, it is the income effect due to changes in consumer prices and the savings response to changes in the real interest rate, \(\left(1 + \Lambda_{\hat{\bar{S}}, \hat{R}_{ER}}\right)(1 - \gamma)\), which (in case of intergovernmental transfers) come as an additional source of employment fluctuation. Thus, intergovernmental transfers dominate transfers among private households.
as long as the direct labor demand fluctuations are sufficiently high. Note that if the two regions are relatively closely integrated and the price income and savings effect vanish, the condition in the proposition above is satisfied.

Stabilizing Employment

We now turn to the analysis of employment stabilization. The transfers among private households to stabilize employment are

\[
\hat{T}_{hh} = -\frac{\Lambda Y_{TT} + 1}{\Lambda C_{DI}(1 - \gamma)(1 - \bar{g})}(-\hat{a}).
\]

In contrast, transfers between the national fiscal authorities look

\[
\hat{T}_{gg} = -\frac{\Lambda Y_{TT} + 1}{\Lambda C_{DI}(1 - \gamma) + 1}(1 - \bar{g})(-\hat{a}).
\]

Both transfers have to offset the effect of productivity shocks on private consumption levels, the expenditure switching effect and the change in national government consumption as described in (3.36). Furthermore, they have to correct the direct impact of the shift on labor productivity. Comparing the two transfer schemes yields that they are of equal sign, but intergovernmental transfers are always smaller than federal transfers among private households. From (3.31) we know that both transfers affect consumption equally. Nevertheless, whether intergovernmental transfers lead to less fluctuations in consumption is again ambiguous. For federal transfers among households we get

\[
\hat{C} \hat{T}_{hh} = \rho(2 - \gamma)\gamma - 1(1 - \gamma)\left(\Lambda C_{DI}(1 - \gamma) + 1\right)(1 - \bar{g})(-\hat{a}).
\]

As federal transfers redistribute consumption-disposable-income by more than the change in the national consumption level in order to offset the direct impacts of productivity shocks on labor demand, an additional source of consumption fluctuations needs to be created. When the two regions are in trade autarky and no substitution effect matters at all, fluctuations in public consumption following the change in domestic goods prices and the shift in labor productivity are the only source of private consumption deviations. In the full trade scenario, we encounter the effect equivalent to demand shocks where transfers among private households cannot stabilize output and employment. By the expenditure switching effect, each unit of payment received will be spent proportionately on both domestic and foreign goods. Since production is completely demand determined, stabilization cannot be achieved.

Intergovernmental transfers induce consumption to fluctuate by

\[
\hat{C} \hat{T}_{gg} = -(1 - \Phi)\Lambda C_{DI}\Lambda S_{RE} \left(\Lambda \hat{S}_{RE}(1 - \gamma)\right)(1 - \gamma)(-\hat{a}).
\]

where \( \Phi \) is a constant, \( 0 < \Phi < 1 \).

\[\text{It is defined as } \Phi = \frac{\Lambda C_{DI}(1 - \gamma)}{\Lambda C_{DI}(1 - \gamma)}14.\]

In order to stabilize labor demand fluctuations,
3.3. Federal Fiscal Arrangements

Intergovernmental transfers redistribute the direct effects on labor demand as well as the change in national consumption level induced by the change in consumption-disposable-income. As intergovernmental transfers directly shift labor demand, these direct effects are perfectly offset. What remains as consumption fluctuations is the additional shift in labor income in order to also offset the change in national consumption caused by the price income effect and the savings response to changes in the real interest rate. It is important to recognize that the magnitude of consumption fluctuations depends on the net equilibrium effects of savings and the price income effect. Whereas the emphasis in case of consumption stabilization lies completely on the response of the change in consumption-disposable-income, it is of course equilibrium net effect that determine the change in the consumption level itself.

**Proposition 3.4** In case of productivity shocks, stabilizing employment fluctuations implies that transfers between national fiscal authorities are always of lower volume than transfers among private households. Regarding consumption fluctuations, transfers between the national fiscal authorities dominate transfers among private households if

\[
\left| \rho (2 - \gamma) \gamma - 1 \right| > \Phi \left( 1 + \Lambda \bar{R} \bar{E} \bar{R} \right) (1 - \gamma).
\]

They lead to smaller fluctuations in consumption and in welfare.

To gain more intuition, let us again discuss the two extremes of economic integration. In trade autarky, there are no savings. As we know from the discussion about the impacts of terms of trade innovations on production, a productivity shock in autarky leads to an equal increase in production. The fall in prices leads households and national fiscal authorities to adjust their consumption by exactly the same magnitude. But since labor productivity prompts a one-to-one decrease in labor demand in addition to the goods demand effect, employment doesn’t change. As a consequence, transfers are needless. In the full trade scenario, there are neither savings on account of deviations in relative prices and the real interest rate nor an income effect due to changes in consumption prices. The transfers have to redistribute changes in labor demand due to changes in goods demand in addition to the shift in labor productivity only. Intergovernmental transfers do so directly via the goods markets. Because consumption-disposable-income is altered by changes in labor income, intergovernmental transfer also implicitly stabilize consumption fluctuations. This case is again reminiscent of the intergovernmental transfer stabilizing demand shocks.

**Perfect Insurance through Federal Transfer Scheme**

By now the differences in working and effectiveness of the two transfer schemes have become clear. As we can deduce from the change in utility in (3.41), a transfer scheme that stabilizes consumption and employment also stabilizes welfare. In this sense this transfer scheme provides perfect insurance. Because neither transfer can fully stabilize consumption, production and employment at the same time, none of the two transfers generally provides
perfect insurance alone. The differences between the two transfers, however, indicates that a combination of both will do.

Proposition 3.5 In case of productivity shocks, we can find a combination of transfers between the fiscal authorities and among private households such that consumption and employment fluctuations are fully stabilized. Furthermore, fluctuations in welfare are extinguished and perfect insurance is provided.

The combined use of transfers to stabilize welfare fluctuations require that intergovernmental transfers correct the direct labor demand effect induced by the expenditure switching, the change in national public demand by balancing national budgets, and the shift in labor productivity, namely

\[ \hat{T}_{gg} = \rho(2 - \gamma)\gamma - 1) (-\tilde{a}), \quad (3.55) \]

whereas the direct transfers among households correct the shift in consumption-disposable-income caused by the savings response to changes in the real interest rate and changes in consumer prices, ie.

\[ \hat{T}_{hh} = -\Lambda_{DI,TOT}(-\tilde{a}) - \hat{T}_{gg} = \left( \Lambda_{S,RER} + 1 \right) (1 - \gamma)(-\tilde{a}). \quad (3.56) \]

The insight of how the combination of federal transfers among households and intergovernmental transfers works is as follows: In combination, the sum of the two transfers must delete any change in consumption-disposable-income. This implies that the consumption level is stabilized. The division of transfers depend on the different working in the goods markets. Since the two transfers jointly correct deviations of the demand for non-tradable goods induced by changes in consumption-disposable-income, the direct impact of intergovernmental transfers on goods markets enables the correction of the fluctuations of labor demand resulting from the expenditure switching effects and the change in national government consumption as well as the direct shift of labor demand caused by the change in labor productivity. What thus remains to be redistributed by the federal transfers among households is the change in savings and the income effect due to the change in consumer prices.

3.4 Conclusion of Chapter 3

The aim of this paper was to study the role of federal fiscal transfers as stabilizers in monetary unions. In particular, we considered two different transfer schemes already proposed by the MacDougall Report on the feasibility of EMU: A direct transfer among the households within different member states and an indirect transfer that involves payments between national fiscal authorities.
The properties of the two transfers differ with respect to their transmission. While direct transfers among private households enter households’ budget directly and thus affect the allocation through the households’ optimal choices, intergovernmental transfers affect allocations through the non-tradable goods markets. This implies a very important distinction between the effectiveness of the two transfers.

For the analysis of different federal transfer schemes we set up a dynamic general equilibrium model with preset nominal wages. The monetary union is exposed to unanticipated temporary asymmetric shocks. Specifically, we analyzed technology shocks that alter aggregate national labor productivity and subsequently prices as well as preference shocks that shift the demand from tradable goods produced in one region to tradable goods produced in the other region. This allowed us to revisit the example Mundell used in his seminal work.

The key insight of our analysis is that economic fluctuations can be best targeted by the type of transfer that directly affects the source of these fluctuations. To be more precise, consider first a shift in demand. Prices remain unaltered because wages are preset. The only disturbance thus stems from the shift in production and thereby labor income. As we have seen, channelling back the shift in demand through intergovernmental transfers that directly affect goods demand leads to perfect insurance. On the contrary, technology shocks lead to changes in labor productivity and subsequently alter prices and thereby the terms of trade. Now we encounter two effects. First, a change in productivity and the terms of trade leads to a labor income effect. Thus, similarly to the preference shock, labor demand is directly affected and can be targeted by the intergovernmental transfer. Second, a change in prices and thus in the real interest rates leads to changes in savings and to a consumer price effect that both change consumption-disposable-income and following private goods demand. Since federal transfers among households redistribute directly through private budgets, they are more suited to stabilize fluctuations than intergovernmental transfers. In fact, none of the two transfers alone can stabilize national fluctuations induced by productivity shocks. An adequate combination of federal transfers among households and intergovernmental transfers, however, deletes fluctuations in employment and consumption. Welfare fluctuations are completely avoided and in this sense the combination of the two transfers provide perfect insurance against asymmetric productivity shocks to both member states.

Our results give a clear notion of the desirability of a federal fiscal arrangement consisting of federal transfers among private households and intergovernmental transfers among national fiscal authorities as proposed in the MacDougall Report since it can stabilize economic impacts of asymmetric shocks. As such, this federal insurance scheme reduces fluctuations and in particular cyclical unemployment within the member states. As a consequence, a federal fiscal insurance arrangement definitely remedies potential tensions among member
states.
Concluding Remarks

In the academic literature on international coordination of macroeconomic policies, two striking observations can be made: First, the paradigm in the literature on monetary policy coordination is that gains from coordination are fairly small. Monetary policy is considered to be concerned with the stabilization of macroeconomic fluctuations only. Gains from policy coordination arise then from the inability of a certain exchange rate system to react appropriately to asymmetric shocks or from preventing countries from the use of stabilization policies to strategically manipulate the terms of trade. Second, the analysis of international monetary policy coordination and the analysis of fiscal policy coordination in strategic setups are effectively uncoupled. That is, the question of how the international coordination of only a part of national macroeconomic policies does change the strategic behavior of the independently conducted remaining part of national policies has been largely ignored.

This dissertation contributes to these observations and comes up with two important results: First, in Chapter 1 I demonstrate that gains from international monetary policy cooperation can be substantial when policymakers jointly prevent structural inefficiencies in the supply of labor and hence production arising from strategic considerations rather than coordinate the stabilization of exogenously driven fluctuations. I demonstrate that the welfare gains from stabilizing macroeconomic fluctuations are generically quite limited because they are of second-order whereas the gains from preventing strategically motivated conflicts over inflation targets are of first-order. The second contribution presented in Chapter 2 is that international policy coordination requires to include both monetary as well as fiscal policy because both sides dispose of effective policy instruments that enable the strategic manipulation of the country’s terms of trade. Hence, the coordination of only one part of national macroeconomic policies through an international agreement or institutional framework still leaves room for national authorities to still unilaterally manipulate the terms of trade by means of other uncoordinated policy instruments. As a result, the first two chapters yield a clear proposition: Gains from policy coordination can be substantial as they are of first-order, but policy coordination must in principle include all instruments that can be used by national authorities to unilaterally manipulate the
country’s terms of trade. In Chapter 3, I analyze the properties of a specific form of policy coordination: National monetary policies are merged within a monetary union and national fiscal policies are coordinated by means of a federal fiscal transfer scheme. By giving up the exchange rate flexibility, the member countries of the monetary union sacrifice an effective policy instrument that can be used to switch expenditures between member countries in order to offset economic shocks that have adverse impacts on these countries. The built-in fiscal transfer system collects taxes from some member countries and pays transfers to other member countries in order to alleviate the economic consequences of adverse shocks. The key result of Chapter 3 is that an appropriate chosen system of federal fiscal transfers indeed provides perfect insurance.

The relevance of these results can be best highlighted in the context of the European Economic and Monetary Union (EMU). According to the analysis in Chapter 1, gains from the convergence of the member countries’ national inflation targets down to a common inflation target spelled out by the European Central Bank of about 2% are of first-order. As the analysis predicts, losses implied by the supposed increase in the fluctuation of macroeconomic variables due to the lack of national monetary policy instruments as well as the exchange rate flexibility are rather of second-order. Consequently, given that national fiscal stances didn’t respond in a strategic manner to the formation of EMU itself, there is a - theoretical - prospect of substantial gains to the member countries that even might be larger than the suspected losses of giving up the independent national monetary policy.

To ensure that member countries really seize the gains from forming the monetary union, however, the EMU also requires the coordination of a presumably wide range of fiscal policies which might in principle be used to strategically manipulate the individual country’s terms of trade. Actually, there is already a special form of national fiscal policy coordination established within the EMU. Europes fiscal rules are well known, but they solely focus on national budgetary deficits and public debts. The Maastricht Treaty (as implemented by the Stability and Growth Pact (SGP)) requires the member countries to keep budget deficit below 3 percent of GDP and the government debt to not exceed 60 percent of GDP. The results of Chapter 2 indicate the requirement of the coordination of other fiscal policy instruments including labor income taxes as well as taxes on consumption expenditures, too. Whereas the result of strategic shifts of national fiscal policy conduct in response to the formation of the EMU itself is rather new, the policy implications are not. In fact, in his report, Delors (1989) recommended not only the binding limits on national budget deficits and public debts but also the coordination of national fiscal policies. In contrast to the strategic aspects raised in Chapter 2, his concern revolved around the lack of exchange rate flexibility which would cause tensions within the monetary union that may even lead to a breakdown of the union if no adequate federal fiscal institutions provide the
member states of the EMU with insurance against asymmetric shocks among the member states. As demonstrated in Chapter 3, a built-in system of federal fiscal transfers among member states is indeed able to perfectly insure the member states against adverse shocks. The results in Chapters 2 and 3 put Delors’ recommendation into a new light as they make a good case for the coordination of national fiscal policy conduct going beyond the SGP.

Nevertheless, with regard to the EMU, this thesis only takes a first step in the direction of the analysis of the need for more fiscal policy coordination in Europe. On the one hand, the results in Chapter 3 suggest that in the European context an important role could be attributed to a system of federal fiscal transfers. In particular, two important questions are worth to be answered: First, in addition to its capability to provide perfect insurance to the member states, does there exist an additional role of federal fiscal transfers to effectively correct incentives of national fiscal authorities to behave strategically? Second, and more relevant against the background of the ongoing controversy over the pros and cons of SGP, does an adequate system of federal fiscal transfers render the deficit rules redundant? On the other hand, future research must identify the empirical importance of the strategic conduct of national fiscal policies in the EMU. In particular, the central empirical question is whether Europe has indeed experienced a shift in national fiscal policy conduct as theoretical results in Chapter 2 predict. As the propositions made in this thesis would call for major political changes in Europe, it is indispensable to thoroughly determine the empirical relevance of the underlying game-theoretic arguments. Moreover, the theoretical postulation of a European federal fiscal transfer mechanism necessitates the quantitative assessment of different transfer schemes. These empirical and quantitative assessments only can form the basis of proper recommendation of federal fiscal policy coordination in Europe through an appropriately designed federal fiscal transfer scheme for the EMU.
Appendix

A Appendix to Chapter 1

A.1 Model Specification

Technology and Production The technologies to produce the tradable and the non-tradable goods are identical within the region:

\[ Y_{j,s} = A_s \left( \int_0^1 L_{j,s} \frac{e^{-d}}{\theta} \, di \right)^{\frac{\theta}{\theta - 1}}, \quad (A.1) \]

where \( \theta > 1 \), for \( j \in \{HT, HN\} \). The aggregate productivity of labor \( A_s \) is subject to shocks. Foreigners share the same aggregation technology but with a different productivity level. Factor cost minimization implies

\[ L_{i,j,s} = \frac{1}{A_s} \left( \frac{W_{i,s}}{W_s} \right)^{-\theta} Y_{j,s} \quad (A.2) \]

to be the optimal demand for labor input of quality \( i \) at wage \( W_{i,s} \) and an overall wage level

\[ W_s = \left( \int_0^1 W_{s}(1-\theta) \, di \right)^{\frac{1}{1-\theta}}. \quad (A.3) \]

The associated aggregate demand for a specific type of labor is

\[ L_{j,s}^i = \left( \frac{W_{i,H,s}}{W_{H,s}} \right)^{-\theta} L_{s}, \quad (A.4) \]

where \( L_s \) is Home aggregate demand for labor. By perfectly competitive goods markets the prices for Home tradable and non-tradable goods are

\[ P_{H,s} = P_{N,s} = \frac{W_s}{A_s}. \quad (A.5) \]

Total aggregate output of the Home tradable good is \( Y_{HT,s} \) and of the Home non-tradable good \( Y_{HN,s} \). For the Foreign region the corresponding expressions apply.
Households  The household’s problem is to maximize its expected lifetime utility

\[ U_t = E_t \sum_{s=t}^{\infty} \beta^{s-t} u_s, \]

where

\[ u_s = \left( C_s^{1-\rho} - \frac{1}{\rho} L_s^{\nu} \right) \]

with

\[ C_s = \frac{C_{T,s} C_{H,s N,s}^{1-\gamma}}{\gamma(1-\gamma)(1-\gamma)}, \]

\[ C_{T,s} = 2C_{H,s T,s}^{1/2}, \]

\[ C_{F,s} = 2C_{F,s T,s}^{1/2}, \]

and \( 0 < \beta < 1, \rho > 0, \nu \geq 1, \) and \( 0 \leq \gamma \leq 1, \) by deciding over bond and cash holdings, consumption, and their monopolistic wages subject to the period asset market constraint

\[ M_s^i + B_s^i \leq W_s^i + X_s^i \quad \forall s \geq t, \quad \text{(A.7a)} \]

the period cash constraints

\[ P_s C_s^i \leq M_s^i \quad \forall s \geq t, \quad \text{(A.7b)} \]

the law of motion for nominal wealth

\[ W_{s+1}^i = M_s^i + R_s B_s^i - P_s C_s^i + W_s^i L_s + T_s^i \quad \forall s \geq t, \quad \text{(A.7c)} \]

and subject to the demand for their specific type of labor

\[ L_s^i = \left( \frac{W_s^i}{W_s} \right)^{-\theta} L_s \quad \forall s \geq t. \quad \text{(A.7d)} \]

Optimal Decisions  Individual optimization yields for any person \( i \) the following composition of the Home real consumption index between the tradable commodities index and the non-tradable good

\[ C_{T,s} = \gamma \left( \frac{P_{T,s}}{P_s} \right)^{-1} C_s, \quad \text{and} \quad C_{H,s N,s} = (1 - \gamma) \left( \frac{P_{H,s}}{P_s} \right)^{-1} C_s, \quad \text{(A.8)} \]

and between Home and Foreign tradable commodities

\[ C_{H,s T,s} = \frac{1}{2} \left( \frac{P_{H,s}}{P_{T,s}} \right)^{-1} C_{T,s}, \quad \text{and} \quad C_{F,s T,s} = \frac{1}{2} \left( \frac{P_{F,s}}{P_{T,s}} \right)^{-1} C_{T,s}. \quad \text{(A.9)} \]

The corresponding Home consumption-based price indices are

\[ P_s = P_{T,s}^{1-\gamma} P_{H,s N,s}^{(1-\gamma)} \quad \text{and} \quad P_{T,s} = P_{H,s T,s}^{1/2} P_{F,s T,s}^{1/2}. \quad \text{(A.10)} \]

Next, let \( \kappa \) and \( \iota \) denote the Lagrange-multipliers on the asset market constraint (A.7a) and the cash constraint (A.7b), respectively. Then the optimality conditions for all states and all periods \( s \) read

\[ \iota_s + \beta E_s \kappa_{s+1} = \frac{u_s C_s^i}{P_s} \quad \text{(A.11)} \]

for the optimal consumption,

\[ \kappa_s = \iota_s + \beta E_s \kappa_{s+1} \quad \text{(A.12)} \]
for optimal cash holdings, and
\[ \kappa_s = \beta R_s E_s \kappa_s + 1 \]  
(A.13)

for optimal bond holdings. Furthermore, by the complementary slackness of the cash constraints, condition (A.7b) is binding if \( \iota_s > 0 \). Making use of the optimality conditions for consumption and cash holdings which must hold in all dates and for all states yields
\[ \kappa_s = \frac{u_{C_s^i}}{P_s} \quad \text{and} \quad \kappa_{s+1} = \frac{u_{C_{s+1}^i}}{P_{s+1}}. \]  
(A.14)

As a result, optimal bond holdings condition then implies the standard Euler equation, namely
\[ \frac{1}{R_s} = \beta E_s \left( \left( \frac{C_{s+1}}{C_s} \right)^{-\rho} \frac{P_s(1 + t_{C,s})}{P_{s+1}(1 + t_{C,s+1})} \right). \]  
(A.15)

Note that the nominal interest rate relates to optimal cash holdings via the opportunity costs of holding an interest free asset. To be more precise, optimal cash holdings (A.12) together with optimal bond holdings imply
\[ \frac{\iota_s}{\kappa_s} = 1 - \frac{1}{R_s}. \]  
(A.16)

Thus, \( \iota_s > 0 \) holds true if and only if \( R_s > 1 \). In the following, the assumption is made that \( R_s > 1 \) but reaches 1 arbitrarily close (compare also Adao et al. (2003)). In the main text it is therefore still written as \( R_s = 1 \). As a consequence, consumption is implicitly determined by household’s cash holdings.

The optimal wage is constrained to be set one period in advance, i.e. the money wage in period \( s \) is determined by
\[ E_{s-1} \left[ u_{L_s^i} L_s^i + \frac{\theta - 1}{\theta} W_s^i L_s^i \beta E_s \kappa_{s+1} \right] = 0. \]  
(A.17)

The condition on optimal bond holdings (A.13) and (A.14) yield
\[ E_{s-1} \left[ u_{L_s^i} L_s^i + \frac{\theta - 1}{\theta} \frac{1}{R_s} W_s^i L_s^i u_{C_s^i} \right] = 0. \]  
(A.18)

Solving for the preset nominal wage level then results in the condition as stated in the main text.

**Market Clearing and Ex Post Equilibrium**

In equilibrium, the markets for tradable and non-tradable goods must clear. For the Home tradable good we thus have that total production \( Y_{HT,s} \) must meet the sum of total Home demand for the Home tradable good, \( C_{HT,s} \), and total Foreign demand of the Home tradable good, \( C_{HT,s}^* \), i.e.
\[ Y_{HT,s} = C_{HT,s} + C_{HT,s}^* \quad \text{and} \quad Y_{FT,s}^* = C_{FT,s} + C_{FT,s}^*. \]  
(A.19)
We can substitute the optimal fraction of total consumption expenditures that is allotted to consumption of the Home tradable good in (A.8) and (A.9). Total Home and Foreign production of tradable goods in terms of overall Home and Foreign tradable goods consumption are

\[ P_{H,s}Y_{HT,s} = \frac{1}{2}P_{T,s}C_{T,s} + \frac{1}{2}P_{T,s}C_{T,s}^* \quad \text{and} \quad P_{F,s}Y_{FT,s} = \frac{1}{2}P_{T,s}C_{T,s} + \frac{1}{2}P_{T,s}C_{T,s}^*, \]  

(A.20) respectively. Thus, the ratio of domestic and Foreign sale revenues is

\[ P_{H,s}Y_{HT,s} = P_{F,s}Y_{FT,s}. \]  

(A.21)

Put differently, nominal trade must always be balanced. Observe that in equilibrium with identical households no net trade in bonds can occur, ie.

\[ B_{i,s} = B_{j,s} = 0 \quad \text{and} \quad B_{i,s}^* = B_{j,s}^* = 0 \quad \forall s, i, j, i \neq j. \]  

(A.22)

Thus, all nominal wealth is in cash holdings, ie. \( W_s = M_{s-1} \) and \( W_s^* = M_{s-1}^* \). Using the aggregate version of the law of motion for wealth an plugging in the government’s budget constraints, one ends up with \( P_sC_s = W_sL_s = P_{H,s}Y_s \) which is equivalent to \( P_{T,s}C_{T,s} + P_{H,s}C_{HN,s} = P_{H,s}(Y_{HT,s} + Y_{HN,s}) \). Together with the market clearing for domestic non-tradables, \( P_{T,s}C_{T,s} = P_{H,s}Y_{H,s} \), and its Foreign equivalent yields \( P_{T,s}C_{T,s} = P_{T,s}C_{T,s}^* \). Since the law of one price holds for Home and Foreign tradable goods, PPP holds for Home and Foreign tradable goods baskets. Hence,

\[ C_{T,s} = C_{T,s}^* \]  

(A.23)

If one expresses overall consumption expenditures in terms of tradable consumption, ie. \( Z_s = \frac{P_s}{P_{T,s}}C_s \), we can use the optimal division of tradable and non-tradable consumption goods to find

\[ Z_s = \frac{P_s}{P_{T,s}}C_s = C_{HT,s} + \frac{P_{H,s}}{P_{T,s}}C_{HN,s} = \left( 1 + \frac{P_{H,s}C_{HN,s}}{P_{T,s}C_{HT,s}} \right) C_{HT,s}. \]  

(A.24)

Recall that \( \frac{C_{HN,s}}{C_{HT,s}} = \frac{(1 - \gamma)P_{T,s}}{P_{H,s}} \). Thus the overall consumption expenditure expressed in terms of tradable goods is

\[ Z_s = \frac{P_s}{P_{T,s}}C_s = \left( 1 + \frac{(1 - \gamma)}{\gamma} \right) C_{T,s} = \frac{1}{\gamma}C_{T,s}. \]  

(A.25)

The Foreign counterpart is \( Z_s^* = \frac{P_s}{P_{T,s}}C_s^* = \frac{1}{\gamma}C_{T,s}^* \). Then, together with (A.23), the equilibrium ratio of overall consumption expenditures in terms of tradable is unity, ie.

\[ Z_s = Z_s^*. \]  

(A.26)

In order to identify the periods nominal exchange rate, one only needs to take the ratio of Home and Foreign nominal consumption spending (cash-in-advance constraint):
\[ \frac{M_s}{\bar{M}_s} = \frac{P_s C_s}{P^*_s C^*_s} = \frac{Z_s}{Z^*_s} \frac{P_{T,s}}{P^*_{T,s}} \mathcal{E}_s = \mathcal{E}_s. \]  

(A.27)

\( Z_s \) can now be solved for by the product of Home and Foreign aggregate consumption spending. By the definition of the price indices and the equilibrium goods prices it follows that \( Z_s = A_{w,s} \frac{W_{w,s}}{M_{w,s}} \). As it turns out to be more convenient to express the equilibrium allocation in terms of the terms of trade \( \text{ToT}_s \) rather than the nominal exchange rate, we summarize the results as follows

\[ Z_s = A_{w,s} \frac{W_{w,s}}{M_{w,s}} \text{ and } \text{ToT}_s = \left( \frac{W_{d,s}}{A_{d,s} M_{d,s}} \right)^2. \]  

(A.28)

With the levels of \( Z_s \) and \( \mathcal{E}_s \) the equilibrium is uniquely determined. It easy to compute the country specific levels of consumption, output and labor by means of the definition of \( Z_s \) and the aggregate budget constraints. This is left to the reader. The resulting allocation is summarized in Table 1 in the main text. Note that equilibrium as determined by the two equations in (A.28) is true irrespective of whether wages are flexible or not. In case of flexible wages, however, the equilibrium wages can be directly plugged in and this results to the expression in the main text. Easiest to calculate the flexible wage equilibrium levels of \( Z_s \) and \( \text{ToT}_s \) is to use the equations (A.32) and (A.33) in the following section where the explicit equilibrium distribution is derived. Note that when wage are flexible, the two conditions have to hold contemporaneously for all instances. The expectations operators drop out. The explicit calculation is left to the reader.

A.2 The Explicit Distribution of the Equilibrium Allocation

In this section, the explicit distribution of the equilibrium allocation is calculated. First, I calculate the expected log-levels of \( Z \) and \( \text{ToT} \). The second step is to calculate the endogenous variance and covariance terms. Thereafter, I develop the expression of the objectives, ie. Home and Foreign expected period utilities, in terms of the closed-form solution. The derivations are carried out for the case of sticky wages only because the case of flexible wages is actually implicitly covered in that derivation. Because the single steps should be clear by then, the explicit calculation of the equilibrium distribution in case of flexible wages is left to the reader.

Calculating \( \mathbf{E}(z) \) and \( \mathbf{E(\text{tot})} \)

Home and Foreign equilibrium preset wages can be used to calculate the expected log-levels of \( Z \) and \( \text{ToT} \) in terms of expected levels of exogenous variables and in terms of endogenous and exogenous variances and covariances. Home and Foreign optimal preset wages are set according to

\[ W = \frac{\theta}{\theta - 1} R \frac{E(L^\nu)}{E(L_{PC}^\rho)} \text{ and } W^* = \frac{\theta}{\theta - 1} R^* \frac{E(L^\nu)}{E(L_{PC}^\rho)}. \]  

(A.29)
where it is already used that w.l.o.g. \( R \) and \( R^* \) are non-stochastic. Aggregate labor demand can be written as
\[
L = \frac{1}{A} (Y_H + Y_N) = \frac{Y}{A}.
\] (A.30)

The aggregate output level \( Y \) determines also the national aggregate budget constraint \( PC = P_H Y \). Plugging the equilibrium output \( Y = ZT o T^{-\frac{1}{2}} \) into the equilibrium wage level yields
\[
W = \frac{\theta}{\theta - 1} R \frac{E \left( \left( \frac{Y}{A} \right)^{\nu} \right)}{E \left( \frac{Y}{A} \right)} = \frac{\theta}{\theta - 1} R \frac{E \left( \left( \frac{ZT o T^{-\frac{1}{2}}}{A} \right)^{\nu} \right)}{E \left( \frac{ZT o T^{-\frac{1}{2}}}{A} \right)}.
\] (A.31)

Home equilibrium wage setting then implies
\[
1 = \frac{\theta}{\theta - 1} R \frac{E \left( \left( \frac{1}{A} ZT o T^{-\frac{1}{2}} \right)^{\nu} \right)}{E \left( \frac{ZT o T^{-\frac{1}{2}}}{A} \right)}.
\] (A.32)

The Foreign equilibrium wages setting reads
\[
1 = \frac{\theta}{\theta - 1} R^* \frac{E \left( \left( \frac{1}{A} ZT o T^{\frac{1}{2}} \right)^{\nu} \right)}{E \left( \frac{ZT o T^{\frac{1}{2}}}{A} \right)},
\] (A.33)

where \( Z^* = Z \) has already been used. Effectively, there are thus two equations in two unknowns, \( Z \) and \( T o T \). By the joint log-normal distribution of the equilibrium allocation, \( E(z) = \mu_z \) and \( E(tot) = \mu_{tot} \) can be derived by using (A.32) and (A.33) and explicitly solving for the log expressions of the expected values.\(^{15} \) After some algebra, the Home and Foreign wage setting in logs yield for \( E(z) = \mu_z \) and \( E(tot) = \mu_{tot} \) in matrix notation
\[
\begin{pmatrix}
-X & \frac{1}{2}Y \\
-X^* & -\frac{1}{2}Y^*
\end{pmatrix}
\begin{pmatrix}
E(z) \\
E(tot)
\end{pmatrix}
= \begin{pmatrix}
K \\
K^*
\end{pmatrix}.
\] (A.34)

\(^{15} \text{As a reminder: Suppose that the variable } Y \text{ is log-normally distributed, i.e. } Y \sim \log\mathcal{N}(\mu_y, \sigma^2). \text{ This means that the log of } Y, \text{ log} Y = y \text{ is } \log \mathcal{N}(\mu_y, \sigma^2) \text{ with } EY = e^{\mu + \frac{\sigma^2}{2}}. \text{ Moreover, if } X \text{ and } Y \text{ are jointly log-normal, i.e.}
\begin{pmatrix}
X \\
Y
\end{pmatrix}
\sim \log\mathcal{N}\left(\begin{pmatrix}
\mu_x \\
\mu_y
\end{pmatrix}, \begin{pmatrix}
\sigma_x^2 & \sigma_{xy} \\
\sigma_{yx} & \sigma_y^2
\end{pmatrix}\right),
\text{where by symmetry } \sigma_{xy} = \sigma_{yx}. \text{ We then have that}
EXY = e^{\mu_x + \mu_y + \frac{1}{2}(\sigma_x^2 + \sigma_y^2 + 2\sigma_{xy})}
= e^{\mu_x + \frac{1}{2}\sigma_x^2 + \mu_y + \frac{1}{2}\sigma_y^2 + \sigma_{xy}}
= EXY e^{\sigma_{xy}}.
\right.\)
where $K = ^\nu \mathcal{X} = \nu - (1 - \rho)\mathcal{Y} = \nu - (1 - \rho)(1 - \gamma)$, and where
\[
K = r + \ln \frac{\theta}{\theta - 1} - E(a_w)
+ \frac{\nu^2}{2} (\sigma_{aw}^2 + \sigma_{ad}^2) + \frac{1}{2} \left[ (\nu^2 - (1 - \rho)^2) \sigma_{w}^2 + \frac{1}{4} (\nu^2 - (1 - \rho)^2(1 - \gamma)^2) \sigma_{tot}^2 \right]
- \frac{1}{2} (\nu^2 - (1 - \rho)^2(1 - \gamma)) \sigma_{z,tot} - \frac{\nu^2}{2} [2(\sigma_{z,aw} + \sigma_{z,ad}) - (\sigma_{tot,aw} + \sigma_{tot,ad})],
\]
and
\[
K^* = r^* + \ln \frac{\theta}{\theta - 1} - E(a_w)
+ \frac{\nu^2}{2} (\sigma_{aw}^2 + \sigma_{ad}^2) + \frac{1}{2} \left[ (\nu^2 - (1 - \rho)^2) \sigma_{w}^2 + \frac{1}{4} (\nu^2 - (1 - \rho)^2(1 - \gamma)^2) \sigma_{tot}^2 \right]
+ \frac{1}{2} (\nu^2 - (1 - \rho)^2(1 - \gamma)) \sigma_{z,tot} - \frac{\nu^2}{2} [2(\sigma_{z,aw} - \sigma_{z,ad}) + (\sigma_{tot,aw} - \sigma_{tot,ad})].
\]
By Cramer’s Rule, one gets
\[
E(z) = -\frac{1}{2\mathcal{X}} (K + K^*) \quad \text{and} \quad E(tot) = \frac{1}{\mathcal{Y}} (K - K^*). \tag{A.35}
\]
It is easily verified that
\[
(K + K^*) = (r + r^*) + 2 \ln \frac{\theta}{\theta - 1} + \nu^2 (\sigma_{aw}^2 + \sigma_{ad}^2) + (\nu^2 - (1 - \rho)^2) \sigma_{w}^2
+ \frac{1}{2} (\nu^2 - (1 - \rho)^2(1 - \gamma)^2) \sigma_{tot}^2 - \nu^2 (2\sigma_{z,aw} - \sigma_{tot,aw}),
\]
and
\[
(K - K^*) = (r - r^*) - (\nu^2 - (1 - \rho)^2(1 - \gamma)) \sigma_{z,tot} - \nu^2 (2\sigma_{z,ad} - \sigma_{tot,aw}),
\]
where the assumption that $E(a_w) = 0$ has already been exploited. Note that this assumption is of no other purpose or importance than to simplify the algebra. Hence, plugging $(K + K^*)$ and $(K - K^*)$ into the solutions to $E(z)$ and $E(tot)$ yields
\[
E_z = -\frac{1}{\mathcal{X}} \left( r_w + \ln \frac{\theta}{\theta - 1} + \Sigma_z \right), \tag{A.36}
\]
where
\[
\Sigma_z = \frac{1}{2} (\nu^2 - (1 - \rho)^2) \sigma_{w}^2 + \frac{1}{8} (\nu^2 - (1 - \rho)^2(1 - \gamma)^2) \sigma_{tot}^2
- \frac{\nu^2}{2} (2\sigma_{z,aw} - \sigma_{tot,aw}) + \frac{\nu^2}{2} (\sigma_{aw}^2 + \sigma_{ad}^2), \tag{A.37}
\]
and
\[
E_{tot} = \frac{2}{\mathcal{Y}} (\Sigma_{tot} - \Sigma_z), \tag{A.38}
\]
with
\[
\Sigma_{tot} = \frac{1}{2} (\nu^2 - (1 - \rho)^2(1 - \gamma)) \sigma_{z,tot} + \frac{\nu^2}{2} (2\sigma_{z,ad} - \sigma_{tot,aw}). \tag{A.39}
\]
Calculating the endogenous variance and covariance terms

Next, the endogenous variance and covariance terms are derived. As wages are predetermined, the period equilibrium allocation can only be altered by changes in productivity and money supply:

\[ \hat{z} \equiv z - E(z) = \hat{a}_w + \hat{m}_w \quad \text{and} \quad \hat{t}_{ot} \equiv \text{tot} - E(\text{tot}) = -2(\hat{a}_d + \hat{m}_d). \] (A.40)

The national money supply feedback rules are given by

\[ \hat{m} = \mu_{a_w} \hat{a}_w + \mu_{a_d} \hat{a}_d \quad \text{and} \quad \hat{m}^* = \mu_{a_w}^* \hat{a}_w - \mu_{a_d}^* \hat{a}_d. \] (A.41)

In terms of world average and difference components, the feedback rules can be stated as

\[ \hat{m}_w = \mu_{w,a_w} \hat{a}_w + \mu_{d,a_d} \hat{a}_d \quad \text{and} \quad \hat{m}_d = \mu_{d,a_w} \hat{a}_w + \mu_{w,a_d} \hat{a}_d. \] (A.42)

where

\[ \mu_{w,a_w} = \frac{1}{2}(\mu_{a_w} + \mu_{a_w}^*), \quad \mu_{d,a_w} = \frac{1}{2}(\mu_{a_w} - \mu_{a_w}^*), \]
\[ \mu_{w,a_d} = \frac{1}{2}(\mu_{a_d} + \mu_{a_d}^*), \quad \text{and} \quad \mu_{d,a_d} = \frac{1}{2}(\mu_{a_d} - \mu_{a_d}^*). \]

Solving for \( \sigma^2_z, \sigma^2_{\text{tot}}, \) and \( \sigma_{z,\text{tot}} \)

The two equations in (A.40) can be rewritten in matrix notation as

\[
\begin{pmatrix}
\hat{z} \\
\hat{t}_{ot}
\end{pmatrix}
= \begin{pmatrix}
\hat{a}_w + \hat{m}_w \\
-2(\hat{a}_d + \hat{m}_d)
\end{pmatrix}
= \begin{pmatrix}
(1 + \mu_{w,a_w}) & \mu_{d,a_d} \\
-2\mu_{d,a_w} & -2(1 + \mu_{w,a_d})
\end{pmatrix}
\begin{pmatrix}
\hat{a}_w \\
\hat{a}_d
\end{pmatrix}
\equiv \Lambda
\]

Consequently,

\[
\begin{bmatrix}
\sigma^2_z & \sigma_{z,\text{tot}} \\
\sigma_{\text{tot},z} & \sigma^2_{\text{tot}}
\end{bmatrix}
= E\Lambda \begin{pmatrix}
\hat{a}_w \\
\hat{a}_d
\end{pmatrix}^\prime
\Lambda = \Lambda
\begin{bmatrix}
\sigma^2_{a_w} & 0 \\
0 & \sigma^2_{a_d}
\end{bmatrix}
\Lambda^\prime
\]

As a result, one gets

\[ \sigma^2_z = (1 + \mu_{w,a_w})^2 \sigma^2_{a_w} + \mu_{d,a_d}^2 \sigma^2_{a_d} \] (A.44)
\[ \sigma^2_{\text{tot}} = 4\mu_{d,a_w}^2 \sigma^2_{a_w} + 4(1 + \mu_{w,a_d})^2 \sigma^2_{a_d} \] (A.45)
\[ \sigma_{\text{tot},z} = -2(1 + \mu_{w,a_w}) \mu_{d,a_d} \sigma^2_{a_d} - 2\mu_{d,a_d} (1 + \mu_{w,a_d}) \sigma^2_{a_d} \] (A.46)

Moreover, it also follows that

\[
\begin{pmatrix}
\sigma_{z,a_w} \\
\sigma_{z,a_d}
\end{pmatrix}
= \begin{pmatrix}
(1 + \mu_{w,a_w})\sigma^2_{a_w} \\
\mu_{d,a_d} \sigma^2_{a_d}
\end{pmatrix}
\quad \text{and} \quad
\begin{pmatrix}
\sigma_{\text{tot},a_w} \\
\sigma_{\text{tot},a_d}
\end{pmatrix}
= \begin{pmatrix}
-2\mu_{d,a_w} \sigma^2_{a_d} \\
-2(1 + \mu_{w,a_d}) \sigma^2_{a_d}
\end{pmatrix}
\] (A.47)
The Objectives in Closed Form

First note that from the optimal wage setting condition together with the budget constraint it directly follows that

\[ EL' = E^{\theta - 1} \frac{1}{1 - \rho} C^{(1 - \rho)} \quad \text{and} \quad EL'^* = E^{\theta - 1} \frac{1}{R^*} C^{*(1 - \rho)}. \]  

(A.48)

Therefore, Home and Foreign expected period utility can be expressed by

\[ Eu = \left( \frac{1}{1 - \rho} - \frac{(\theta - 1)}{\nu \theta} \frac{1}{R} \right) EC^{1-\rho} \quad \text{and} \quad Eu^* = \left( \frac{1}{1 - \rho} - \frac{(\theta - 1)}{\nu \theta} \frac{1}{R^*} \right) EC'\rho^*, \]

respectively. Since \( C = ZT o T^{-(1 - \gamma)} \) and \( C^* = ZT o T^{1/2(1 - \gamma)} \) are log-normal,

\[ EC'\rho = (1 - \rho) \left( E(z) - \frac{1}{2}(1 - \gamma)E(tot) \right) + \frac{(1 - \rho)^2}{2} \left( \sigma_z^2 + \frac{1}{4}(1 - \gamma)^2 \sigma_{tot}^2 - (1 - \gamma)\sigma_{z,tot} \right) \]

\[ EC'\rho^* = (1 - \rho) \left( E(z) + \frac{1}{2}(1 - \gamma)E(tot) \right) + \frac{(1 - \rho)^2}{2} \left( \sigma_{z,tot}^2 + \frac{1}{4}(1 - \gamma)^2 \sigma_{tot}^2 + (1 - \gamma)\sigma_{z,tot} \right) \]

Consequently, plugging the solutions for \( E(z) \) and \( E(tot) \) into the two equations above allows to express Home expected utility after some algebra as

\[ Eu = \bar{u}(R_w, R_d) \exp \left\{ \frac{\nu(1 - \rho)}{2X} \Omega(\mu_{w,a}, \mu_{d,a}, \mu_{w,a}, \mu_{d,a}) \right\}, \]

where

\[ \bar{u}(R_w, R_d) = \left( \frac{1}{1 - \rho} - \frac{\theta - 1}{\nu \theta} \frac{1}{R_w R_d} \right) \left( \frac{\theta - 1}{\nu \theta} \frac{1}{R_w} \right)^{(\gamma - 1)/(1 - \gamma)} \frac{1}{\gamma} \]

and

\[ \Omega(\mu_{w,a}, \mu_{d,a}, \mu_{w,a}, \mu_{d,a}) = -X\sigma_z^2 - \frac{\gamma}{4}\sigma_{tot}^2 + \frac{(1 - \gamma)}{2}\chi X^2 \sigma_{z,tot} + \nu(2\sigma_{z,a} - \sigma_{tot,a}) + \nu X(2\sigma_{z,a} - \sigma_{tot,a}) + \nu X (\sigma_{a,w}^2 + \sigma_{a,d}^2). \]

Foreign expected utility reads

\[ Eu^* = \bar{u}^*(R_w, R_d) \exp \left\{ \frac{\nu(1 - \rho)}{2X} \Omega^*(iC_{w,a}, iC_{d,a}, iC_{d,a}, iC_{d,a}) \right\}, \]

where

\[ \bar{u}^*(R_w, R_d) = \left( \frac{1}{1 - \rho} - \frac{\theta - 1}{\nu \theta} \frac{1}{R_w} \right) \left( \frac{\theta - 1}{\nu \theta} \frac{1}{R_w} \right)^{(\gamma - 1)/(1 - \gamma)} \frac{1}{\gamma} \]

and

\[ \Omega^*(\mu_{w,a}, \mu_{d,a}, \mu_{w,a}, \mu_{d,a}) = -X\sigma_z^2 - \frac{\gamma}{4}\sigma_{tot}^2 + \frac{(1 - \gamma)}{2}\chi X^2 \sigma_{z,tot} + \nu(2\sigma_{z,a} - \sigma_{tot,a}) + \nu X (\sigma_{a,w}^2 + \sigma_{a,d}^2). \]
Proof of Proposition 1

For clarification, the terms first and second order are understood to refer to a Taylor polynomial of first and second order. In principle, a formal proof could be given here with the exact derivation of the second order approximation to the solution of the model and the welfare function. The proof of Proposition 1, however, is trivial by the separation of expected utility into the components that are non-stochastic and the components depending on variances and covariances as in equations (A.49 and A.50) and in the main text. In particular, note that the log-linear distribution of the model implies that the second order approximations to the log-linear model is exact because the equilibrium and the welfare function in logs are 2nd order polynomials. Moreover, the second order terms are clearly the variance and covariance terms of the equilibrium. Consequently, an approximation of the equilibrium and the welfare function up to first order leaves out the second order variance and covariance terms completely and hence the terms describing uncertainty are lost.

A.3 Optimal Monetary Policy Coordination

When Home and Foreign policymakers coordinate optimally on both ex-post and ex-ante monetary policy, they do so as to maximize the sum of Home and Foreign expected utilities, i.e. \( \text{Eu} + \text{Eu}^* \), over \( \{R, \mu_{aw}, \mu_{ad}\} \) and \( \{R^*, \mu_{aw}^*, \mu_{ad}^*\} \), respectively. In an alternative expression, Home and Foreign authorities jointly set \( \{R_w, R_d\}, \{\mu_{w,a_w}, \mu_{d,a_w}\} \), and \( \{\mu_{w,a_d}, \mu_{d,a_d}\} \) in order to maximize the joint welfare.

Proof of Proposition 2

There are two first order conditions for setting \( \{R_w, R_d\} \), namely

\[
\frac{d\bar{u}(R_w, R_d)}{dR_w} \exp\left\{\frac{\nu(1-\rho)}{2\chi} \Omega(\cdot)\right\} + \frac{d\bar{u}^*(R_w, R_d)}{dR_w} \exp\left\{\frac{\nu(1-\rho)}{2\chi} \Omega^*(\cdot)\right\} = 0, \quad \text{and (A.51)}
\]

\[
\frac{d\bar{u}(R_w, R_d)}{dR_d} \exp\left\{\frac{\nu(1-\rho)}{2\chi} \Omega(\cdot)\right\} + \frac{d\bar{u}^*(R_w, R_d)}{dR_d} \exp\left\{\frac{\nu(1-\rho)}{2\chi} \Omega^*(\cdot)\right\} = 0, \quad \text{(A.52)}
\]

respectively. In choosing the optimal nominal interest rates \( R_w \) and \( R_d \), policymakers take into account that the net nominal interest rates must be non-negative, i.e. \( R_w \geq 1 \) and \( R_d \geq 1 \).

Symmetry: \( (R_w^{Opt} = R_d^{Opt}) \). Multiplying the FOC to \( R_w \) (equation (A.51)) by \( \frac{1}{R_w} \), and dividing through \( \left(\frac{\theta - 1}{\theta \nu}\right)\frac{1}{R_w} \), yields

\[
0 = \left[\frac{\theta - 1}{\theta \nu} \left(\frac{1}{R_w a_w} \right) - \frac{(1-\rho)}{\chi} \right] \times R_w \frac{(1-\gamma)(1-\rho)}{\chi} \exp\left\{\frac{\nu(1-\rho)}{2\chi} \Omega(\cdot)\right\} + \left[\frac{\theta - 1}{\theta \nu} \left(\frac{R_d}{R_w} \right) - \frac{(1-\rho)}{\chi} \right] \times R_d \frac{(1-\gamma)(1-\rho)}{\chi} \exp\left\{\frac{\nu(1-\rho)}{2\chi} \Omega^*(\cdot)\right\}. \quad \text{(A.53)}
\]
A similar manipulation to the first order condition to \( R_d \) (equation (A.52)), ie. multiplying by \( R_d \) and dividing through \( \left( \frac{\theta - 1}{\theta} \right)^{\frac{(1-\rho)}{\theta}} \), yields

\[
0 = \left[ \frac{\theta - 1}{\theta} \left( \frac{1}{\kappa_w \kappa_d} \right) - \frac{(1-\rho)(1-\gamma)}{\theta} \left( \frac{1}{(1-\rho)} - \frac{\theta - 1}{\theta} \right) \right] \times R_d^{\frac{(1-\gamma)(1-\rho)}{\gamma}} \exp \left\{ \frac{\nu(1-\rho)}{2\chi} \Omega^*(\cdot) \right\}.
\]

Thus, equations (A.53) and (A.54) denote two equations in the two unknowns \( R_w \) and \( R_d \). By taking ratios of the two conditions, one gets rid of the factors \( R_d^{\frac{(1-\gamma)(1-\rho)}{\gamma}} \) and \( R_d^{\frac{(1-\gamma)(1-\rho)}{\gamma}} \) and arrives at

\[
\frac{\theta - 1}{\theta} + \frac{(1-\rho)(\theta - 1)(1-\gamma)}{\gamma \theta} - \frac{1}{\gamma} \left( \frac{1}{\kappa_w \kappa_d} \right) = \frac{\theta - 1}{\theta} + \frac{(1-\rho)(\theta - 1)(1-\gamma)}{\gamma \theta} - \frac{1}{\gamma} \left( \frac{R_d}{R_w} \right).
\]  

(A.55)

After some algebra one can solve for \( R_d \) and it becomes obvious that the first order conditions in (A.51) and (A.52) are satisfied if and only if \( R_d = 1 \), or equivalently \( R^{\text{Opt}} = R^{\text{Opt}^*} \), and \( \gamma > 0 \). For \( \gamma = 0 \), the two countries are fully separated and coordination makes no sense at all as the jointly optimal outcome must necessarily coincide with individually optimal interest rate policy.

**Optimal Policy Intervention:** \( (R^{\text{Opt}} \text{ and } R^{\text{Opt}^*}) \) If one plugged the symmetry result into one of the two FOCs, it would follow that \( R^{\text{Opt}} = \frac{\theta - 1}{\theta} \). This, however, violates the non-negativity constraint of the net nominal interest rates. It is easy to demonstrate that the two FOCs are negative for \( R > \frac{\theta - 1}{\theta} \). If the second order conditions hold, the objectives are indeed maximized for

\[
R^{\text{Opt}} = 1.
\]  

(A.56)

**Second Order Conditions** The verification of the second order conditions for \( R^{\text{Opt}} \) and \( R^{\text{Opt}^*} \) to be optimal is straightforward and left to the reader.

**Proof of Proposition 3**

The two first order conditions for setting \( \{\mu_w, k \geq w \} \) for \( k = w, d \), ie. in case of both the aggregate and the asymmetric shocks, are

\[
0 = \frac{\nu(1-\rho)}{2\chi} \hat{u} (\cdot) \exp \left\{ \frac{\nu(1-\rho)}{2\chi} \Omega^*(\cdot) \right\} + \frac{\nu(1-\rho)}{2\chi} \hat{u}^* (\cdot) \exp \left\{ \frac{\nu(1-\rho)}{2\chi} \Omega^*(\cdot) \right\} + \frac{\nu(1-\rho)}{2\chi} \hat{u} (\cdot) \exp \left\{ \frac{\nu(1-\rho)}{2\chi} \Omega^*(\cdot) \right\} \]  

(A.57)

and

\[
0 = \frac{\nu(1-\rho)}{2\chi} \hat{u} (\cdot) \exp \left\{ \frac{\nu(1-\rho)}{2\chi} \Omega^*(\cdot) \right\} + \frac{\nu(1-\rho)}{2\chi} \hat{u}^* (\cdot) \exp \left\{ \frac{\nu(1-\rho)}{2\chi} \Omega^*(\cdot) \right\} + \frac{\nu(1-\rho)}{2\chi} \hat{u} (\cdot) \exp \left\{ \frac{\nu(1-\rho)}{2\chi} \Omega^*(\cdot) \right\} \]  

(A.58)

for \( k = w, d \), respectively. Note that \( \frac{\nu(1-\rho)}{2\chi} \hat{u} \) and \( \frac{\nu(1-\rho)}{2\chi} \hat{u}^* \) are strictly positive and by Proposition 1 they must be identical. Reordering the two FOCs yields

\[
0 = \frac{d\Omega(\cdot)}{d\mu_w, k} + \frac{d\Omega^*(\cdot)}{d\mu_w, k} \exp \left\{ -\frac{\nu(1-\rho)}{2\chi} \left( \Omega^*(\cdot) - \Omega^*(\cdot) \right) \right\}, \]  

(A.59)

and

\[
0 = \frac{d\Omega(\cdot)}{d\mu_d, k} + \frac{d\Omega^*(\cdot)}{d\mu_d, k} \exp \left\{ -\frac{\nu(1-\rho)}{2\chi} \left( \Omega(\cdot) - \Omega^*(\cdot) \right) \right\}, \]  

(A.60)
for $k = w, d$. Next, define $-\nu (1 - \rho) (\Omega (\cdot) - \Omega^*(\cdot))$ as $\Delta$ and express it in terms of policy coefficients, ie.

$$
\Delta \equiv -\frac{\nu (1 - \rho)}{2} (\Omega (\cdot) - \Omega^*(\cdot))
$$

(A.61)

By the additive structure, $\Delta$ can be split into the components referring to the aggregate shock and the asymmetric shock, respectively. Consequently,

$$
\Delta = \Delta_{aw} + \Delta_{ad},
$$

(A.62)

where

$$
\Delta_{aw} = -\frac{\nu (1 - \rho)(1 - \gamma)}{\gamma} (\nu - \mathcal{X}(1 + \mu_{w,a} \omega)) \mu_{d,a} \sigma_{a,w}^2,
$$

and

$$
\Delta_{ad} = -\frac{\nu (1 - \rho)(1 - \gamma)}{\gamma} (\nu - \mathcal{X}(1 + \mu_{w,a} \omega)) \mu_{d,a} \sigma_{a,d}^2.
$$

It is important to observe that $\Delta$ fully captures the difference between Home and Foreign utility. Moreover, for the two components $\Delta_{aw}$ and $\Delta_{ad}$ the difference in utility is zero only if either $(\nu - \mathcal{X}(1 + \mu_{w,a} \omega)) = 0$ or $\mu_{d,a} = 0$ for $k = w, d$. For expositional convenience I define the term $(\nu - \mathcal{X}(1 + \mu_{w,a} \omega))$ as $\mathcal{A} (\mu_{w,a} \omega)$, ie. for $k = w, d$

$$
\mathcal{A} (\mu_{w,a} \omega) \equiv (\nu - \mathcal{X}(1 + \mu_{w,a} \omega)) .
$$

(A.63)

Proof of Proposition 3: Aggregate Shocks

The two first order conditions for the optimal response to aggregate productivity shocks are

$$
0 = 2 \left( \mathcal{A} (\mu_{w,a} \omega) - \frac{(1 - \gamma)}{\gamma} \mathcal{X}^2 \mu_{d,a} \omega \right) + 2 \left( \mathcal{A} (\mu_{w,a} \omega) + \frac{(1 - \gamma)}{\gamma} \mathcal{X}^2 \mu_{d,a} \omega \right) \exp\{\Delta_{aw}\}
$$

(A.64)

for $\mu_{w,a} \omega$ and

$$
0 = 2 \left( \frac{(1 - \gamma)}{\gamma} \mathcal{X} \mathcal{A} (\mu_{w,a} \omega) - \mathcal{Z} \mu_{d,a} \omega \right) - 2 \left( \frac{(1 - \gamma)}{\gamma} \mathcal{X} \mathcal{A} (\mu_{w,a} \omega) + \mathcal{Z} \mu_{d,a} \omega \right) \exp\{\Delta_{aw}\}
$$

(A.65)

for $\mu_{d,a} \omega$, respectively.

Symmetry: ($\mu_{aw}^{Opt} = \mu_{aw}^{*Opt}$) The proof that the optimal solution entails symmetry is carried out in two steps: First, it is demonstrated that $\mu_{d,a} = 0$ is indeed a solution to the problem. Second, it is shown that this solution is unique. Rearranging the two FOCs in (A.64) and (A.65) yields

$$
\mathcal{A} (\mu_{w,a} \omega) (1 + \exp\{\Delta_{aw}\}) = \frac{(1 - \gamma)}{\gamma} \mathcal{X}^2 (1 - \exp\{\Delta_{aw}\}) \mu_{d,a} \omega
$$

(A.66)

for $\mu_{w,a} \omega$ and

$$
\frac{(1 - \gamma)}{\gamma} \mathcal{X} \mathcal{A} (\mu_{w,a} \omega) (1 - \exp\{\Delta_{aw}\}) = \mathcal{Z} (1 + \exp\{\Delta_{aw}\}) \mu_{d,a} \omega
$$

(A.67)

16Note that for $\rho = 1$, ie. in case of log-utility, the term $(1 - \rho)$ doesn’t appear in (A.64) and (A.65) and the same logic goes through.
for $\mu_{d,a_w}$, respectively. Recall that $\Delta_{a_w}$ is zero only if $A(\mu_{w,a_w}) = 0$ or $\mu_{d,a_w} = 0$. Consequently, equation (A.67) is indeed solved by $\mu_{d,a_w} = 0$. Plugging this into the rearranged FOC to $\mu_{w,a_w}$ (A.66) implies that $A(\mu_{w,a_w}) = (\nu - \lambda'(1 + \mu_{w,a_w})) = 0$. Hence, $\mu_{d,a_w} = 0$ is part of a valid solution to the maximization problem.

The uniqueness of this solution is proven by showing that $\mu_{d,a_w} \neq 0$ contradicts the FOCs. This is undertaken in two intermediary steps for $\rho > 1$ and $\rho \leq 1$.

(i) $\rho > 1$: Suppose that there exist another solution to the FOCs and $\mu_{d,a_w} \neq 0$ and $\gamma < 1$.\footnote{When $\gamma < 1$ since $\gamma = 1$ trivially implies $\mu_{d,a_w} = 0$ by equation (A.67).}

From equation (A.67) follows that $A(\mu_{w,a_w}) \neq 0$ and hence $\Delta_{a_w} \neq 0$. The two rearranged FOCs in (A.67) and (A.66) imply for the sign of $\mu_{d,a_w}$, $\Delta_{a_w}$, and $A(\mu_{w,a_w})$ that

$$\text{sgn}\{(1 - \exp\{\Delta_{a_w}\}\} = \text{sgn}\{A(\mu_{w,a_w})\mu_{d,a_w}\} \quad (A.68)$$

and

$$\text{sgn}\{\mu_{d,a_w}\} = \text{sgn}\{A(\mu_{w,a_w}) (1 - \exp\{\Delta_{a_w}\})\}. \quad (A.69)$$

Note that $\text{sgn}\{1 - \exp\{\Delta_{a_w}\}\} = -\text{sgn}\{\Delta_{a_w}\}$. Consequently, the only combinations that satisfy the two first order conditions are $\Delta_{a_w} < 0$ and $\text{sgn}\{\mu_{d,a_w}\} = \text{sgn}\{A(\mu_{w,a_w})\}$ or $\Delta_{a_w} > 0$ and $\text{sgn}\{\mu_{d,a_w}\} \neq \text{sgn}\{A(\mu_{w,a_w})\}$. However, by the definition of $\Delta_{a_w}$, this is only consistent with $\rho \leq 1$. Hence, for all $\rho > 1$, there exists only a unique symmetric equilibrium with $\mu_{d,a_w} = 0$.

(ii) $\rho \leq 1$: The next step is to show that $\mu_{d,a_w} \neq 0$ is inconsistent for $\rho \leq 1$, too. First, look at the case $\text{sgn}\{\mu_{d,a_w}\} = \text{sgn}\{A(\mu_{w,a_w})\}$. Let $\mu_{d,a_w}$ and $A(\mu_{w,a_w})$ both positive. Then the two FOCs to holds imply that

$$0 > \left(\frac{1 - \gamma}{\lambda} \lambda^2 \mu_{d,a_w}\right)$$

and

$$0 < \left(\frac{1 - \gamma}{\lambda} \lambda A(\mu_{w,a_w}) - \lambda \mu_{d,a_w}\right).$$

The two conditions are, however mutually exclusive for $1 \geq \rho$. In particular, the two condition require that $A(\mu_{w,a_w}) < \frac{(1-\gamma)\lambda^2}{\mu_{d,a_w}}$ and $A(\mu_{w,a_w}) > \frac{\lambda^2}{(1-\gamma)\lambda}$. For $1 \geq \rho$, $\frac{\lambda^2}{(1-\gamma)\lambda} \geq \frac{(1-\gamma)\lambda^2}{\mu_{d,a_w}}$ and therefore both conditions cannot be satisfied at the same time. For $\mu_{d,a_w}$ and $A(\mu_{w,a_w})$ both negative, the same conditions apply.

However, $\text{sgn}\{\mu_{d,a_w}\} \neq \text{sgn}\{A(\mu_{w,a_w})\}$ is incompatible with the two FOCs and $1 \geq \rho$, too.
Let \( \mu_{d,a} \) be positive and \( A(\mu_{w,a}) \) negative. Then the two FOCs to hold implies that

\[
0 > \left( A(\mu_{w,a}) + \frac{(1-\gamma)}{\gamma} \lambda^2 \mu_{d,a} \right)
\]

and

\[
0 < \left( \frac{(1-\gamma)}{\gamma} \lambda A(\mu_{w,a}) + Z \mu_{d,a} \right).
\]

Again, the two conditions are mutually exclusive for \( 1 \geq \rho \). In particular, the two conditions require that \( A(\mu_{w,a}) < -\frac{(1-\gamma)}{\gamma} \lambda \mu_{d,a} \) and \( \frac{Z}{\gamma} \mu_{d,a} < 0 \), because for \( 1 \geq \rho \) it is true that \( -\frac{Z}{\gamma} \mu_{d,a} \leq -\frac{(1-\gamma)}{\gamma} \lambda \mu_{d,a} \) and the ratio \( \frac{A(\mu_{w,a})}{\mu_{d,a}} < 0 \).

Optimal Policies: \( (\mu_{w,a}^{Opt} \text{ and } \mu_{a}^{Opt}) \) As a result, since \( \mu_{d,a}^{Opt} = 0 \) it must be true that

\[
A(\mu_{w,a}) = \left( \nu - \lambda (1 + \mu_{w,a}^{Opt}) \right) = 0
\]

it follows that \( \mu_{a}^{Opt} = \mu_{d,a}^{Opt} = \frac{(1-\rho)}{\lambda} \).

Second Order Conditions It is easily verified that the second-order conditions are satisfied. Consequently, the solution to the first-order conditions will depict the global optimum to the maximization problem.

Proof of Proposition 3: Asymmetric Shocks

In case of asymmetric shocks, the proof goes in almost the same way. The two first order conditions are

\[
0 = 2 \left( \nu - Z(1 + \mu_{w,a}) - \frac{(1-\gamma)}{\gamma} \lambda^2 \mu_{d,a} \right) + 2 \left( \nu - Z(1 + \mu_{w,a}) + \frac{(1-\gamma)}{\gamma} \lambda^2 \mu_{d,a} \right) \exp\{\Delta_{a} \}
\]

for \( \mu_{w,a} \) and

\[
0 = 2 \left( \frac{(1-\gamma)}{\gamma} \lambda A(\mu_{w,a}) - \lambda \mu_{d,a} \right) - 2 \left( \frac{(1-\gamma)}{\gamma} \lambda A(\mu_{w,a}) + \lambda \mu_{d,a} \right) \exp\{\Delta_{a} \}
\]

for \( \mu_{d,a} \), respectively.

Symmetry: \( (\mu_{d,a}^{Opt} = \mu_{a}^{Opt}) \) The proof that the optimal policy response to asymmetric shocks is unique and entails symmetry is carried out again in two steps: First it is shown that \( \mu_{d,a} = 0 \) indeed is a solution to the FOCs. Second, it is demonstrated that \( \mu_{d,a} \neq 0 \) is not compatible with the FOCs. Rearranging the two FOCs in (A.71) and (A.72) yields

\[
(\nu - Z(1 + \mu_{a})) (1 + \exp\{\Delta_{a} \}) = \frac{(1-\gamma)}{\gamma} \lambda (1 - \exp\{\Delta_{a} \}) \mu_{d,a}
\]
for \( \mu_{w,a_d} \) and
\[
\frac{1 - \gamma}{\gamma} \mathcal{X}(\nu - \mathcal{X}(1 + \mu_{w,a_d}))(1 - \exp(\Delta_{a_d})) = (1 + \exp(\Delta_{a_d})) \mu_{d,a_d} \quad (A.74)
\]
for \( \mu_{d,a_d} \), respectively. Recall that \( \Delta_{a_d} \) is zero only if \( \mathcal{A}(\mu_{w,a_d}) = 0 \) or \( \mu_{d,a_d} = 0 \). Consequently, equation (A.74) is indeed solved by \( \mu_{d,a_d} = 0 \). Plugging this into the rearranged FOC to \( \mu_{w,a_d} \) (A.73) implies that \((\nu - Z(1 + \mu_{w,a_d})) = 0 \). Hence, \( \mu_{d,a_d} = 0 \) is part of a valid solution to the maximization problem.

The uniqueness of this solution is proven again by showing that \( \mu_{d,a_d} \neq 0 \) contradicts the FOCs. This is done in two intermediary steps for \( \rho > 1 \) and \( \rho \leq 1 \). Suppose to the contrary that there exist another solution to the FOCs and \( \mu_{d,a_d} \neq 0 \) and \( \gamma < 1 \).18 From equation (A.74) follows that \( \mathcal{A}(\mu_{w,a_d}) \neq 0 \) and hence \( \Delta_{a_d} \neq 0 \). The two rearranged FOCs in (A.74) and (A.73) imply for the signs of \( \mu_{d,a_d}, \Delta_{a_d}, \mathcal{A}(\mu_{w,a_d}), \) and \((\nu - Z(1 + \mu_{w,a_d})) \) that
\[
\text{sgn}\{(\nu - Z(1 + \mu_{w,a_d}))\} = \text{sgn}\{(1 - \exp(\Delta_{a_d})) \mu_{d,a_d}\} \quad (A.75)
\]
and
\[
\text{sgn}\{\mu_{d,a_d}\} = \text{sgn}\{\mathcal{A}(\mu_{w,a_d})(1 - \exp(\Delta_{a_d}))\}. \quad (A.76)
\]

Note that \( \text{sgn}\{1 - \exp(\Delta_{a_d})\} = -\text{sgn}\{\Delta_{a_d}\} \) and moreover that
\[
\nu - Z(1 + \mu_{w,a_d}) > 0 \quad \Rightarrow \quad \mathcal{A}(\mu_{w,a_d}) > 0 \quad (A.77)
\]
and
\[
\mathcal{A}(\mu_{w,a_d}) < 0 \quad \Rightarrow \quad \nu - Z(1 + \mu_{w,a_d}) < 0. \quad (A.78)
\]

(i) \( \rho > 1 \): It is easily shown that the condition A.76 is satisfied if either \( \Delta_{a_d} > 0 \) and \( \text{sgn}\{\mu_{d,a_d}\} \neq \text{sgn}\{\mathcal{A}(\mu_{w,a_d})\} \) or \( \Delta_{a_d} < 0 \) and \( \text{sgn}\{\mu_{d,a_d}\} = \text{sgn}\{\mathcal{A}(\mu_{w,a_d})\} \). By the definition of \( \Delta_{a_d} \) follows that this condition is always violated for \( \rho > 1 \).

(ii) \( \rho \leq 1 \): When \( \mu_{d,a_d} \) is positive as is \( \nu - Z(1 + \mu_{w,a_d}) \) and hence \( \mathcal{A}(\mu_{w,a_d}) \) is positive, too, the two FOCs to need to satisfy
\[
0 > \left(\nu - Z(1 + \mu_{w,a_d}) - \frac{1 - \gamma}{\gamma} \mathcal{X}^2 \mu_{d,a_d}\right) \quad (A.79)
\]
and
\[
0 < \left(\frac{1 - \gamma}{\gamma} \mathcal{X}^2 \mathcal{A}(\mu_{w,a_d}) - Z \mu_{d,a_d}\right). \quad (A.80)
\]

The two conditions are mutually exclusive for \( 1 \geq \rho \). In particular, the two condition require that
\[
\frac{\mathcal{Y}}{1 - \gamma} \mathcal{X}^2 \left(\nu - Z(1 + \mu_{w,a_d})\right) < \mu_{d,a_d} \quad \text{and} \quad \frac{1 - \gamma}{\gamma} \mathcal{X}^2 \left(\nu - \mathcal{X}(1 + \mu_{w,a_d})\right) > \mu_{d,a_d}. \quad \text{If} \quad \frac{\mathcal{Y}}{1 - \gamma} \mathcal{X}^2 \left(\nu - Z(1 + \mu_{w,a_d})\right) \leq \frac{1 - \gamma}{\gamma} \mathcal{X}^2 \mathcal{A}(\mu_{w,a_d}), \text{the two conditions above are mutually exclusive. This condition can be restated as} \quad \frac{\mathcal{Y}}{1 - \gamma} \mathcal{X}^2 \leq \frac{\nu - Z(1 + \mu_{w,a_d})}{\mathcal{A}(\mu_{w,a_d})}. \text{ Importantly, this condition is always}
\]

18Again, \( \gamma = 1 \) trivially implies \( \mu_{d,a_d} = 0 \) by equation (A.74).
satisfied. To see this, take the ratio of the two rearranged FOCs in (A.73) and (A.74) to eliminate \(\mu_{d,a_d}\). This results to

\[
\frac{Y}{(1 - \gamma)X^2} \frac{1 - \exp\{\Delta_{a_d}\}}{1 + \exp\{\Delta_{a_d}\}} = \frac{(\mu - Z(1 + \mu_{w,a_w}))}{A(\mu_{w,a_d})}.
\]

(A.81)

Because \(\frac{1 - \exp\{\Delta_{a_d}\}}{1 + \exp\{\Delta_{a_d}\}} < 1\) for \(\Delta_{a_d} \neq 0\), the two FOCs cannot be simultaneously satisfied. The same exercise leads to the identical conditions when \(\mu_{d,a_d}\) is negative as is \(\nu - Z(1 + \mu_{w,a_w})\) and hence \(A(\mu_{w,a_d})\).

The other case is depicted by alternating signs, i.e. \(\mu_{d,a_d}\) is positive but \(\nu - X(1 + \mu_{w,a_d})\) and hence \(\nu - Z(1 + \mu_{w,a_d})\) are negative. Again, it can be shown that the conditions for the two FOCs to hold simultaneously are violated. This is, however, left to the reader since by now the way to prove should be clear. As a result, it follows that the only solution that satisfies the FOCs must entail symmetry. Hence, the solution is also unique.

**Optimal Policies: \((\mu_{d,a_d}^{Opt} \text{ and } \mu_{a_d}^{Opt})\)** Consequently, since \(\mu_{d,a_d}^{Opt} = 0\) it must be true that \(\left(\nu - Z(1 + \mu_{d,a_d}^{Opt})\right) = 0\) it follows that

\[
\mu_{d,a_d}^{Opt} = \mu_{a_d}^{Opt} = \frac{(1 - \rho)(1 - \gamma)^2}{Z}.
\]

(A.82)

**Second Order Conditions** The second-order conditions are satisfied independent of the values of \(\mu_{w,a_d}\) and \(\mu_{d,a_d}\). Consequently, the solution to the first-order conditions will depict the global optimum to the maximization problem.

**A.4 Non-cooperative Monetary Policy: Nash**

**Proof of Proposition 4**

When Home and Foreign authorities set the nominal interest rate independently, they choose \(\{R, R^*\}\) so as to maximize their respective residents’ expected welfare. The two first order conditions are

\[
\frac{d\bar{u}(R, R^*)}{dR} \exp\left\{\frac{\nu(1 - \rho)}{2\lambda} \Omega(\cdot)\right\} = 0, \quad \text{and}
\]

\[
\frac{d\bar{u}(R, R)}{dR^*} \exp\left\{\frac{\nu(1 - \rho)}{2\lambda} \Omega^*(\cdot)\right\} = 0,
\]

respectively. Taking the explicit derivatives and dividing the FOC to \(R\) (equation (A.83)) through \(\exp\left\{\frac{\nu(1 - \rho)}{2\lambda} \Omega(\cdot)\right\}\), \(\left(\frac{\theta - 1}{\theta - \frac{1}{\pi_w}}\right) \frac{(1 - \rho)}{\lambda}\), and \(R_d \frac{(1 - \gamma)(1 - \rho)}{2\gamma}\) yields

\[
0 = \frac{\theta - 1}{\theta - \frac{1}{\pi_w}} \left(\frac{1}{R} - \frac{(1 - \rho)}{2\lambda} \left(\frac{1}{(1 - \rho)} - \frac{1}{\pi_w} \frac{1}{R}\right) \frac{1}{R} \right) - \frac{(1 - \rho)(1 - \gamma)}{2\gamma} \left(\frac{1}{(1 - \rho)} - \frac{\theta - 1}{\theta - \frac{1}{\pi_w}} \frac{1}{R}\right) \frac{1}{R}.
\]

(A.85)
A similar manipulation to the FOC to $R^*$ (equation (A.84)) yields
\[ 0 = \frac{\theta - 1}{\nu} \left( \frac{1}{R^*} \right) \left( \frac{1}{(1 - \rho)} - \frac{\theta - 1}{\nu} \right) \frac{1}{R^*} - \frac{(1 - \rho)(1 - \gamma)}{2\nu} \left( \frac{1}{1 - \rho} - \frac{\theta - 1}{\nu} \right) \frac{1}{R^*}. \]  
(A.86)

Existence of Symmetric Equilibria only: ($R^{Nash} = R^{*Nash}$) The two FOCs in (A.85) and (A.86) are linear in $R$ and $R^*$, respectively, and they are both independent of the other countries action. Because the two conditions are identical, it follows immediately that Home and Foreign authorities take the same action. Hence, $R^{*Nash} = R^{Nash}$.

Nash Policy Intervention: ($R^{Nash}$ and $R^{*Nash}$) Solving the two equations (A.85) and (A.86) results to
\[ R^{Nash} = R^{*Nash} = \frac{\theta - 1}{\theta} \left( 1 + \frac{\gamma X}{Y + (1 - \gamma)A} \right). \]  
(A.87)

Second Order Conditions The verification of the second order conditions for $R^{Nash}$ and $R^{*Nash}$ is straightforward and left to the reader.

Proof of Proposition 5

The two first order conditions when national authorities set their policy feedback coefficients to shocks non-cooperatively, i.e. $\{\mu_a, \mu^*_a\}$ for $k = w, d$ are
\[ 0 = \frac{\nu(1 - \rho)}{2\lambda} \left( \frac{d \Omega(\cdot)}{d \mu_a} \right) \exp \left\{ \frac{\nu(1 - \rho)}{2\lambda} \Omega(\cdot) \right\} \]  
and \[ 0 = \frac{\nu(1 - \rho)}{2\lambda} \left( \frac{d \Omega^*(\cdot)}{d \mu^*_a} \right) \exp \left\{ \frac{\nu(1 - \rho)}{2\lambda} \Omega^*(\cdot) \right\} \]  
for $k = w, d$, respectively. First note that $\frac{\nu(1 - \rho)}{2\lambda} \frac{d \Omega(\cdot)}{d \mu_a}$ and $\frac{\nu(1 - \rho)}{2\lambda} \frac{d \Omega^*(\cdot)}{d \mu^*_a}$ are strictly positive and they are by Proposition 4 identical. Furthermore, $\exp \left\{ \frac{\nu(1 - \rho)}{2\lambda} \Omega(\cdot) \right\}$ and $\exp \left\{ \frac{\nu(1 - \rho)}{2\lambda} \Omega^*(\cdot) \right\}$ are strictly positive, too. Hence, the problem reduces to find the maximand of $\Omega(\cdot)$ and $\Omega^*(\cdot)$.

Proof of Proposition 5: Aggregate Shocks

Home and Foreign first order conditions to the policy rule setting-game in case of aggregate productivity shocks read
\[ 0 = A(\mu_{w,a,w}) \left( 1 + \frac{(1 - \gamma)}{Y} \lambda \right) - \left( Z + \frac{(1 - \gamma)}{Y} \lambda^2 \right) \mu_{d,a,w} \]  
(A.90)
for $\mu_{a,w}$ and
\[ 0 = A(\mu_{w,a,w}) \left( 1 + \frac{(1 - \gamma)}{Y} \lambda \right) + \left( Z + \frac{(1 - \gamma)}{Y} \lambda^2 \right) \mu_{d,a,w} \]  
(A.91)
for $\mu^*_{a,w}$, respectively.

Existence of Symmetric Equilibria only: ($\mu^{Nash}_{a,w} = \mu^*_{a,w}$) Taking the difference of the two FOCs (A.90) and (A.91) yields
\[ 0 = - \left( Z + \frac{(1 - \gamma)}{Y} \lambda^2 \right) \mu_{d,a,w}. \]  
(A.92)
Consequently, because \( Z + \frac{(1-\gamma)}{\mathcal{Y}} \mathcal{X}^2 > 0 \) it follows that equation (A.92) requires \( \mu_{d,a}^{Nash} = 0 \) and hence a symmetric equilibrium.

**Nash Policy Intervention to Aggregate Shocks:** \( (\mu_{a}^{Nash,a} \text{ and } \mu_{a}^{sNash,a}) \) Making use of the symmetry, conditions (A.90) and (A.91) directly imply that \( \mathcal{A}(\mu_{w,a}) = 0 \) because \( \left( 1 + \frac{(1-\gamma)}{\mathcal{Y}} \mathcal{X} \right) > 0 \). As a result,

\[
\mu_{a}^{Nash} = \mu_{a}^{sNash} = \frac{(1-\rho)}{\mathcal{X}}. \tag{A.93}
\]

**Second Order Conditions** The second-order conditions for the solution to be the maximum are satisfied and the verification is left to the reader.

**Proof of Proposition 5: Asymmetric Shocks**

Home and Foreign first order conditions to the policy rule setting-game in case of asymmetric productivity shocks read

\[
0 = \nu \left( 1 + \frac{(1-\gamma)}{\mathcal{Y}} \mathcal{X} \right) - \left( Z + \frac{(1-\gamma)}{\mathcal{Y}} \mathcal{X}^2 \right) \left( 1 + \mu_{w,a} \right) - \mathcal{X} \left( 1 + \frac{(1-\gamma)}{\mathcal{Y}} \mathcal{X} \right) \mu_{d,a} \tag{A.94}
\]

for \( \mu_{a} \) and

\[
0 = \nu \left( 1 + \frac{(1-\gamma)}{\mathcal{Y}} \mathcal{X} \right) - \left( Z + \frac{(1-\gamma)}{\mathcal{Y}} \mathcal{X}^2 \right) \left( 1 + \mu_{w,a} \right) + \mathcal{X} \left( 1 + \frac{(1-\gamma)}{\mathcal{Y}} \mathcal{X} \right) \mu_{d,a} \tag{A.95}
\]

for \( \mu_{a}^s \), respectively.

**Existence of Symmetric Equilibria only:** \( (\mu_{a}^{Nash} = \mu_{a}^{sNash}) \) Taking the difference of the two FOCs (A.94) and (A.95) yields

\[
0 = -\mathcal{X} \left( 1 + \frac{(1-\gamma)}{\mathcal{Y}} \mathcal{X} \right) \mu_{d,a}. \tag{A.96}
\]

Consequently, because \( \mathcal{X} \left( 1 + \frac{(1-\gamma)}{\mathcal{Y}} \mathcal{X} \right) > 0 \) it follows that equation (A.96) requires \( \mu_{d,a} = 0 \) and hence a symmetric equilibrium.

**Nash Policy Intervention to Asymmetric Shocks:** \( (\mu_{a}^{Nash} \text{ and } \mu_{a}^{sNash}) \) Making use of the symmetry, conditions (A.94) and (A.95) imply that

\[
\nu \left( 1 + \frac{(1-\gamma)}{\mathcal{Y}} \mathcal{X} \right) = \left( Z + \frac{(1-\gamma)}{\mathcal{Y}} \mathcal{X}^2 \right) \left( 1 + \mu_{w,a} \right). \tag{A.97}
\]

Solving for \( \mu_{w,a} \) then results to

\[
\mu_{a}^{Nash} = \mu_{a}^{sNash} = (1-\rho)(1-\gamma) \mathcal{X} \left( \frac{1+\gamma}{Z} \frac{1+\gamma}{\mathcal{Y}} \right). \tag{A.98}
\]
Second Order Conditions  The second-order conditions for the solution to be the maximum are satisfied and the verification is left to the reader.

B  Appendix to Chapter 2

The model in Chapter 2 builds on the model in Chapter 1. In particular, the structure of the two models is identical. The only difference between the two models is the introduction of fiscal policy. As a consequence, the setup, the optimality conditions and the equilibrium determination is equivalent to the model in Chapter 1. Moreover, by Lemma 1 and 2, it immediately follows that one can simply substitute policy variables of the version in Chapter 1 for the compound expression in Chapter 2. To be specific, substitute the nominal interest rates \( R \) and \( R^* \) for the ex ante compound policy terms \( I_W \) and \( I_W^* \). For the logs of the ex post compound public interventions follows that \( \hat{i}_C \) and \( \hat{i}_C^* \) correspond to \( \hat{m} \) and \( \hat{m}^* \). The world average and difference components of Home and Foreign compound public interventions can easily be derived from national Home and Foreign money supply and consumption tax feedback rules. The compound national public policy rules are given by

\[
\hat{i}_C = i_{C,a_w} \hat{a}_w + i_{C,a_d} \hat{a}_d \quad \text{and} \quad \hat{i}_C^* = i_{C,a_w}^* \hat{a}_w - i_{C,a_d}^* \hat{a}_d,
\]

(B.1)

where \( i_{C,a_w} = \mu_{a_w} - \tau_{a_w} \), \( i_{C,a_w}^* = \mu_{a_w}^* - \tau_{a_w}^* \), \( i_{C,a_d} = \mu_{a_d} - \tau_{a_d} \), and \( i_{C,a_d}^* = \mu_{a_d}^* - \tau_{a_d}^* \). In terms of world average and difference components, the feedback rules can be stated as

\[
\hat{i}_w = i_{w,a_w} \hat{a}_w + i_{d,a_d} \hat{a}_d \quad \text{and} \quad \hat{i}_d = i_{d,a_w} \hat{a}_w + i_{w,a_d} \hat{a}_d,
\]

(B.2)

where

\[
\begin{align*}
i_{w,a_w} &= \frac{1}{2} \left( (\mu_{a_w} + \mu_{a_w}^*) - (\tau_{a_w} + \tau_{a_w}^*) \right), \\
i_{d,a_w} &= \frac{1}{2} \left( (\mu_{a_w} - \mu_{a_w}^*) - (\tau_{a_w} - \tau_{a_w}^*) \right), \\
i_{w,a_d} &= \frac{1}{2} \left( (\mu_{a_d} + \mu_{a_d}^*) - (\tau_{a_d} + \tau_{a_d}^*) \right), \quad \text{and} \\
i_{d,a_d} &= \frac{1}{2} \left( (\mu_{a_d} - \mu_{a_d}^*) - (\tau_{a_d} - \tau_{a_d}^*) \right).
\end{align*}
\]

From these definitions, the corresponding terms of Chapter 1 should be self-explaining. Therefore, the proofs of the first four Propositions are identical to the proofs of the last four Propositions in Chapter 1.

B.1 Optimal Public Policy Coordination

When Home and Foreign policymakers optimally coordinate on both ex-post and ex-ante fiscal and monetary policy interventions, they do so as to maximize the sum of Home and Foreign expected utilities, i.e., \( Eu + Eu^* \), over \( \{I_W, i_{C,a_w}, i_{C,a_d}\} \) and \( \{I_W^*, i_{C,a_w}^*, i_{C,a_d}^*\} \), respectively. In an alternative expression, Home and Foreign authorities jointly set \( \{I_{W,w}, I_{W,d}\} \), \( \{i_{w,a_w}, i_{d,a_w}\} \), and \( \{i_{w,a_d}, i_{d,a_d}\} \) in order to maximize the joint welfare.
Proof of Proposition 1

There are two first order conditions for setting \( \{I_W, I_d\} \), namely
\[
\frac{d\bar{u}(I_W, I_d)}{dI_W} \exp \left\{ \frac{\nu(1 - \rho)}{2\lambda} \Omega(\cdot) \right\} + \frac{d\bar{u}^*(I_W, I_d)}{dI_W} \exp \left\{ \frac{\nu(1 - \rho)}{2\lambda} \Omega^*(\cdot) \right\} = 0, \quad \text{and (B.3)}
\]
\[
\frac{d\bar{u}(I_W, I_d)}{dI_d} \exp \left\{ \frac{\nu(1 - \rho)}{2\lambda} \Omega(\cdot) \right\} + \frac{d\bar{u}^*(I_W, I_d)}{dI_d} \exp \left\{ \frac{\nu(1 - \rho)}{2\lambda} \Omega^*(\cdot) \right\} = 0, \quad \text{(B.4)}
\]
respectively. For the subsequent proof of symmetry (\( I^{Opt}_W = I^{Opt}_d \)), the optimal policy interventions \( I^{Opt}_W \) and \( I^{Opt}_d \), and the SOCs, see proof of Proposition 1.2..

Proof of Proposition 2

The two first order conditions for setting \( \{i_{C_w,a_k}, i_{C_d,a_k}\} \) for \( k = w, d \), i.e. in case of both the aggregate and the asymmetric shocks, are
\[
0 = \frac{\nu(1 - \rho)}{2\lambda} \bar{u}(\cdot) \exp \left\{ \frac{\nu(1 - \rho)}{2\lambda} \Omega(\cdot) \right\} \frac{d\Omega(\cdot)}{di_{C_w,a_k}} + \frac{\nu(1 - \rho)}{2\lambda} \bar{u}^*(\cdot) \exp \left\{ \frac{\nu(1 - \rho)}{2\lambda} \Omega^*(\cdot) \right\} \frac{d\Omega^*(\cdot)}{di_{C_w,a_k}}, \quad \text{(B.5)}
\]
\[
0 = \frac{\nu(1 - \rho)}{2\lambda} \bar{u}(\cdot) \exp \left\{ \frac{\nu(1 - \rho)}{2\lambda} \Omega(\cdot) \right\} \frac{d\Omega(\cdot)}{di_{C_d,a_k}} + \frac{\nu(1 - \rho)}{2\lambda} \bar{u}^*(\cdot) \exp \left\{ \frac{\nu(1 - \rho)}{2\lambda} \Omega^*(\cdot) \right\} \frac{d\Omega^*(\cdot)}{di_{C_d,a_k}}, \quad \text{(B.6)}
\]
for \( k = w, d \), respectively.

Proof of Proposition 2: Aggregate Shocks

See proof of Proposition 1.3.

Proof of Proposition 2: Asymmetric Shocks

See proof of Proposition 1.3.

B.2 Non-cooperative Public Policy: Nash

Proof of Proposition 3

When Home and Foreign authorities set the ex-ante policy intervention to the wage setting independently, they choose \( \{I_W, I_{W}^*\} \) so as to maximize their respective residents’ expected welfare. The two first order conditions are
\[
\frac{d\bar{u}(I_W, I_{W}^*)}{dI_W} \exp \left\{ \frac{\nu(1 - \rho)}{2\lambda} \Omega(\cdot) \right\} = 0, \quad \text{and (B.7)}
\]
\[
\frac{d\bar{u}(I_W, I_{W}^*)}{dI_{W}^*} \exp \left\{ \frac{\nu(1 - \rho)}{2\lambda} \Omega^*(\cdot) \right\} = 0, \quad \text{(B.8)}
\]
respectively. For the subsequent proof of symmetry (\( I^{Nash}_W = I^{Nash}_W \)), the optimal policy interventions \( I^{Nash}_W \) and \( I^{Nash}_W \), and the SOCs, see proof of Proposition 1.4..
Proof of Proposition 4

The two first order conditions when national authorities set their policy feedback coefficients to shocks non-cooperatively, i.e. \( \{i_{C,a_k}, i_{C^*,a_k}\} \) for \( k = w, d \) are

\[
0 = \frac{\nu(1-\rho)}{2\omega} \bar{u}(\cdot) \exp \left\{ \frac{\nu(1-\rho)}{2\omega} \Omega(\cdot) \right\} \frac{d\Omega(\cdot)}{di_{C,a_k}}, \tag{B.9}
\]

and

\[
0 = \frac{\nu(1-\rho)}{2\omega} \bar{u}^*(\cdot) \exp \left\{ \frac{\nu(1-\rho)}{2\omega} \Omega^*(\cdot) \right\} \frac{d\Omega^*(\cdot)}{di_{C,a_k}}, \tag{B.10}
\]

for \( k = w, d \), respectively.

Proof of Proposition 4: Aggregate Shocks

See proof of Proposition 1.5.

Proof of Proposition 4: Asymmetric Shocks

See proof of Proposition 1.5.

B.3 Monetary Cooperation and Fiscal Independence

When national monetary authorities coordinate their policies but national fiscal authorities stay independent, the game-theoretic setup is described by a game of three players: one coordinating "global" monetary authority that seeks to maximize the sum of Home and Foreign welfare and two independent fiscal authorities that seek to maximize the respective residents' welfare.

Proof of Proposition 5

Proposition 5 is proven in two steps. First, the only pure strategy equilibrium is derived. Second, it is argued that there cannot be any other equilibrium in mixed strategies so that the pure strategy equilibrium is unique.

The Only Pure Strategy Equilibrium  In case of the ex ante wage setting intervention, recall from Definition 2 that

\[
I_W = \frac{1 + t_C}{1 - t_W} R \quad \text{and} \quad I_W^* = \frac{1 + t_C^*}{1 - t_W^*} R^*.	ag{B.11}
\]

For notational convenience define the part of Home and Foreign fiscal policy as \( T = \frac{1 + t_C}{1 - t_W} \) and \( T^* = \frac{1 + t_C^*}{1 - t_W^*} \). Then, in terms of world and difference components of policy interventions, ie. \( I_{W_{w}} \) and \( I_{W_{d}} \), the compound policy variables are

\[
I_{W_{w}} = T_w R_w \quad \text{and} \quad I_{W_{d}} = T_d R_w. \tag{B.12}
\]
Home and Foreign monetary authorities jointly decide over \( \{R_w, R_d\} \) to maximize \( Eu + Eu^* \). Monetary authorities are, of course, constrained to set the net nominal interest rates to be non-negative, i.e. for the gross nominal interest rate \( R \geq 1 \) and \( R^* \geq 1 \). The Home fiscal authority sets \( T \) so as to maximize \( Eu \) and the Foreign authority sets \( T^* \) so as to maximize \( Eu^* \). Importantly, considering pure strategy equilibria only, the first order conditions of the monetary authorities to this problem coincide with the first order condition to the problem of optimal public policy (B.3) and (B.4) except for the multiplicatively added derivative of the compound policy interaction with respect to the nominal interest rates, i.e. \( \frac{dW_w}{dR_w} \) in equation (B.3) and \( \frac{dW_d}{dR_d} \) in equation (B.4), respectively. As a consequence, the optimality conditions derived in the proof of Proposition 1 carry over. In particular, this implies for the Nash equilibrium in pure strategies that monetary authorities want to implement the optimal level of public policy intervention \( I_{W_{Opt}} = I_{w_{Opt}} = \frac{\theta - 1}{\theta} \) in both countries. For the national fiscal policy authorities, taking as given the monetary policy interventions, the first order conditions coincide with the first order conditions to the problem of non-cooperatively set public policies in the proof of Proposition 3 except for the multiplicatively added derivative of the compound policy interaction with respect to the national fiscal policy terms, i.e. \( \frac{dW_w}{d\Pi_w} \) in equation (B.7) and \( \frac{dW_d}{d\Pi_d} \) in equation (B.8), respectively. Again, the characterization of the optimality conditions carry over and hence national fiscal policymakers seek to implement the Nash levels of compound national interventions \( I_{W_{Nash}} = I_{w_{Nash}} = \frac{\theta - 1}{\theta} \left( 1 + \frac{\gamma Y^*}{Y + (1 - \gamma)X} \right) \). Hence, as an implication of Lemma 2, the pure strategy equilibrium is given by the reduced problem depicted by the choice of the level of compound policy intervention. The symmetry results in Proposition 1 and 3 reduces the problem further to two condition in two unknowns. For the Home policy choices, monetary authority's best response to a given fiscal policy intervention \( T \) is

\[
RT = \frac{\theta - 1}{\theta} \begin{cases} 
1 + \frac{\gamma X}{Y + (1 - \gamma)X} & \text{if } T < \frac{\theta - 1}{\theta}, \\
1 & \text{if } T \geq \frac{\theta - 1}{\theta},
\end{cases}
\]

(B.13)

where the kink in the best response stems from the requirement that the nominal interest rate cannot be less than one. The Home fiscal authority’s best response to a given nominal interest rate is

\[
TR = \frac{\theta - 1}{\theta} \left( 1 + \frac{\gamma X}{Y + (1 - \gamma)X} \right).
\]

(B.14)

Since \( 1 + \frac{\gamma X}{Y + (1 - \gamma)X} \geq 1 \), there is only one fixed point which determines the Nash equilibrium, namely \( T = \frac{\theta - 1}{\theta} \left( 1 + \frac{\gamma X}{Y + (1 - \gamma)X} \right) \) and \( R = 1 \).

**Uniqueness of the Equilibrium** To see that the pure strategy equilibrium is unique, suppose to the contrary that there exists an equilibrium in mixed strategies. Then it must hold true that given the monetary authorities are randomizing over their nominal interest rate, fiscal authorities must be indifferent between choosing any pure strategy over which
they actually randomize (compare eg. Mas-Colell et al. (1995) Proposition 8.D.1). As the monetary authorities are, however, constrained to set the net nominal interest rate non-negatively, fiscal authorities will find it better to set fiscal interventions at level strictly larger than \( \frac{\theta - 1}{\theta} \) as their best responses dictate. This, in turn, leads to the pure strategy best response of setting \( R = R^* = 1 \). As a result, there exists no mixed strategy equilibrium and the pure strategy equilibrium is unique.

**Second Order Conditions** Easy to verify and again left to the reader.

**Proof of Proposition 6**

In case of aggregate shocks, the compound policy interventions to the consumption spending are given by

\[
    i_{C,a} = \mu_{a} - \tau_{a} \quad \text{and} \quad i^{*}_{C,a} = \mu^{*}_{a} - \tau^{*}_{a}.
\]  
\[\text{(B.15)}\]

The policy interventions are defined in terms of world aggregate and difference components as

\[
    i_{C,w,a} = \mu_{w,a} - \tau_{w,a} \quad \text{and} \quad i^{*}_{C,d,a} = \mu^{*}_{d,a} - \tau^{*}_{d,a}.
\]  
\[\text{(B.16)}\]

By the identical reasoning as in the proof of Proposition 5, the first order conditions of the monetary authorities for choosing \( \mu_{w,a} \) and \( \mu_{d,a} \) are fully characterized by the optimality conditions in the proof of Proposition 2 (Aggregate Shocks). The first order conditions of the national fiscal authorities are characterized by the conditions in the proof of Proposition 4 (Aggregate Shocks). In particular, monetary authorities seek to implement \( i^{Opt}_{C,a} = i^{Opt}_{C,a} = \frac{(1 - \rho)}{\chi} \). The national fiscal authorities, however, seek to implement \( i^{Nash}_{C,a} = i^{Nash}_{C,a} = \frac{(1 - \rho)}{\chi} \) which takes the identical target level of intervention. Consequently, for the compound national public policy intervention to aggregate shocks \( i_{C,a} \), the two equations determining \( \mu_{a} \) and \( \tau_{a} \) coincide. As a result, all combinations of \( \mu_{a} \) and \( \tau_{a} \) such that

\[
    \mu_{a} - \tau_{a} = \frac{(1 - \rho)}{\chi}
\]  
\[\text{(B.17)}\]

constitute Nash equilibria. The sole problem of policymaking reduces to a coordination game between national monetary and fiscal policy conduct.

**Second Order Conditions**

Easy to verify and again left to the reader.

**Proof of Proposition 7**

The proof of Proposition 7 evolves in two steps: First, it is demonstrated that there exist only equilibria in pure strategies if \( \rho = 1 \) or \( \gamma \in \{0, 1\} \). This is done in straight analogy to the proofs in the proofs of Propositions 5 and 6. The second step is to show that there is a
unique equilibrium in mixed strategies if $\rho \neq 1$ or $0 < \gamma < 1$. In case of asymmetric shocks, the compound policy interventions to the consumption spending are given by

$$i_{C,a,d} = \mu_{a,d} - \tau_{a,d} \quad \text{and} \quad i^*_{C,a,d} = \mu^*_{a,d} - \tau^*_{a,d}.$$  \hfill (B.18)

The policy interventions are defined in terms of world aggregate and difference components as

$$i_{C,w,a,d} = \mu_{w,a,d} - \tau_{w,a,d} \quad \text{and} \quad i^*_{C,d,a,d} = \mu^*_{d,a,d} - \tau^*_{d,a,d}.$$  \hfill (B.19)

### The Equilibrium in Pure Strategies

The first order conditions of the monetary authorities for choosing $\mu_{w,a,d}$ and $\mu_{d,a,d}$ are fully characterized by the optimality conditions in the proof of Proposition 2 (Asymmetric Shocks). The first order conditions of the national fiscal authorities are characterized by the conditions in the proof of Proposition 4 (Asymmetric Shocks). In particular, monetary authorities seek to implement

$$i_{Opt}^{C,a,d} = i^{Opt}_{C,a,d} = \frac{(1-\rho)(1-\gamma)^2}{Z}. \quad \text{The national fiscal authorities, however, seek to implement}$$

$$i^{Nash}_{C,a,d} = i^{Nash}_{C,a,d} = (1-\rho)(1-\gamma) \left( \frac{X + (1-\gamma)Y}{2Y + (1-\gamma)X^2} \right). \quad \text{These are two conditions in two unknowns that determine in the pure strategy equilibrium the level of Home and Foreign compound public policy intervention, respectively.}$$

For a given fiscal policy parameter $\tau_{a,d}$, the best response of Home money supply in pure strategies is to set

$$\mu_{a,d} = \tau_{a,d} + \frac{(1-\rho)(1-\gamma)^2}{Z}. \quad \text{(B.20)}$$

For a given money supply coefficient $\mu_{a,d}$, the best response of Home fiscal policy in pure strategies is

$$\tau_{a,d} = \mu_{a,d} - (1-\rho)(1-\gamma) \left( \frac{X + (1-\gamma)Y}{2Y + (1-\gamma)X^2} \right). \quad \text{(B.21)}$$

As a first result, there exists only a pure strategy fixed point if either $\rho = 1$ or $\gamma = 0$ or $\gamma = 1$. In this case, the condition coincide and there exists a continuum of equilibria. All that remains is a coordination problem between the monetary and the fiscal authorities.

### The Equilibrium in Mixed Strategies

The proof that there exists a Nash equilibrium even in case of $\rho \neq 1$ or $0 < \gamma < 1$ follows the standard fixed point argument (See Mas-Colell et al. (1995) Proposition 8.D.3). In particular, one has to show that first the strategy sets of all policy authorities are non-empty, convex, and a compact subset of $\mathbb{R}^1$. The first two conditions are trivially given. Compactness can be achieved by deleting strategies which are actually not rationalizable. For example, the bounds of the sets are chosen by the two cases when either monetary or fiscal policymakers myopically set their desired levels and the best responses of the respective other authorities to these levels. To be specific, suppose the monetary authorities set $\mu_{a,d} = i^{Opt}_{C,a,d}$ and $\mu^*_{a,d} = i^{Opt*}_{C,a,d}$. The best responses of the independent fiscal authorities according to equation (B.21) are $\tau_{a,d} = i^{Opt}_{C,a,d} - i^{Nash}_{C,a,d}$ and $\tau^*_{a,d} = i^{Opt*}_{C,a,d} - i^{Nash}_{C,a,d}$. In turn, suppose next that independent fiscal authorities simply set
coefficient to their desired levels, i.e. \( \tau_{ad} = -i_{Nash}^{C,ad} \) and \( \tau^*_{ad} = -i^*_{Nash}^{C,ad} \). Monetary authorities’ best responses are according to equation (B.20) \( \mu_{ad} = i^{Opt}_{C,ad} - i_{Nash}^{C,ad} \) and \( \mu^*_{ad} = i^{Opt}_{C,ad} - i^*_{Nash}^{C,ad} \). Consequently, \( \mu_{ad}, \mu^*_{ad} \in [i^{Opt}_{C,ad} - i_{Nash}^{C,ad}, i^{Opt}_{C,ad} - i^*_{Nash}^{C,ad}] \) and \( \tau_{ad}, \tau^*_{ad} \in [i_{Nash}^{C,ad}, i^{Opt}_{C,ad} - i_{Nash}^{C,ad}] \). The second set of conditions deals with the objectives. In particular, they must be continuous in all arguments and they must be concave in each single policy action. Continuity is trivially given and concavity reduces to checking whether the second order conditions are satisfied, too. This is the case and corresponds to the checks above. Hence, there exists a Nash equilibrium in the game with coordinating monetary authorities and independently acting fiscal authorities.

Continuum of Equilibria in Mixed Strategy with Unique Policy Outcome

Next, the following two claims are proven: First, if there exist any mixed strategy equilibrium in this policy setting game, then there must be continuum of mixed strategies leading to the same equilibrium distribution over the policy outcomes. Second, There exist no other mixed strategy equilibrium that entails a different distribution over the policy outcomes and hence different expected welfare.

To prove the first claim is fairly easy. The important insight is that due to Lemma 1 monetary and fiscal policy actions are perfect substitutes. Hence, the combination of national monetary and fiscal policy is indeterminate as to yield a certain level of the compound policy intervention. To be more precise, suppose that the two pure strategies \( \tilde{\mu}_{ad} \) and \( \tilde{\tau}_{ad} \) are played with positive probability. Then it must be true that \( \tilde{\mu}_{ad} + k \) and \( \tilde{\tau}_{ad} + k \) for any \( k \in R^1 \) is also valid for this equilibrium. Consequently, the two sets (for Home monetary and fiscal) of pure strategies that are played with positive probability are determined up to a jointly additive component. The result is that as there exist an equilibrium in the policy setting game, there must be a continuum of equilibria leading to the same probability distribution over the compound policy interventions and hence the same expected welfare.

The proof of the second claim that there exist no other mixed strategy equilibrium that entails a different policy outcome and hence different expected welfare is only slightly more involved and builds on the first claim. The crucial part is to see that given a mixed strategy equilibrium, the equilibrium distribution over the pure strategies played must be unique. In particular, recall that in any mixed strategy equilibrium in a simultaneous move game it holds true that players must be indifferent between choosing any of the pure strategies that are played with positive probability (Compare again Mas-Colell et al. (1995) Proposition 8.D.1). Moreover, recall that for all pure strategies played by their opponents, monetary and fiscal authorities have a unique best response. As a consequence, in order to leave any player indifferent between choosing any pure strategy that is played with positive probability, the
strict concavity of the objectives and the uniqueness of the best responses imply that the probability distribution must be unique as there cannot be any degree of freedom to alter the distribution. Suppose this were not true. Suppose Home monetary authority changes the probability distribution and puts more weight on, say, $\mu_{ad}$. The uniqueness of the best response and concavity of the fiscal authorities objective implies that the fiscal authority can no longer be indifferent as it wants to put more weight on the best response to $\mu_{ad}$, say $\tau_{ad}$. Hence, the fiscal authorities best response would be to play $\tau_{ad}$ with probability one. As a result, the equilibrium probability distribution for a given mixed strategy equilibrium is unique.

To see that there can be no other mixed strategy equilibrium that entails a different policy outcome, suppose to the contrary that there exists an equilibrium with different expected payoffs. Due to the first claim it must be true that there also exists a continuum of mixed strategy equilibria that entail the identical probability distribution over the compound policy outcomes and hence expected payoffs. Therefore, given the compact strategy space, any pure strategy can be played with positive probability in both sets of mixed strategy equilibria leading to different expected payoffs. Because given any opponent’s pure strategy that is played with positive probability in equilibrium the players best response is unique, the level of compound policy interventions must be the same in both sets of equilibria. Consequently, the difference in the expected payoffs can only be due to different equilibrium probability distributions. As demonstrated above, however, the probability distribution is unique given the set of pure strategies played with positive probability. As a result, there exists a unique distribution over policy outcomes and hence the expected welfare of the continuum of mixed strategy equilibria is unique, too.

**Implementation in the Numerical Example** In this Section, I briefly explain how the mixed strategy equilibrium is calculated. As a simplification, the strategy space is convexified via a two-point distribution over the specific boundaries as defined above. To be more precise, Home and Foreign monetary authorities set

$$\mu_{ad} = \frac{i_{Opt}^{C,ad}}{\alpha} \quad \text{with probability} \quad \alpha$$

and

$$\mu_{ad}^* = \frac{i_{Opt}^{C,ad} - i_{Nash}^{C,ad}}{\beta} \quad \text{with probability} \quad 1 - \alpha.$$ 

Consequently, there are $2^2$ different realizations of world aggregate and difference components of monetary policy interventions to asymmetric shocks. Similarly, for independently acting fiscal authorities one gets

$$\tau_{ad} = \frac{-i_{Nash}^{C,ad}}{\beta} \quad \text{with probability} \quad \beta$$

and

$$\tau_{ad}^* = \frac{i_{Opt}^{C,ad} - i_{Nash}^{C,ad}}{1 - \beta} \quad \text{with probability} \quad 1 - \beta.$$ 

As a consequence, there are $2^2$ different realizations of the world and different components of fiscal policy coefficients, too. There are thus $2^4$ combinations of monetary and fiscal policy coefficients for $i_{C,w,ad}$ and $i_{C,d,ad}$. As a consequence, there are 16 realizations of the
compound public policy interventions to asymmetric shocks which can be each associated to the probability over the respective actions. Hence, there are in principle 16 different realization of policy rules that entail 16 different terms of expected utility. Because all authorities are expected-utility-maximizer, they maximize the sum of the value of their respective objective weighted by the probability of the realization of the compound policy interaction with respect to the authorities own probability of playing the strategy. This leads to four first order conditions. For the numerical example, they are implemented in MatLab where I use the routine by Broydn to find the root of this problem.

C  Appendix to Chapter 3

In this technical appendix all the results and equations for the analysis of federal fiscal transfer arrangements are derived. The appendix is structured naturally as it begins with repeating the equilibrium conditions and the definitions and calculating the log-linearization of these conditions. The next step is to take union-wide aggregates and regional differences of all variables and express the equilibrium conditions in these terms. By Aoki’s method we then can solve for the long run and short responses of regional variables to asymmetric demand and supply shocks. Using the solution of the equilibrium responses to asymmetric demand and supply shocks allows us to determine the transfers and their consequences in the policy analysis. We conclude this appendix by showing how the welfare can be approximated.

C.1  Summarizing the Equilibrium Conditions

The long-run equilibrium for given regional and federal public policies is determined by the per capita versions of the Euler condition (C.1), optimal money demand (C.2), Fisher parity (C.4), optimal labor supply, i.e. the wage setting equation (C.3), aggregate production \( Y_s = a_s L_s \), aggregate demand (C.8), the price level (C.6, C.7), all the foreign counterparts of these conditions, home intertemporal budget constraint (C.11) and the transversality condition, and market clearing for the nominal bonds and for money (C.10).

In the short-run, however, households cannot adjust their wages to unanticipated shocks. With preset nominal wages, employment will be completely determined by the domestic labor demand. Thus in the short-run the optimal labor supply that equates marginal utility derived from consuming additional earnings to disutility from labor need not hold. Consequently, the set of short-run equilibrium conditions is the same as the set of long-run equilibrium conditions but without (C.3) and its foreign counterpart.
**Private Sector and Market Clearing**

Consumption Euler equation:

\[ C_{s+1} = C_s \beta (1 + r_{s+1}) \quad \text{and} \quad C^*_s = C^*_s \beta (1 + r^*_s); \]  

(C.1)

Optimal money demand

\[ \frac{M_s}{P_s} = \chi C_s \frac{1 + i_{s+1}}{i_{s+1}} \quad \text{and} \quad \frac{M^*_s}{P_s} = \chi C^*_s \frac{1 + i_{s+1}}{i_{s+1}}; \]  

(C.2)

Optimal wage setting

\[ W_s = \frac{\theta}{\theta - 1} L_s C_s P_s \quad \text{and} \quad W^*_s = \frac{\theta}{\theta - 1} L^*_s C^*_s P^*_s; \]  

(C.3)

Nominal interest:

\[ 1 + i_{s+1} = (1 + r_{s+1}) P_{s+1}^s P_s = (1 + r^*_s) P_{s+1}^s P^*_s; \]  

(C.4)

Labor demand:

\[ L_s = \frac{1}{a_s} Y_s \quad \text{and} \quad L^*_s = \frac{1}{a^*_s} Y^*_s; \]  

(C.5)

Goods prices:

\[ P_{H,s} = P_{N,s} = \frac{W_s}{a_s} \quad \text{and} \quad P^*_{F,s} = P^*_{N,s} = \frac{W^*_s}{a^*_s}; \]  

(C.6)

Consumption-based price indices:

\[ P_s = \left( \gamma P_{T,s}^{(1-\rho)} + (1 - \gamma) P_{N,s}^{(1-\rho)} \right)^{\frac{1}{1-\rho}}, \quad P_{T,s} = \left( \alpha P_{H,s}^{(1-\rho)} + (1 - \alpha) P_{F,s}^{(1-\rho)} \right)^{\frac{1}{1-\rho}}, \]  

\[ P^*_s = \left( \gamma P^*_{T,s}^{(1-\rho)} + (1 - \gamma) P^*_{N,s}^{(1-\rho)} \right)^{\frac{1}{1-\rho}}, \quad \text{and} \quad P^*_{T,s} = P_{T,s}; \]  

(C.7)

Total aggregate goods demand:

\[ Y_s = (1 - \gamma (1 - \alpha)) ToT_s^{-\rho (1-\alpha) \gamma} C_s + \alpha \gamma ToT_s^{-\rho (1-\alpha) \gamma} RER_s \rho C^*_s + G_s, \]  

and \[ Y^*_s = (1 - \alpha) ToT_s^{-\rho (1-\alpha) \gamma} C_s + (1 - \alpha \gamma) ToT_s^{-\rho (1-\alpha) \gamma} RER_s^{* \rho} C^*_s + G^*_s \]  

(C.8)

Terms of trade and real exchange rate:

\[ ToT_s = \left( \frac{P_{H,s}}{P^*_{F,s}} \right) = \left( \frac{a_s^* W_s}{a_s W^*_s} \right), \quad \text{and} \quad RER_s = \left( \frac{P_s}{P^*_s} \right) = ToT_s^{(1-\gamma)}; \]  

(C.9)

Bonds and money market market clearing conditions:

\[ B_s + B^*_s = 0, \quad \text{and} \quad M_s + M^*_s = M^{MU}_s; \]  

(C.10)

Budget constraint:

\[ B_{s+1} + M_s + P_s C_s \leq (1 + i_s) B_s + M_{(s-1)} + P_{H,s} Y_s + T_{loc,s} + T_{hh,s}, \]  

and \[ B^*_{s+1} + M^*_s + P^*_s C^*_s \leq (1 + i_s) B^*_s + M^*_{(s-1)} + P^*_{F,s} Y^*_s + T^*_{loc,s} + T^*_{hh,s} \]  

\[ \forall s \geq t. \]  

(C.11)
Local Governments and Federal Fiscal Arrangements

Local governments’ budget constraints:

\[ T_{\text{loc},s} + P_{N,s}G_s = T_{gg,s} + T_{cb,s} \quad \text{and} \quad T^*_{\text{loc},s} + P^*_{N,s}G^*_s = T^*_{gg,s} + T^*_{cb,s}; \quad (C.12) \]

Payments by the central bank:

\[ T_{cb,s} + T^*_{cb,s} = (M_s^{MU} - M_{s-1}^{MU}); \quad (C.13) \]

Federal government’s budget:

\[ T_{hh,s} + T^*_{hh,s} = 0 \quad \text{and} \quad T_{gg,s} + T^*_{gg,s} = 0, \quad (C.14) \]

respectively.

The Symmetric Steady State

There exists a closed form solution to a steady state if home and foreign households have the same initial wealth level. This implies first zero net foreign assets and second that \( \alpha = \frac{1}{2} \).

In this steady state regions are identical with respect to their per capita production, labor supply and money holdings. If we furthermore assume that steady state local government expenditures are equal, the regions also have identical per capita consumption, wage and price levels. We denote the steady state levels with overbars. So we have \( \bar{B} = \bar{B}^* = 0, \bar{G} = \bar{G}^*, \bar{a} = \bar{a}^*, \) and \( \bar{\alpha} = \frac{1}{2} \). The steady state levels are

\[
\begin{align*}
(\text{Consumption}) & \quad \bar{C} = \bar{C}^* = \bar{Y} - \bar{G} = \bar{Y}^* - \bar{G}^* = (1 - \bar{g})\bar{Y} \\
(\text{Production}) & \quad \bar{Y} = \bar{Y}^* = \bar{a} \left( \frac{\theta - 1}{\theta} \right)^{\frac{1}{2}} \\
(\text{Labor}) & \quad \bar{L} = \bar{L}^* = \frac{1}{\bar{a}} \bar{Y} = \frac{1}{\bar{a}^*} \bar{Y}^* \\
(\text{Money holding}) & \quad \bar{M} = \bar{M}^* = \frac{M^U}{2} \\
(\text{Wages}) & \quad \bar{W} = \bar{W}^* = \left( \frac{\theta - 1}{\theta} \right)^{\frac{1}{2}} (1 - \beta) \bar{M}^U \\
(\text{Prices Levels}) & \quad \bar{P} = \bar{P}^* = \left( \frac{\theta - 1}{\theta} \right)^{\frac{1}{2}} (1 - \beta) \bar{M}^U 
\end{align*}
\]

C.2 The Log-linearized Equilibrium Conditions

Now we approximate the equilibrium conditions linearly and express them in terms of log-deviations around the symmetric steady state. Log-deviations are denoted in a standard way by \( \frac{dX}{X} = \bar{X} \). Deviations of variables that are zero in steady state are expressed in terms of steady state consumption expenditures, eg. \( \bar{B} = \frac{dB}{BC} \).
Private Sector and Market Clearing

Consumption Euler equation:
\[
\hat{C}_{s+1} = \hat{C}_s + (1 - \beta)\hat{r}_{s+1} \quad \text{and} \quad \hat{C}^*_{s+1} = \hat{C}^*_s + (1 - \beta)\hat{r}^*_{s+1}
\]  
(C.15)

Optimal money demand:
\[
\hat{M}_s - \hat{P}_s = \hat{C}_s - \frac{\beta}{(1 - \beta)} (\hat{P}_{s+1} - \hat{P}_s) = \hat{C}^*_s - \frac{\beta}{(1 - \beta)} (\hat{P}^*_{s+1} - \hat{P}^*_s)
\]  
(C.16)

Optimal wage setting:
\[
\hat{W}_s = \hat{L}_s + \hat{C}_s + \hat{P}_s \quad \text{and} \quad \hat{W}^*_s = \hat{L}^*_s + \hat{C}^*_s + \hat{P}^*_s
\]  
(C.17)

Nominal interest rate:
\[
\hat{\gamma}_{s+1} = \hat{r}_{s+1} + \frac{1}{(1 - \beta)} (\hat{P}_{s+1} - \hat{P}_s) = \hat{r}^*_{s+1} + \frac{1}{(1 - \beta)} (\hat{P}^*_{s+1} - \hat{P}^*_s)
\]  
(C.18)

Labor demand:
\[
\hat{L}_s = -\hat{a}_s + \hat{Y}_s \quad \text{and} \quad \hat{L}^*_s = -\hat{a}^*_s + \hat{Y}^*_s
\]  
(C.19)

Goods prices:
\[
\hat{P}_{H,s} = \hat{P}_{N,s} = \hat{W}_s - \hat{a}_s \quad \text{and} \quad \hat{P}^*_{F,s} = \hat{P}^*_{N,s} = \hat{W}^*_s - \hat{a}^*_s
\]  
(C.20)

Consumption-based price indices:
\[
\hat{P}_s = \hat{P}_{H,s} - \frac{\gamma}{2} \hat{T}_{oT} \quad \text{and} \quad \hat{P}^*_s = \hat{P}^*_{F,s} + \frac{\gamma}{2} \hat{T}_{oT}
\]  
(C.21)

Total aggregate goods demand
\[
\hat{Y}_s = (1 - \frac{\gamma}{2})(1 - \bar{g}) \left( \hat{C}_s - \rho \frac{\gamma}{2} \hat{T}_{oT} \right) \\
+ \frac{\gamma}{2} (1 - \bar{g}^*) \left( \hat{C}^*_s - \rho (1 - \frac{\gamma}{2}) \hat{T}_{oT} \right) \\
+ \frac{\gamma}{2} (2 - \bar{g} - \bar{g}^*) \hat{a}_s + \bar{g} \hat{G}_s
\]  
(C.22)

and for foreigners
\[
\hat{Y}^*_s = \frac{\gamma}{2} (1 - \bar{g}) \left( \hat{C}_s + \rho (1 - \frac{\gamma}{2}) \hat{T}_{oT} \right) \\
(1 - \frac{\gamma}{2}) (1 - \bar{g}^*) \left( \hat{C}^*_s + \rho \frac{\gamma}{2} \hat{T}_{oT} \right) \\
- \frac{\gamma}{2} (2 - \bar{g} - \bar{g}^*) \hat{a}_s + \bar{g}^* \hat{G}^*_s
\]  
(C.23)

Terms of trade and real exchange rate:
\[
\hat{T}_{oT} = \hat{P}_{H,s} - \hat{P}^*_{F,s}, \quad \text{and} \quad \hat{R} \hat{E} R_s = (1 - \gamma) \hat{T}_{oT}.
\]  
(C.24)
Appendix to Chapter 3

Bonds and money market market clearing conditions:
\[ B_s + B_s^* = 0, \quad \text{and} \quad M_s + \hat{M}_s = \hat{M}_s^{MU}. \] (C.25)

Budget constraints:
\[ dB_{s+1} + \hat{M}_s + \hat{P}C \left( \hat{P}_s + \hat{C}_s \right) = \hat{P}_H \hat{Y}_s \left( \hat{P}_{H,s} + \hat{Y}_s \right) + \hat{T}_{loc} \hat{T}_{loc,s} + dT_{hh,s} \]

Where we already used that \( B = 0 \) and \( \hat{M}_{s-1} = 0 \). Expressing the deviations in terms of the consumption expenditures in the initial steady state, \( \hat{P}C \), we get
\[ \hat{B}_{s+1} + \hat{C}_s = \left( \frac{1}{1-g} \right) \left( \hat{P}_{H,s} + \hat{Y}_s \right) - \hat{P}_s - \frac{x}{1-\beta} \hat{M}_s + \hat{T}_{loc} \hat{T}_{loc,s} + \hat{T}_{hh,s} \]
and \( \hat{B}_{s+1}^* + \hat{C}_s^* = \left( \frac{1}{1-g} \right) \left( \hat{P}_{F,s} + \hat{Y}_s^* \right) - \hat{P}_s^* - \frac{x}{1-\beta} \hat{M}_s^* + \hat{T}_{loc}^* \hat{T}_{loc,s} + \hat{T}_{hh,s}^* \) (C.26)

Local Governments and Federal Fiscal Arrangements

Local governments’ budget constraints:
\[ \hat{T}_{loc} \hat{T}_{loc,s} + \hat{P}N \hat{G} \left( \hat{P}_{N,s} + \hat{G}_s \right) = dT_{gg,s} + dT_{cb,s} \]

Following the expression of initially zero values in terms of steady state consumption expenditures and noting that \( \frac{\hat{T}_{loc}}{\hat{P}C} = \frac{\hat{P}N \hat{G}}{\hat{P}C} = \frac{\hat{g}}{1-g} \) we end up with
\[ \frac{\hat{g}}{1-g} \hat{T}_{loc,s} + \frac{\hat{g}}{1-g} \left( \hat{P}_{N,s} + \hat{G}_s \right) = \hat{T}_{gg,s} + \hat{T}_{cb,s} \]
and \( \frac{\hat{g}}{1-g} \hat{T}_{loc,s}^* + \frac{\hat{g}}{1-g} \left( \hat{P}_{N,s}^* + \hat{G}_s^* \right) = \hat{T}_{gg,s}^* + \hat{T}_{cb,s}^* \). (C.27)

Seignorage revenues from issuing new money
\[ dT_{cb,s} + dT_{cb,s}^* = \bar{M}^{MU} \bar{M}_s^{MU} \Leftrightarrow \hat{T}_{cb,s} + \hat{T}_{cb,s}^* = \frac{\bar{M}^{MU}}{\hat{P}C} \bar{M}_s^{MU} = \frac{2x}{1-\beta} \bar{M}_s^{MU}, \] (C.28)

since in the initial symmetric steady state \( \hat{P}C = \hat{P}^* \hat{C}^* \) and \( \bar{M} = \bar{M}^* = \frac{\bar{M}^{MU}}{2} \).
The federal fiscal transfers among private households across regions and the transfers between the two local governments follow
\[ \hat{T}_{hh,s} + \hat{T}_{hh,s}^* = 0 \quad \text{and} \quad \hat{T}_{gg,s} + \hat{T}_{gg,s}^* = 0, \] (C.29)
respectively.

C.3 Aggregates and Country Differences of the Log-linearized Equilibrium Conditions

Using the method put forward by Aoki and nicely employed in Obstfeld and Rogoff (1995, 1996), the next step is to take the differences of the steady state deviations. Recall that using Aoki’s method means to express individual levels in terms of expressions of union-wide meanings, namely aggregate and relative terms. For instance \( x = \frac{1}{2} \left( x^{MU} + \Delta x \right) \) and \( x^* = \frac{1}{2} \left( x^{MU} - \Delta x \right) \), where \( x^{MU} = x + x^* \) and \( \Delta x = x - x^* \).
Taking Country Differences of the Log-linearized Equilibrium Conditions

Private Sector and Market Clearing

Consumption Euler equation:

$$\Delta \hat{C}_{s+1} = \Delta \hat{C}_s + (1 - \beta) \Delta \hat{r}_{s+1};$$  \hspace{1cm} (C.30)

Optimal money demand:

$$\Delta \hat{M}_s - \hat{RER}_s = \Delta \hat{C}_s - \frac{\beta}{(1 - \beta)} \left( \hat{RER}_{s+1} - \hat{RER}_s \right) - \beta \Delta \hat{r}_{t+1};$$  \hspace{1cm} (C.31)

Optimal wage setting:

$$\Delta \hat{W}_s = \Delta \hat{L}_s + \Delta \hat{C}_s + \hat{RER}_s;$$  \hspace{1cm} (C.32)

Note, that since \(i_{s+1} = i^*_s\), the above and its foreign counterpart imply

$$\Delta \hat{r}_{s+1} = \frac{1}{(1 - \beta)}(\hat{RER}_s - \hat{RER}_{s+1}).$$  \hspace{1cm} (C.33)

Labor demand:

$$\Delta \hat{L}_s = -\Delta \hat{a}_s + \Delta \hat{Y}_s;$$  \hspace{1cm} (C.34)

Total aggregate goods demand:

$$\Delta \hat{Y}_s = (1 - \gamma)(1 - \bar{g}) \Delta \hat{C}_s - \rho(2 - \gamma\gamma)(1 - \bar{g}) \hat{ToT}_s + 2(1 - \bar{g})\gamma \hat{a}_s + \bar{g} \Delta \hat{G}_s;$$  \hspace{1cm} (C.35)

Goods prices: The difference of goods price is, of course, simply the terms of trade, ie.

$$\hat{ToT}_s = (\Delta \hat{W}_s - \Delta \hat{a}_s);$$  \hspace{1cm} (C.36)

Bonds and money market clearing conditions:

$$\Delta \hat{B}_s = 2 \hat{B}_s, \quad \text{and} \quad \Delta \hat{M}_s = 2 \left( \hat{M}_s - \hat{M}_s^{MU} \right);$$  \hspace{1cm} (C.37)

Budget constraints:

$$\Delta \hat{B}_{s+1} + \Delta \hat{C}_s = \left( \frac{1}{1 - \bar{g}} \right) \left( \hat{ToT}_s + \Delta \hat{Y}_s \right) - \hat{RER}_s - \frac{\chi}{1 - \beta} \Delta \hat{M}_s + \frac{\bar{g}}{1 - \bar{g}} \Delta \hat{T}_{loc,s} + \Delta \hat{T}_{hh,s}$$  \hspace{1cm} (C.38)

Local Governments and Federal Fiscal Arrangements

Governments’ budget constraints:

$$\frac{\bar{g}}{1 - \bar{g}} \Delta \hat{T}_{loc,s} + \frac{\bar{g}}{1 - \bar{g}} \left( \hat{ToT}_s + \Delta \hat{G}_s \right) = \Delta \hat{T}_{gg,s} + \Delta \hat{T}_{cb,s};$$  \hspace{1cm} (C.39)

Central bank payments:

$$dT_{cb,s} + dT^*_{cb,s} = \hat{M}^{MU} M^*_s^{MU} \quad \leftrightarrow \quad \hat{T}_{cb,s} + \hat{T}^*_{cb,s} = \frac{\hat{M}^{MU}}{PC} M^*_s^{MU} = \frac{2 \chi}{1 - \beta} M^*_s^{MU},$$  \hspace{1cm} (C.40)
since in the initial symmetric steady state $P\bar{C} = P^*\bar{C}^*$ and $\bar{M} = \bar{M}^* = \frac{M_{MU}}{2}$. Consequently, $\Delta \hat{T}_{cb,s} = 0$.

Since the federal fiscal transfers must net out, we have

$$\Delta \hat{T}_{hh,s} = 2\hat{T}_{hh,s}$$ and $$\Delta \hat{T}_{gg,s} = 2\hat{T}_{gg,s},$$

respectively.

**Aggregate Levels of the Log-linearized Equilibrium Conditions**

Since the two regions are of equal size and symmetric in the initial steady state, aggregate levels are the equally weighted sum of national levels.

**Private Sector and Market Clearing**

Consumption Euler equation:

$$\hat{C}_{s+1}^{MU} = \hat{C}_s^{MU} + (1 - \beta)\hat{r}_{s+1}^{MU};$$

(C.42)

Optimal money demand:

$$\hat{M}_s^{MU} - \hat{p}_s^{MU} = \hat{C}_s^{MU} - \frac{\beta}{(1 - \beta)}(\hat{p}_{s+1}^{MU} - \hat{p}_s^{MU}) - \beta M_{t+1};$$

(C.43)

Optimal wage setting:

$$\hat{W}_s^{MU} = \hat{L}_s^{MU} + \hat{C}_s^{MU} + \hat{p}_s^{MU};$$

(C.44)

Labor demand:

$$\hat{L}_s^{MU} = -a_s^{MU} + \hat{Y}_s^{MU};$$

(C.45)

Total aggregate goods demand:

$$\hat{Y}_s^{MU} = (1 - \bar{g})(\hat{C}_s^{MU} + \bar{g}\hat{G}_s^{MU});$$

(C.46)

Goods prices and consumption-based price indices:

$$\hat{P}_{H,s} + \hat{P}_{F,s} = \hat{P}_{N,s}^{MU} = (\hat{W}_s^{MU} - a_s^{MU})$$ and $$\hat{p}_s^{MU} = \hat{P}_{N,s}^{MU};$$

(C.47)

Bonds and money market clearing conditions:

$$\hat{B}_s^{MU} = 0$$ and $$\hat{M}_s + \hat{M}_s^* = 2\hat{M}_s^{MU};$$

(C.48)

Flow budget constraints:

$$\hat{C}_s^{MU} = \left( \frac{1}{1 - \bar{g}} \right) (\hat{P}_{N,s}^{MU} + \hat{Y}_s^{MU}) - \hat{p}_s^{MU} - \frac{2\chi}{1 - \beta}\hat{M}_s^{MU} + \frac{\bar{g}}{1 - \bar{g}}\hat{T}_{loc,s} + \hat{T}_{hh,s}$$

(C.49)
Local Governments and Federal Fiscal Arrangements

Local governments’ budget constraints:
\[
\frac{\hat{g}}{1 - \hat{g}} \hat{T}_{loc,s} + \frac{\hat{g}}{1 - \hat{g}} \left( \hat{P}_{N,s}^{MU} + \hat{G}_{s}^{MU} \right) = \hat{T}_{gg,s} + \hat{T}_{cb,s},
\]  
(C.50)

Payments by central bank:
\[
\hat{T}_{cb,s}^{MU} = \frac{2\chi}{1 - \beta} \hat{M}_{s}^{MU},
\]  
(C.51)

The transfers among private households across regions and for transfers between the two local governments must net out:
\[
\hat{T}_{hh,s}^{MU} = 0 \quad \text{and} \quad \hat{T}_{gg,s}^{MU} = 0,
\]  
(C.52)

respectively.

C.4 Comparing Steady States

The long run deviations implied by the temporary shocks are expressed by \( \hat{x} \). To alleviate the economic interpretation of the transmission processes, we introduce the following notation for the constant terms that stem from the log-linear approximations: Each constant is dubbed \( \Lambda_{\hat{x}, \hat{y}} \) and its two indices indicate the effect of a change in the latter on the former, i.e. \( \Lambda_{\hat{x}, \hat{y}} \) is the constant for \( \hat{x} = \Lambda_{\hat{x}, \hat{y}} \hat{y} \).

The long run new steady state allocation is determined by the long-run budget constraint, where actually the transversality condition has been imposed, total aggregate goods demand, the optimal labor supply determined by the optimal wage setting, and the long-run change in national public expenditures.

Relative deviations in the long-run budget constraints:
\[
\Delta \hat{C} = 2\delta \hat{B} + \frac{1}{1 - \hat{g}} \Delta \hat{Y} + \left( \frac{1}{1 - \hat{g}} - (1 - \gamma) \right) \hat{ToT} + \Delta \hat{T}_{loc};
\]  
(C.53)

Relative deviations in long-run total goods demand:
\[
\Delta \hat{Y} = (1 - \gamma)(1 - \hat{g})\Delta \hat{C} - \rho(2 - \gamma)\gamma(1 - \hat{g}) \hat{ToT} + \hat{g} \Delta \hat{C};
\]  
(C.54)

Relative deviations in long-run optimal wage setting:
\[
\Delta \hat{W} = \Delta \hat{L} + \Delta \hat{C} + \hat{RER};
\]

The long-run change in labor demand is simply the change in goods demand, i.e. \( \Delta \hat{L} = \Delta \hat{Y} \).

We can thus rewrite the optimal wage setting equation above using the long-run change in the terms of trade \( \hat{ToT} = \Delta \hat{W} \) and thus \( \hat{RER} = (1 - \gamma)\Delta \hat{W} \) as
\[
\gamma \hat{ToT} = \Delta \hat{Y} + \Delta \hat{C}.
\]  
(C.55)
The long-run change in local governments’ budget constraints are
\[ \Delta \hat{T}_{loc} = -\frac{\bar{g}}{1 - \bar{g}} \left( \overline{\sigma_{TT}} + \Delta \hat{G} \right) \]  
(C.56)

Substituting the government’s budget constraint in the intertemporal budget constraints yields
\[ \Delta \hat{C} = 2\hat{\delta} \hat{B} + \Delta \hat{Y} + \gamma \overline{T \sigma T}. \]

The assumption is that in the long-run, the share of government expenditures in aggregate domestic output reaches again the share it had in the initial steady state, i.e. \( \bar{g} = \frac{\bar{G}_0}{\bar{Y}_0} = \frac{\bar{G}_1}{\bar{Y}_1} \), where subscripted indices denote the initial (0) and new (1) steady state. Thus, \( \Delta \hat{Y} = \Delta \hat{G} \).

The relation between relative national consumption and relative national production is given by aggregate goods demand (C.54) and wage setting (C.55), namely
\[ \Delta \hat{Y} = \frac{(1 - \gamma) - \rho(2 - \gamma)}{1 + \rho(2 - \gamma)} \Delta \hat{C} = \Lambda \hat{Y} \hat{C} \Delta \hat{C}. \]  
(C.57)

From the long-run intertemporal budget constraints follow directly the new long-run relation between the relative net foreign asset positions and the relative changes in consumption levels,
\[ \Delta \hat{C} = \frac{1 + \rho(2 - \gamma)}{\rho(2 - \gamma) - (1 - \gamma)} \hat{\delta} \hat{B} = 2\Lambda \hat{C} \hat{C} \hat{B}, \]  
(C.58)

and consequently \( \Delta \hat{Y} = \Delta \hat{L} = -\hat{\delta} \hat{B} \).

**Aggregate Levels**

The aggregate long-run deviation in goods demand is \( \hat{Y}^{MU} = (1 - \bar{g}) \hat{C}^{MU} + \bar{g} \hat{G}^{MU} \). Recall the assumption that in the long-run the share of government expenditures in output is adjusted to \( \bar{g} \). Consequently,
\[ \hat{Y}^{MU} = \hat{C}^{MU} = \hat{G}^{MU}. \]  
(C.59)

Optimal labor supply gives the aggregate wage setting
\[ \hat{W}^{MU} = \hat{L}^{MU} + \hat{C}^{MU} + \hat{P}^{MU}. \]

With \( \hat{P}^{MU} = \hat{P}^{MU}_N = \hat{W}^{MU} \) and the long-run change in aggregate labor demand \( \hat{L}^{MU} = \hat{Y}^{MU} \) we get that
\[ \hat{Y}^{MU} = -\hat{C}^{MU}, \]  
(C.60)

which, however, contradicts the long-run change in aggregate goods demand in sign. Thus
\[ \hat{Y}^{MU} = \hat{L}^{MU} = \hat{C}^{MU} = \hat{G}^{MU} = 0. \]  
(C.61)

For wages and prices we get
\[ \hat{W}^{MU} = \hat{P}^{MU} = \hat{M}^{MU}. \]  
(C.62)
Regional Levels

Now we can determine the regional deviations by using the aggregates and differences in changes of the variables. Recall that $x = \frac{1}{2} (x^{MU} + \Delta x)$ and $x^* = \frac{1}{2} (x^{MU} - \Delta x)$.

$$\hat{C} = \Lambda \hat{c}_B \hat{B},$$

and consequently

$$\hat{C} = \Lambda \hat{c}_B \hat{Y} \quad \text{and} \quad \hat{Y} = \hat{L} = -\frac{\delta}{2} \hat{B}$$

(C.63)

(C.64)

C.5 Short-run equilibrium responses

Following Obstfeld and Rogoff (1995), we separate the short-run equilibrium conditions into two equations, one determining the IS schedule (GG-schedule), and the LM-schedule, ie. the money market equilibrium (MM-schedule). Recall that in our setting as wages are preset, the new steady state after an unanticipated shock is reached the period thereafter. In particular, we can set the log-deviations of tomorrow’s variables as the new steady state responses, ie. $\Delta \hat{c}_{s+1} = \Delta \hat{c}$.

The MM-schedule

The difference in the short-run consumption Euler equations reads

$$\Delta \hat{C} = \Delta \hat{C} + (1 - \beta) \Delta \hat{r}.$$ 

Since changes in the nominal interest rates are identical for home and foreign private households, (C.33) implies

$$\Delta \hat{r} = \frac{1}{(1 - \beta)} (\hat{RER} - \hat{RER}),$$

and thus

$$\Delta \hat{C} = \Delta \hat{C} + (\hat{RER} - \hat{RER}).$$

(C.65)

The optimal money demand today is

$$\Delta \hat{M} = \Delta \hat{C} + \hat{RER} - \beta \frac{(\hat{RER} - \hat{RER})}{(1 - \beta)} - \beta \Delta \hat{r}$$

$$= \Delta \hat{C} + \hat{RER}.$$ 

(C.66)

Using the same relation for the long-run change in relative money holdings, ie. $\Delta \hat{M} = \Delta \hat{C} + \hat{RER}$ and subtracting (C.66), we get

$$\Delta \hat{M} - \Delta \hat{M} = \Delta \hat{C} + \hat{RER} - \hat{RER}$$

$$= (\hat{RER} - \hat{RER}) + \hat{RER} - \hat{RER}$$

$$= 0.$$
The second equations uses (C.65). The intuition is straightforward: Households directly adjust their relative money holdings to the new steady state ratio. This is the equivalent in the monetary union with an integrated money market to the direct jump of the exchange rate in the Obstfeld and Rogoff Redux setup. Combining optimal money holding in the new steady state with the short-run Euler equation using the identity above we get

$$\Delta \hat{M} = \Delta \hat{C} + \hat{RER}.$$  \hspace{1cm} (C.67)

The GG-schedule

Again, the first step is to recognize that from the next period on the economy is in the new steady state. In particular, households adjust their net foreign assets immediately to the new steady state levels. Thus, for the relative deviations of the short-run budget constraints

$$\Delta \hat{B} + \Delta \hat{C} = \left( \frac{1}{1 - \gamma} \right) (T_{oT} + \Delta \hat{Y}) - \hat{RER} - \frac{1}{1 - \beta} \Delta \hat{M} + \tilde{T}_{loc} \Delta \tilde{T}_{loc} + \Delta \hat{T}_{hh}$$  \hspace{1cm} (C.68)

we use the new steady state relation between net foreign assets and consumption in (C.58) and get a relationship that depends on contemporaneous ratios and the new steady state consumption ratio. We now take an intermediate step to derive an expression of $\Delta \hat{C}$ in terms of short-run deviations only.

Recall the alternative expression of the consumption Euler equation (C.65), namely $\Delta \hat{C} = \Delta \hat{C} + (\hat{RER} - \hat{RER})$. The only term we have to get rid of is $\hat{RER}$ which is determined by the new steady state wage levels, i.e. $\hat{RER} = (1 - \gamma)T_{oT} = (1 - \gamma)\Delta \hat{W}$. Since the long-run ratio of wage levels is determined by (C.55),

$$\gamma T_{oT} = \Delta \hat{Y} + \Delta \hat{C} = \left( 1 + \Lambda \hat{Y}, \hat{C} \right) \Delta \hat{C}.$$  \hspace{1cm} (C.68)

Using this equation in (C.65) we end up with

$$\Delta \hat{C} = \Delta \hat{C} + \hat{RER} - \frac{(1 - \gamma)}{\gamma} \left( 1 + \Lambda \hat{Y}, \hat{C} \right) \Delta \hat{C}$$

$$\iff \left( 1 + \frac{(1 - \gamma)}{\gamma} \left( 1 + \Lambda \hat{Y}, \hat{C} \right) \right) \Delta \hat{C} = \Delta \hat{C} + \hat{RER}$$

$$\iff \Delta \hat{C} = \frac{1}{(1 + (1 - \gamma) \Lambda \hat{Y}, \hat{C})} \hat{RER}$$

$$\iff \Delta \hat{C} = \Lambda_{\hat{C}, \hat{Y}, \hat{C}} \Delta \hat{C} + \Lambda_{\hat{C}, \hat{RER}, \hat{C}} \hat{RER}.$$  \hspace{1cm} (C.69)

The notation in (C.69) is particularly useful as it shows the decomposed effects of short-run deviations in the endogenous choice of current consumption and the exogenous component of the change in the real exchange rate on the the long-run relative consumption levels.
We can use (C.69), (C.65) and the MM-schedule (C.67) to express (C.68) as

\[
\left(\Lambda_{\hat{B},\hat{C}} \Lambda_{\hat{C},\hat{C}} + \frac{\gamma}{1 - \beta} + 1\right) \Delta \hat{C} = \left(\frac{1}{1 - \beta}\right) (\hat{T}_0 \hat{T} + \Delta \hat{Y}) - \left(1 + \Lambda_{\hat{B},\hat{C}} \Lambda_{\hat{C},\hat{R}_{ER}} + \frac{\gamma}{1 - \beta}\right) \hat{R}_{ER} \hat{T} + \hat{T}_{loc} \Delta \hat{T}_{loc} + \Delta \hat{T}_{hh}
\]

(C.70)

The last step is to substitute for output fluctuations: It is here where we incorporated the assumption that national fiscal authorities cannot adjust their national lump-sum taxes in the short-run, i.e. \(\hat{T}_{loc} = \hat{T}^*_loc = 0\). As already mentioned in the text, this assumption serves only to distinguish the two different transfer schemes since we want to concentrate on the role and effectiveness of different federal fiscal transfers. Otherwise, intergovernmental transfers might be redistributed to households via regional lump-sum taxes and is thus equivalent to federal transfers among households. This can easily be seen if one simply plugs (C.39) into households’ budget constraint (C.68).

Thus, using (C.39) together with (C.40) and (C.41) and plugging into the the equation of differences in log-deviations of total aggregate national demands (C.35) we get

\[
\Delta \hat{Y} = (1 - \gamma)(1 - \bar{g})\Delta \hat{C} - \rho(2 - \gamma)(1 - \bar{g}) + \bar{g}) \hat{T}_0 \hat{T} + 2(1 - \bar{g})\gamma \hat{\alpha} + (1 - \bar{g})2\hat{T}_{gg}. \tag{C.71}
\]

Substituting into (C.71) yields

\[
\left(\Lambda_{\hat{B},\hat{C}} \Lambda_{\hat{C},\hat{C}} + \frac{\gamma}{1 - \beta} + 1 - (1 - \gamma)\right) \Delta \hat{C} = \left(\frac{1}{1 - \beta}\right) \hat{T}_0 \hat{T} - \left(1 + \Lambda_{\hat{B},\hat{C}} \Lambda_{\hat{C},\hat{R}_{ER}} + \frac{\gamma}{1 - \beta}\right) \hat{R}_{ER} \hat{T} + 2\gamma \hat{\alpha} + 2\hat{T}_{gg} + 2\hat{T}_{hh}
\]

\[
\Leftrightarrow \left(\Lambda_{\hat{S},\hat{C}} + (1 - (1 - \gamma))\right) \Delta \hat{C} = \left(\frac{1 - \rho(2 - \gamma)\gamma - 1}{1 - \rho(2 - \gamma)\gamma - 1} \hat{T}_0 \hat{T} + 2\gamma \hat{\alpha} + 2\hat{T}_{gg} + 2\hat{T}_{hh}
\]

(C.72)

Using our notation this can then be rewritten as

\[
\Delta \hat{C} = \Lambda_{\hat{C},\hat{D}_I}\left((1 - \rho(2 - \gamma)\gamma - 1) \hat{T}_0 \hat{T} + 2\gamma \hat{\alpha} + 2\hat{T}_{gg} + 2\hat{T}_{hh}\right) = \Lambda_{\hat{C},\hat{D}_I}\left((1 - \rho(2 - \gamma)\gamma - 1) \hat{T}_0 \hat{T} + 2\gamma \hat{\alpha} + 2\hat{T}_{gg} + 2\hat{T}_{hh}\right)
\]

(C.73)

Plugging the relative log-deviations of consumption levels into the relative log-deviations of production (C.71) yields

\[
\Delta \hat{Y} = \left(\frac{1}{1 - \beta}\right)(1 - \bar{g}) \Lambda_{\hat{C},\hat{D}_I}\Lambda_{\hat{D}_I,\hat{T}_0 \hat{T}} - \rho(2 - \gamma)(1 - \bar{g}) + \bar{g}) (1 - \bar{g})\gamma (1 - \bar{g}) + \bar{g}) \hat{T}_0 \hat{T}
\]

\[
+ \left((1 - \gamma)\Lambda_{\hat{C},\hat{D}_I} + 1\right) (1 - \bar{g})\gamma \hat{\alpha}
\]

\[
+ (1 - \gamma)(1 - \bar{g})\Lambda_{\hat{C},\hat{D}_I} 2\hat{T}_{hh}
\]

\[
+ \left((1 - \gamma)\Lambda_{\hat{C},\hat{D}_I} + 1\right) (1 - \bar{g})2\hat{T}_{gg}
\]

\[
\Leftrightarrow \Delta \hat{Y} = \Lambda_{\hat{Y},\hat{D}_I}\hat{T}_0 \hat{T} + \Lambda_{\hat{Y},\hat{\alpha}}\hat{\alpha} + \Lambda_{\hat{Y},\hat{T}_{gg}} 2\hat{T}_{gg} + \Lambda_{\hat{Y},\hat{T}_{hh}} 2\hat{T}_{hh}. \tag{C.74}
\]
The change in net foreign assets is simply calculated by using (C.68) and substituting for \( \Delta \hat{C}, \Delta \hat{Y} \), and for \( \Delta \hat{M} \). However, it should already be clear that it it simply the adjustment to the changes in future consumption as households smooth consumption pattern over time and by the change in the real exchange rate induced by the Euler equation, namely

\[
2\hat{B} = \Lambda_{\bar{B},\hat{C}} \Delta \hat{C} + \Lambda_{\bar{B},RER} (1 - \gamma) \hat{T}_{oT}.
\]  

(C.75)

Finally, recall that changes in labor demand are simply given by \( \Delta \hat{L} = -\Delta \hat{a} + \Delta \hat{Y} \).

**Aggregate Levels**

From aggregate consumption Euler equations follow that

\[
\hat{C}^{MU} = \hat{C}^{MU} + (1 - \beta) \hat{r}^{MU} \quad \Leftrightarrow \quad \hat{r}^{MU} = \frac{-1}{(1 - \beta)} \hat{C}^{MU}.
\]

(C.76)

Using this for the optimal aggregate money demand implies

\[
\hat{M}^{MU} = \hat{C}^{MU} + \frac{1}{(1 - \beta)} \hat{p}^{MU} - \frac{\beta}{(1 - \beta)} \hat{r}^{MU} - \frac{\beta}{(1 - \beta)} \hat{C}^{MU} \\
\Leftrightarrow \quad \hat{C}^{MU} = \hat{M}^{MU} + \hat{a}^{MU}.
\]

(C.77)

With \( \hat{G}^{MU} = \frac{1 - \bar{g}}{\bar{g}} \hat{C}^{MU} + \hat{a}^{MU} = \frac{1 - \bar{g}}{\bar{g}} \hat{M}^{MU} + \hat{a}^{MU} \), the short run aggregate goods demand deviations are

\[
\hat{Y}^{MU} = (1 - \bar{g}) \left( \hat{M}^{MU} + \hat{a}^{MU} \right) + (1 - \bar{g}) \left( 1 - \frac{2}{1 - \beta} \right) \hat{M}^{MU} + \bar{g} \hat{a}^{MU} \\
\Leftrightarrow \quad \hat{Y}^{MU} = (1 - \bar{g}) \left( 1 + \frac{2}{1 - \beta} \right) \hat{M}^{MU} + \bar{a}^{MU}.
\]

(C.78)

Aggregate short-run labor demand fluctuations are then given by

\[
\hat{L}^{MU} = -\hat{a}^{MU} + \hat{Y}^{MU} = (1 - \bar{g}) \left( 1 + \frac{2}{1 - \beta} \right) \hat{M}^{MU}.
\]

(C.79)

**Regional Levels**

The regional short-run deviations are then for consumption

\[
\hat{C} = \Lambda_{\hat{C}, DI} \left( \frac{1}{2} \Lambda_{\hat{C}, DI} \hat{T}_{oT} + \gamma \hat{\alpha} + \hat{T}_{gg} + \hat{T}_{hh} \right) + \frac{1}{2} \left( \hat{M}^{MU} + \hat{a}^{MU} \right),
\]

(C.80)

production

\[
\hat{Y} = \frac{1}{2} \Lambda_{\hat{Y}, T_{oT}} \hat{T}_{oT} + \Lambda_{\hat{Y}, \hat{a}} \hat{\alpha} + \Lambda_{\hat{Y}, \hat{T}_{gg}} \hat{T}_{gg} + \Lambda_{\hat{Y}, \hat{T}_{hh}} \hat{T}_{hh} + \frac{1}{2} \left( 1 - \bar{g} \right) \left( 1 + \frac{2}{1 - \beta} \right) \hat{M}^{MU} + \hat{a}^{MU},
\]

(C.81)
employment

$$\hat{L} = \frac{1}{2} \Lambda_{Y,\hat{ToT}} \hat{ToT} - \Delta \hat{\alpha} + \Lambda_{Y,\hat{\alpha}} \hat{\alpha} + \Lambda_{Y,\hat{T}_{gg}} \hat{T}_{gg} + \Lambda_{Y,\hat{T}_{hh}} \hat{T}_{hh} + \frac{(1 - \bar{g})}{2} \left( 1 + \frac{2\chi}{1 - \beta} \right) \hat{M}^{MU},$$

(C.82)

and net foreign assets

$$\hat{B} = \Lambda_{\hat{B},\hat{C}} \Delta \hat{C} + \Lambda_{\hat{B},\hat{RER}} (1 - \gamma) \hat{ToT}.$$  

(C.83)

C.6 Policy Analysis: Calculating the Transfers

Let us now turn to derivations of first the different transfers and second the implications of the macroeconomic variables not targeted. We will, as outlined in the paper, concentrate on consumption and employment only. For the welfare approximation, see the last section of the appendix. Furthermore, aggregate deviations are not considered. In particular, we concentrate on completely asymmetric productivity shocks only and set $\hat{\alpha}^{MU} = 0$. Consequently, we also set $\hat{M}^{MU} = 0$ as the aim of the analysis to study stabilization properties of federal fiscal transfer arrangements.

Demand Shocks

Demand shocks disturb the composition of private tradable goods consumption. They have no impact on current prices because wages are fixed. Nevertheless, a shift in demand affects overall consumption level since it changes income disposable on consumption through changes in labor income.

Stabilizing Consumption

Stabilizing consumption simply means that we set $\hat{C} = 0$. As the two transfer schemes affect consumption-disposable-income in (C.80) equally, consumption stabilization leads to identical transfer volumes

$$\hat{T}_{hh} = \hat{T}_{gg} = -\gamma \hat{\alpha}.$$  

(C.84)

Plugging the transfers into the short run equilibrium response of employment (C.82) we get for federal transfers among private households

$$\hat{L}_{T_{hh}} = \Lambda_{Y,\hat{\alpha}} \hat{\alpha} - \Lambda_{Y,\hat{T}_{hh}} \gamma \hat{\alpha} = (1 - \bar{g}) \gamma \hat{\alpha},$$  

(C.85)

and for the intergovernmental transfers

$$\hat{L}_{T_{gg}} = \Lambda_{Y,\hat{\alpha}} \hat{\alpha} - \Lambda_{Y,\hat{T}_{gg}} \gamma \hat{\alpha} = 0$$  

(C.86)

as the remaining employment fluctuations.
Stabilizing Employment

Employment fluctuations are stabilized if $\hat{L} = 0$. We thus get that intergovernmental transfers are

$$\hat{T}_{gg} = -\frac{\Lambda_{\hat{Y},\hat{\alpha}}}{\Lambda_{\hat{Y},\hat{T}_{gg}}} \hat{\alpha} = -\gamma \hat{\alpha}. \quad (C.87)$$

Obviously, consumption fluctuations are also completely deleted as can be deduced from above.

The federal transfers among private households are

$$\hat{T}_{hh} = -\frac{\Lambda_{\hat{Y},\hat{\alpha}}}{\Lambda_{\hat{Y},\hat{T}_{hh}}} \hat{\alpha} = \hat{T}_{gg} - \frac{(1 - \tilde{g})}{(1 - \gamma)\Lambda_{\hat{C},\hat{D}I}} \gamma \hat{\alpha}. \quad (C.88)$$

The consumption fluctuations when these transfers are imposed are

$$\hat{C} = \Lambda_{\hat{C},\hat{D}I} \left( \gamma \hat{\alpha} - \left( \gamma \hat{\alpha} + \frac{(1 - \tilde{g})}{(1 - \gamma)\Lambda_{\hat{C},\hat{D}I}} \right) \right) = -\frac{\Lambda_{\hat{C},\hat{D}I}(1 - \tilde{g})}{(1 - \gamma)\Lambda_{\hat{C},\hat{D}I}}. \quad (C.89)$$

Productivity Shocks

Let us next study the stabilization of productivity shocks. As we only consider completely asymmetric productivity shocks, we set $\hat{\alpha} = -\hat{\alpha}^*$. Due to sticky wages, productivity shocks alter marginal costs of producing tradable and nontradable goods and cause a shift in the terms of trade by $\hat{T}_{oT} = -2\hat{\alpha}$.

Stabilizing Consumption

By the reason that the two transfers affect private consumption identically, they coincide in volume if they target consumption fluctuations. For setting $\hat{C} = 0$ we get

$$\hat{T}_{hh} = \hat{T}_{gg} = \left( (1 + \Lambda_{\hat{S},\hat{T}_{oER}})(1 - \gamma) - (1 - \rho(2 - \gamma)\gamma) \right) (-\hat{\alpha}). \quad (C.90)$$

Plugging the transfers into the equation determining short run employment changes (C.82) yields for federal transfers among the privates

$$\hat{L}_{Thh} = - \left( \Lambda_{\hat{Y},\hat{T}_{oT}} + 1 \right) (-\hat{\alpha}) + \hat{T}_{hh} = (1 - \rho(2 - \gamma)\gamma)(1 - \tilde{g})(-\hat{\alpha}), \quad (C.91)$$

and for the intergovernmental transfers

$$\hat{L}_{T_{gg}} = - \left( \Lambda_{\hat{Y},\hat{T}_{oT}} + 1 \right) (-\hat{\alpha}) + \hat{T}_{gg} = \left( 1 + \Lambda_{\hat{S},\hat{T}_{oER}} \right)(1 - \gamma)(1 - \tilde{g})(-\hat{\alpha}), \quad (C.92)$$
Stabilizing Employment

When we look at employment stabilization, ie. $\dot{L} = 0$, the transfers among private households are

$$\dot{T}_{hh} = -\frac{\Lambda_{Y,T,T} + 1}{\Lambda_{Y,T,T}} \ddot{a} = -\frac{\Lambda_{Y,T,T} + 1}{\Lambda_{C,D} (1 - \gamma)(1 - \bar{g})} (-\ddot{a}). \quad (C.93)$$

Consumption then fluctuates by

$$\ddot{C}_{hh} = \Lambda_{C,D} \left( -\Lambda_{D,D,T,T} \ddot{a} + \dot{T}_{hh} \right) = \frac{\rho(2 - \gamma) \gamma - 1}{(1 - \gamma)} (-\ddot{a}). \quad (C.94)$$

In contrast, transfers between the national fiscal authorities look

$$\dot{T}_{gg} = -\frac{\Lambda_{Y,T,T} + 1}{\Lambda_{Y,T,T}} \ddot{a} = -\frac{\Lambda_{Y,T,T} + 1}{\left( \Lambda_{C,D} (1 - \gamma) + 1 \right)(1 - \bar{g})} (-\ddot{a}) \quad (C.95)$$

which leads consumption to fluctuate by

$$\ddot{C}_{gg} = \Lambda_{C,D} \left( -\Lambda_{D,D,T,T} \ddot{a} + \dot{T}_{gg} \right) = -\Lambda_{C,D} \left( 1 - \frac{\Lambda_{C,D}(1 - \gamma)}{\Lambda_{C,D}(1 - \gamma) + 1} \right) (1 - \gamma) (-\ddot{a}), \quad (C.96)$$

where $\Phi$ is a constant and defined as $\Phi = \frac{\Lambda_{C,D}(1 - \gamma)}{\Lambda_{C,D}(1 - \gamma) + 1}$, $0 < \Phi < 1$.

Perfect Insurance through Federal Transfer Scheme

In order to stabilize consumption, the sum of the two transfer has to offset the change in consumption-disposable-income, ie

$$\left( \dot{T}_{hh} + \dot{T}_{gg} \right) = - \left( (1 - \rho(2 - \gamma) \gamma) - \left( 1 + \Lambda_{S,R,E} \right) (1 - \gamma) \right) (-\ddot{a}). \quad (C.97)$$

Employment stabilization determines the combination of the federal transfer among private households and the intergovernmental transfers. Employment fluctuations in (C.82) rewritten yields

$$\dot{L} = \left( (1 - \rho(2 - \gamma) \gamma) + (1 - \gamma) \Lambda_{C,T,T} \right) (1 - \bar{g}) (-\ddot{a}) + \Lambda_{Y,T,T} \dot{T}_{hh} \left( \dot{T}_{hh} + \dot{T}_{gg} \right) + (1 - \bar{g}) \dot{T}_{gg}, \quad (C.98)$$

where we have use the fact that $\Lambda_{Y,T,T} - \Lambda_{Y,T,T} = (1 - \bar{g})$. Since the sum of the two transfers delete consumption fluctuations, () implies

$$\dot{L} = 0 = \left( (1 - \rho(2 - \gamma) \gamma) + (1 - \gamma) \Lambda_{C,T,T} \right) (1 - \bar{g}) (-\ddot{a}) + \Lambda_{Y,T,T} \dot{T}_{hh} \left( \dot{T}_{hh} + \dot{T}_{gg} \right) + (1 - \bar{g}) \dot{T}_{gg} = (1 - \rho(2 - \gamma) \gamma) (1 - \bar{g}) (-\ddot{a}) + (1 - \bar{g}) \dot{T}_{gg}, \quad (C.99)$$

since $\Lambda_{C,T,T} (1 - \gamma)(1 - \bar{g}) (-\ddot{a}) + \Lambda_{Y,T,T} \left( \dot{T}_{hh} + \dot{T}_{gg} \right) = 0$. Thus, the combined use of transfers to stabilize welfare fluctuations imply that intergovernmental transfer keeps employment constant

$$\dot{T}_{gg} = (\rho(2 - \gamma) \gamma - 1) (-\ddot{a}), \quad (C.100)$$
whereas the direct transfers among households correct the shift in consumption-disposable-income, ie.
\[
\hat{T}_{bh} = -\Lambda_{DI,T\overline{D}}(-\tilde{a}) - \hat{T}_{sg} = \left(\Lambda_{\hat{\delta},RER} + 1\right)(1 - \gamma)(-\tilde{a}). \tag{C.101}
\]

### C.7 Approximating Welfare

In this section, we derive the approximation of home and foreign welfare and show that the optimal reaction to union-wide aggregate welfare indeed is to delete short-run consumption and employment fluctuations. Clearly, as both regions are inhabited by a unit mass of identical households, regional welfare is simply expressed by the life-time utility.

Recall the part home and foreign life-time utility are
\[
U_{t}^{R} = \sum_{s=t}^{\infty} \beta^{s-t} \left(\ln C_{s} - \frac{1}{2} L_{s}^{2}\right) \quad \text{and} \quad U_{t}^{*R} = \sum_{s=t}^{\infty} \beta^{s-t} \left(\ln C_{s}^{*} - \frac{1}{2} L_{s}^{2}\right). \tag{C.102}
\]

The second-order Taylor expansion for home utility around the initial steady state implies
\[
U_{t}^{R} = U_{0}^{R} + \sum_{s=t}^{\infty} \beta^{s-t} \left(\hat{C}_{s} - \frac{1}{2} L_{s}^{2}\hat{L}_{s} - \frac{1}{2} \left(\hat{C}_{s}^{2} - \frac{1}{4} L_{s}^{2}\hat{L}_{s}^{2}\right)\right). \tag{C.103}
\]

Recall that the new steady state is reached after one period. Thus, for \( s \geq t + 1 \) we have that \( \hat{C}_{s} = \hat{C} \) and \( \hat{L}_{s} = \hat{L} \). Consequently, welfare is approximated up to second-order by
\[
U^{R} = \bar{U}^{R} + \left(\hat{C} - L^{2}\hat{L}\right) + \frac{\beta}{1 - \beta} \left(\hat{C} - L^{2}\hat{L}\right) + \left(\hat{C}^{2} - L^{2}\hat{L}^{2}\right) + \frac{\beta}{1 - \beta} \left(\hat{C}^{2} - L^{2}\hat{L}^{2}\right). \tag{C.104}
\]

Using the long-run relationship between consumption and employment in (C.64), \( \hat{L} = \hat{Y} = \Lambda_{\hat{\gamma},\hat{C}} \hat{C} \), with \( \Lambda_{\hat{\gamma},\hat{C}} = \Lambda_{\hat{C},\hat{Y}}^{-1} \), and the relationship between long-run and short-run consumption, \( \hat{C} = \Lambda_{\hat{C},\hat{C}} \hat{C} \), derived in (C.69), we rewrite (C.104) and its foreign counterpart as
\[
U^{R} = \bar{U}^{R} + \left(1 + \frac{\beta}{1 - \beta} \left(1 - L^{2}\Lambda_{\hat{\gamma},\hat{C}}\right) \Lambda_{\hat{C},\hat{C}}\right) \left(\hat{C} - \frac{1}{2} \hat{C}^{2}\right) - L^{2} \left(\hat{L} + \frac{1}{2} \hat{L}^{2}\right) \tag{C.105}
\]
\[
U^{*R} = \bar{U}^{*R} + \left(1 + \frac{\beta}{1 - \beta} \left(1 - L^{2}\Lambda_{\hat{\gamma},\hat{C}}\right) \Lambda_{\hat{C},\hat{C}}\right) \left(\hat{C}^{*} + \frac{1}{2} \hat{C}^{*2}\right) - L^{2} \left(\hat{L}^{*} + \frac{1}{2} \hat{L}^{*2}\right). \tag{C.106}
\]

### Regional Welfare

Following Obstfeld and Rogoff (1995, 1996), we take first-order approximations for regional welfare. Thus, we get for the deviations of welfare, \( dU^{R} = U^{R} - \bar{U}^{R} \)
\[
dU^{R} = \left(1 + \frac{\beta}{1 - \beta} \left(1 - L^{2}\Lambda_{\hat{\gamma},\hat{C}}\right) \Lambda_{\hat{C},\hat{C}}\right) \hat{C} - L^{2}\hat{L} \quad \text{and} \quad dU^{*R} = -dU^{R}. \tag{C.107}
\]

The second box results from looking at completely asymmetric shocks and using log-linearized equilibrium conditions. Recall that \( \hat{C} = -\hat{C}^{*} \) and \( \hat{L} = -\hat{L}^{*} \).
Union-wide Welfare

The union-wide welfare is simply the sum of regional welfare, i.e., $U^{MU,R} = U^{*,R} + U^R$. The fact that up to first-order $dU^{*,R} = -dU^R$ means that a simple first-order Taylor approximation leads to $dU^{MU,R} = dU^{*,R} - dU^R = 0$. Consequently, union-wide welfare must be expressed by using the sum of regional welfare approximated up to second-order. It follows from (C.106) and $\hat{C}^{*,2} = \hat{C}^2$ and $\hat{L}^{*,2} = \hat{L}^2$ that

$$dU^{MU,R} = -\left(1 + \frac{\beta}{1 - \beta} \left(1 - \hat{L}^2 \Lambda_{Y,C} \Lambda_{\hat{C},\hat{C}}\right)\right) \hat{C}^2 - \hat{L}^2 \hat{L}^2.$$  

(C.108)

Clearly, $dU^{MU,R}$ is maximized for $\hat{C} = \hat{L} = 0$. However, this does not imply that $U^{MU,R}$ is maximized as monopolistic distortions on labor markets prevail which leads $U^{MU,R}$ to be suboptimal. However, as it is standard in the New Keynesian literature, this could be corrected by redistributing proportional labor income taxes lump-sum. Compare, eg. Obstfeld and Rogoff (2000, 2002a).
Bibliography


